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Final Report



RELIABILITY PREDICTION - MECHANICAL STRESS/STRENGTH INTERFERENCE

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Research and Technology Division  
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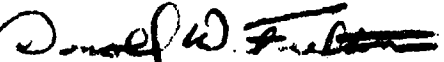
FOREWORD

This final report was prepared by Charles Lipson, Narendra J. Sheth and Ralph L. Disney of the University of Michigan, Ann Arbor, Michigan, under Contract AF30(602)-3684, project number 5519, task number 551902.

This report outlines a nonelectronic reliability prediction technique subject to special export controls.

This report has been reviewed and is approved.

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#### ABSTRACT

This study addressed itself to the development of a stress-strength Interference Theory in the form of a practical engineering tool, to be used for designing and quantitatively predicting the reliability of mechanical parts and components subjected to mechanical loading.

Early practices in stress-strength relationship dealt almost entirely along the lines of factors of safety. Utilization of such factors is justified when they are based on considerable experience with parts not too different from the one under consideration. However, when substantial changes in the geometry, the processing, or the function of the part are contemplated, a major error may result if the old factors of safety are projected to the new set of conditions.

In the present investigation an approach was used which attempted to recognize the above limitations. Instead of an indiscriminate grouping of all the variables affecting stress and strength into one index (factor of safety) these variables were individually recognized. The principal variable is the scatter in the stresses imposed on the part and in the strength of the material resisting these stresses.

The prevailing practice is to use the mean values of the calculated stress and strength, ignoring the natural scatter that each may possess. However, the variability in these two factors results in a statistical distribution of stress and strength. When these two distributions interfere, that is when stress becomes higher than strength, failure results. Means of expressing these distributions, in a practical engineering sense, and means of calculating the resulting interferences, represent the heart of the present study.

The problem of strength distribution was approached with the aid of S-N curves. A great deal of fatigue data was gathered for various materials, heat treatments, surface conditions, etc. About 75% of these data were obtained from the Mechanical Properties Data Center, Traverse City, Michigan, which also provided some tensile and strength rupture information. The rest of the data came from literature and other sources.

The fatigue data thus obtained were then converted into strength data which portrayed the scatter of strength at a given life. Several methods of expressing the resultant distribution were studied and the Weibull distribution was decided on as the most effective means of expressing the strength distribution in the Interference Theory. For each material, heat treatment, surface condition, etc. studied Weibull parameters were calculated, and these are tabulated in Appendix 1. Pertinent information was then plotted and the graphs were incorporated in the body of the report (Section 6).

The problem of stress distribution (Section 7) turned out to be much more involved. When one speaks of "stress distribution" he usually refers to a spectrum of loading or stresses to which a part is subjected.

Indeed, most of the available data on the subject is expressed in this manner. In an engineering sense, this kind of a distribution means number of times that a given part is subjected to a given load or stress. In the Interference Theory, however, this is not what is wanted. For consistency with the strength distribution the number of parts subjected to a given stress is required instead.

In the present study the required stress distribution was obtained by converting the stress spectrum, which generally has some mean stress, into a spectrum with zero mean, with the aid of the Goodman diagram. This was done to facilitate the conversion of the resultant spectrum into an equivalent stress, based on zero mean stress. The conversion was accomplished by means of Miner's rule. The required stress distribution was then expressed in terms of the equivalent stress. This distribution was then compared with the strength distribution to determine the degree of interference.

From an extensive literature survey made in the course of this study, it was found that most investigations have assumed both the stress and strength distribution to be normal. In those cases when they were not normal a Monte-Carlo technique was employed. This involved a sophisticated means of randomly selecting a sample from one distribution and comparing it with a sample from a different distribution.

In view of serious limitations of the Monte-Carlo technique a method of Integrals was developed (Appendix 3 and Appendix 4) and used in the present study. This method involved developing an integral resulting from the interference of two distributions and calculating this complex integral with the IBM - 7090 computer using MAD language.

Although the interference of several distributions was considered the final tabulated interference values were restricted to the cases when the stress distribution was normal and strength distribution was Weibull, and when both distributions were Weibull (Appendix 2). This represents distributions most frequently encountered in actual engineering practice.

In order to show the application of the Interference Theory Technique, developed in the present study, an example was solved, as described in Section 9.

# TABLE OF CONTENTS

Section	Page No.
1. Introduction	1
2. Interference Theory	4
2.1 Two Normal Distributions	4
2.2 Consistency of Two Distributions	7
2.3 Non-Normal Distributions	9
2.4 The Integral Method	9
3. Objectives of the Present Study	11
4. Study Approach	12
4.1 Literature Search	12
4.2 Theoretical Analysis	12
4.3 Empirical Data	12
4.4 Factors Affecting the Statistical Distribution of Fatigue Strength	14
4.5 Factors Affecting the Statistical Distribution of Tensile, Rupture Strength	16
4.6 Analysis of Strength Data	16
5. Analytical Expressions for the Interference	17
5.1 Introduction	17
5.1.1 Interference Probabilities	17
5.1.2 Calculation of Probabilities of Failure	19
5.1.3 Interference Tables	23
5.2 Use of Interference Tables	24
5.2.1 Parameters for the Weibull-Weibull Case	24
5.2.2 Parameters for the Weibull Distributed Strength, Normal Distributed Stress Case	26
5.2.3 Use of the Tables, Explanation of Missing Values and Interpolation	27
5.3 Examples of Use of the Tables	28
6. Statistical Distribution of Strength	29
6.1 Analysis of Strength Data	29
6.1.1 Conversion of Life Data to Strength Data	29
6.1.2 Strength Response Test	29
6.2 Plot of Strength Data	31
6.3 Determination of the Weibull Parameters	37
6.4 Graphs of Weibull Parameters	45
7. Statistical Distribution of Stress	158
7.1 Stress Spectrum vs. Stress Distribution	158
7.2 Conversion of Stress Spectrum to an Equivalent Stress ( $S_{equ}$ )	158
7.2.1 Miner's Rule	160
7.2.2 Corten-Dolan's Rule	164

8. Interference of Stress Distribution with Strength Distribution	169
9. Application of Interference Theory to Design Problems	174
9.1 Weibull Parameters	174
9.2 The Equivalent Stress	174
9.3 Percent Interference	176
9.4 The Effect of Design Factors	179
Conclusions	182
Recommendations	184
References	185
Bibliography	187
Appendix 1 Tables of Weibull Parameters	194
A-1.1 Weibull Parameters for Fatigue Strength	195
Index to Weibull Parameters for Fatigue Strength	
of Various Materials	196
Reference Data for Weibull Parameters	198
Tables	206
References for Fatigue Strength Data	241
A-1.2 Weibull Parameters for Tensile Strength	242
Index to Weibull Parameters for Tensile Strength	
of Various Materials	243
Tables	244
References for Tensile Strength Data	249
A-1.3 Weibull Parameters for Rupture Strength	250
Index to Weibull Parameters for Rupture Strength	
of Various Materials	251
Tables	252
References for Rupture Strength Data	257
Appendix 2 Tables of Interference Values	258
A-2.1 Stress Distribution - Normal, Strength Distribution - Weibull	259
A-2.1.1 Stress Standard Deviation = 0	260
A-2.1.2 Stress Standard Deviation $\neq$ 0	264
A-2.2 Stress Distribution - Weibull, Strength Distribution - Weibull	306
Appendix 3 Mathematical Theories of Analytical Expression	397
A-3.1 Introduction	398

A-3.1.1	The Concept of a Random Variable	396
A-3.1.2	Interference Theory and Random Variable	398
A-3.2	Determination of Probability of Failure	399
A-3.3	Some Examples	409
A-3.3.1	Normally Distributed Strength (X) and Normally Distributed Stress (Y)	409
A-3.3.2	Exponentially Distributed Strength (X) and Exponentially Distributed Stress (Y)	410
A-3.3.3	Gamma Distributed Strength (X) and Gamma Distributed Stress (Y)	411
A-3.3.4	Weibull Distributed Strength (X) and Weibull Distributed Stress (Y)	418
A-3.3.5	Weibull Distributed Strength (X) and Normally Distributed Stress (Y)	422
Appendix 4	Discussion of the Evaluation of Integrals in A-3.3.4 and A-3.3.5	425
A-4.1	Numerical Analysis	426
A-4.1.1	The Problem	426
A-4.1.2	Method of Solution	426
A-4.1.3	Properties of Simpson's Rule	427
A-4.1.4	Error Analysis-Weibull-Weibull Case	427
A-4.1.5	Conclusions Concerning Errors	440
A-4.1.6	A Note on Interpolation	441
A-4.1.7	Error Analysis-Weibull-Normal Case	441
A-4.1.8	Conclusions Concerning Errors	448
A-4.1.9	A Note on Interpolation	448

## EVALUATION

This study was addressed to the development of a practical reliability engineering tool to be used in predicting the reliability of mechanical parts fabricated from ferrous metals and subjected to dynamic loading. The tool was to be based on the Stress-Strength Interference theory and to be usable by design engineers with little or no statistical background.

The study has provided a prediction technique which is almost "cook-book" in nature and requires a minimum amount of computation. Once the values of the parameters of the interfering distributions have been determined, the probability of failure is obtained directly from tables contained in the report. Within the limitations that the material must be ferrous and the failure mechanism must be fatigue, the reliability of an almost unlimited number of parts can be predicted. For example, a few are gears, bolts, shafts, springs, torsion bars, vehicle frames and suspensions, landing gears, forged air frame members, etc. The engineering and statistical basis for the technique are such that there is every reason to believe that predictions based on this technique will be realistic. However, the precision will depend to a large extent on the accuracy of the stress distribution. If this distribution is based on measurements much greater precision will result than for the case where only the mean stress is known and the variability is estimated.



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## SECTION 1 INTRODUCTION

A complete cycle of reliability is made up of four stages: 1. Specification of Reliability; 2. Prediction of Reliability; 3. Verification of Reliability; 4. Preservation of Reliability. The heart of the second stage, namely, the Prediction of Reliability, is the Interference Theory.

The basic idea behind reliability is that a given part has certain physical properties, which, if exceeded, will result in failure. The factor which may cause these properties to be exceeded is the stress imposed by the operating conditions. Thus, in prediction of reliability it is not the stress alone or the strength alone that are the determining factors but the combined effect of the two.

Early practices in stress-strength relationship dealt almost entirely along the lines of factors of safety. Once a part was designed and the ratio of strength to stress was in the range of approximately 5 to 10 it was considered to be safe for service. In certain industries and in certain applications factors of safety as high as 20:1 were employed.

The definition of the factors of safety varied from user to user, depending on the sophistication and the complexity of the problem.

Some of these definitions, found in literature, are listed below:

Factor of Safety	=	$\frac{\text{Ultimate Strength}}{\text{Nominal Stress}}$
Factor of Safety	=	$\frac{\text{Yield Strength}}{\text{Nominal Stress}}$
Factor of Safety	=	$\frac{\text{Ultimate Strength}}{\text{Actual Working Stress}}$
Factor of Safety	=	$\frac{\text{Yield Strength}}{\text{Actual Working Stress}}$
Factor of Safety	=	$\frac{\text{Maximum Safe Load}}{\text{Normal Load}}$
Factor of Safety	=	$\frac{\text{Computed Strength}}{\text{Computed Load}}$
Margin of Safety	=	$\frac{\text{Strength-Stress}}{\text{Stress}}$
Design Factor	=	$\frac{\text{Strength}}{\text{Design Stress}}$
Factor of Utilization	=	$\frac{\text{Stress}}{\text{Strength}}$



$$\text{Functional Reserve Factor} = \frac{\text{Magnitude of Variable Producing Failure}}{\text{Magnitude of Variable at Operating Conditions}}$$

where the variable could be force, power, torque, material, surface finish, fillet radius, etc.

Apparently the factor of safety was meant to account for all the variables which were known to affect the stress and strength of the member. The utilization of a factor of safety of this kind has justification, only when its value is based on considerable experience, with parts not too different from the one under consideration. However, when substantial changes in the geometry, the processing, or the function of the part are contemplated, a major error may result if the old factor of safety is projected to the new set of conditions.

This is illustrated by the problem of automotive axle shafts which have been failing in service in large numbers. These shafts have been fabricated from a steel with a tensile strength of 240,000 psi, and yet, the operating stresses as measured in actual service were found to be only 13,000 psi. This produced an apparent factor of safety of  $240,000/13,000 = 18.5$ . This is obviously a fictitious value, since the shafts were failing in service, and the true factor of safety was less than one. The explanation lies in the fact that axle strength to be compared with the 13,000 psi operating stress should not have been the ultimate strength of the material (240,000 psi) but the fatigue strength corresponding to the surface finish of the shafts, the mode of loading to which the shaft was subjected, etc. When the ultimate strength was reduced by these derating factors the resultant value was found to be 12,000 psi. This strength, when compared with the 13,000 psi stress produced the realistic factor of safety of 0.9.

Examples such as this lead to the next phase in the relationship between stress and strength, namely to the concepts of a significant stress and a significant strength. By significant stress is meant the actual stress imposed on the part and it may include the effect of stress raisers, magnification due to impact loading, residual stresses, etc. By significant strength is meant the actual strength of the part in its fabricated form, under actual operating conditions. A rational approach to significant strength still employs ultimate strength as the basis. However, instead of an indiscriminate grouping of all the factors affecting the ultimate strength into one index, it attempts to evaluate quantitatively the effect of each individual factor pertaining to the part and the conditions under consideration. The result is a value which is strictly applicable to the part under consideration and to the set of loading conditions to which the part is subjected in service. The principal factors affecting strength and which must be considered in determining the significant strength are: life expectancy, type of loading, (axial, bending, torsional, or a combination),

size effect, surface finish, surface treatment, notch effects, mode of loading (static, completely reversed dynamic, or a combination).

These concepts of significant stress and significant strength represent a major step toward a more realistic prediction of reliability and, as such, they have been included in the present investigation. By themselves, however, they are not sufficient. This is because the prevailing practice is to use the mean values of the calculated strength and stress, ignoring the natural scatter that stresses and strengths may have.

The variability in these two factors results in the existence of a statistical distribution function of stress and strength (See Figure 2.1) and is the heart of the Interference Theory. Thus, for proper prediction of reliability, an estimate must be made of both the mean value and the dispersion characteristics of both the strength and stress.

The strength of the part, as all properties of non-homogeneous materials, varies from specimen to specimen, in view of the variation in hardness, surface finish, degree of stress concentration, etc. The operating stress imposed on the part varies too. These stresses vary from time to time in a particular part, from part to part in a particular design, and from environment to environment. Therefore, both the mean value and the dispersion characteristics of stress and strength must be determined.

Once these parameters are found, percent interference and thus probability of failure can be determined from the interference area (shaded area in Figure 2.1). Means of computing these interferences represent one of the principal objectives of the present investigation.

## SECTION 2 INTERFERENCE THEORY

Suppose there are two barrels containing slips of paper, each having a number printed on it. The numbers in barrel Y are distributed according to distribution Y, as in Figure 2.1, and the numbers in barrel X are distributed according to distribution X. If, at random, slips of paper from each barrel are selected and paired, they may be classified into successes and failures. A success is constituted by a strength value exceeding a stress value, as for example, when  $x_1 > y_1$ . Failure will occur if  $x_2 < y_2$  as shown. It will be noted that, although the shaded area is a measure of interference, it is not interference itself: a pair of points  $x_3$  and  $y_3$ , although in the shaded area, will not produce failure. By continued pairing of stresses and strengths, at random, pairs will be found where the stress will exceed the strength. By continued experimentation a good estimate of the probability of interference can be found.

### 2.1 Two Normal Distributions

From an exhaustive survey of literature made during this investigation it was found that most studies have assumed both the stress and the strength distribution to be normal. This is a natural assumption to make in order to solve a practical problem, as no work was found dealing with an analytical expression for two interference distributions when they are not normal.

When the stress and strength distributions are assumed to be normal the probability of interference can be determined from the equation:<sup>1</sup>

$$Z = \frac{\mu_y - \mu_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \quad (2.1)$$

where

$\mu_y$	=	mean stress
$\mu_x$	=	mean strength
$\sigma_y^2$	=	stress variance
$\sigma_x^2$	=	strength variance
Z	=	standardized normal variate determined from standard tables. (See Table 2.1)

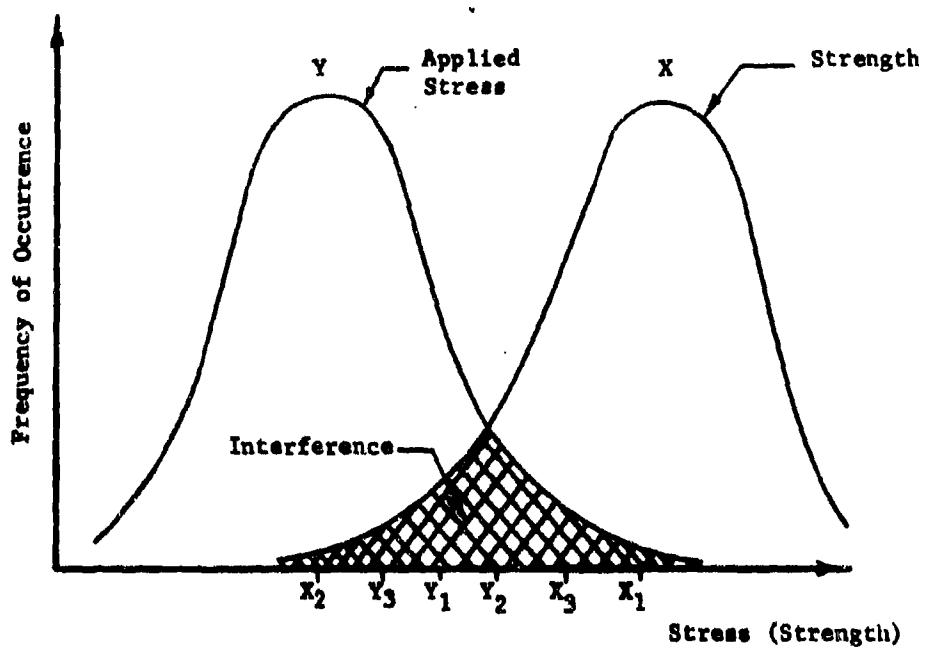
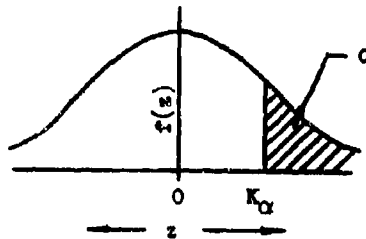


Figure 2.1 Interference of Stress and Strength Distributions

Tabulation of the values of  $\alpha$  versus  $K_\alpha$  for the Standardized Normal Curve.

$$\alpha = P(z > K_\alpha) = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

= Area under the Standardized Normal Curve from  $z = K_\alpha$  to  $z = \infty$



$K_\alpha$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3373	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.9866
1.3	.9688	.9661	.9634	.9608	.9581	.9555	.9529	.9503	.9478	.9453
1.4	.9428	.9403	.9378	.9354	.9329	.9304	.9279	.9255	.9231	.9207
1.5	.9183	.9159	.9135	.9112	.9088	.9064	.9041	.9017	.8994	.8970
1.6	.8946	.8923	.8900	.8877	.8854	.8831	.8808	.8785	.8762	.8739
1.7	.8716	.8693	.8670	.8647	.8624	.8601	.8578	.8555	.8532	.8509
1.8	.8486	.8463	.8440	.8418	.8395	.8372	.8349	.8326	.8304	.8281
1.9	.8259	.8236	.8213	.8191	.8168	.8146	.8123	.8101	.8078	.8056
2.0	.8033	.8011	.7988	.7966	.7943	.7921	.7898	.7876	.7853	.7831
2.1	.7809	.7786	.7764	.7742	.7719	.7697	.7675	.7653	.7631	.7609
2.2	.7587	.7565	.7543	.7521	.7499	.7477	.7455	.7433	.7411	.7389
2.3	.7367	.7345	.7323	.7301	.7279	.7257	.7235	.7213	.7191	.7169
2.4	.7147	.7125	.7103	.7081	.7059	.7037	.7015	.6993	.6971	.6949
2.5	.6927	.6905	.6883	.6861	.6839	.6817	.6795	.6773	.6751	.6729
2.6	.6707	.6685	.6663	.6641	.6619	.6597	.6575	.6553	.6531	.6509
2.7	.6487	.6465	.6443	.6421	.6399	.6377	.6355	.6333	.6311	.6289
2.8	.6267	.6245	.6223	.6201	.6179	.6157	.6135	.6113	.6091	.6069
2.9	.6047	.6025	.6003	.5981	.5959	.5937	.5915	.5893	.5871	.5849

Table 2.1 Normal Distribution<sup>1</sup>

Thus, if the average stress is 30 ksi, with a standard deviation of 3 ksi, and the average strength is 50 ksi with standard deviation of 10 ksi,  $z = 1.91$  and from Table 2.1  $\phi$ , which represents interference, is found to be .0281. Thus, percent interference, (probability of failure) is 2.81%.

In practical applications of the Interference Theory the following problem arises: both distributions under consideration extend to plus and minus infinity. It is apparent, therefore, that any two distributions will overlap and cause interference. This, of course, is erroneous because some distributions, such as strength, must have a finite lower bound of zero. In many situations the physical set-up and the sample size adjust for this lower bound. For example, suppose a part were designed so that the strength distribution is placed  $6\sigma$  away from the stress distribution, both distributions having standard deviation equal to  $\sigma$ . From equation 2.1 it is found:

$$z = \frac{6\sigma}{\sqrt{2}\sigma} = 4.24$$

and the probability of interference comes out to be .00001. This means that only one part will fail in 100,000 parts produced. If actually only 50,000 parts are made, the physical problem has effectively truncated the distributions. However, the probability of one part remains .00001. This means that due to sample size the extreme portions of these distributions are no longer important since the sample size is such that not even one failure can be expected.

For the ease of application of the Interference Theory, a graph was constructed, as shown in Figure 2.2. The example solved previously through equation (2.1) now yields:  $\frac{\mu_1 - \mu_2}{\sigma_{\min}} = \frac{50\text{ksi} - 30\text{ksi}}{3\text{ksi}}$

$$= 6.67 \text{ and } \frac{\sigma_{\max}}{\sigma_{\min}} = \frac{10\text{ksi}}{3\text{ksi}} = 3.3$$

and the probability of interference comes out approximately 0.03, as before.

## 2.2 Consistency of Two Distributions

In the application of the Interference Theory the following important point must be considered; the distribution of stress and the distribution of strength must be consistent with each other. In a fatigue test or in actual application in service, a single part has a single fatigue life for a given loading condition. Subsequent testing of additional parts under the same load will show a scatter in life, leading to a life distribution. Through more extensive testing a strength distribution for a given life can be obtained, such as the distribution in Figure 2.1 (the method

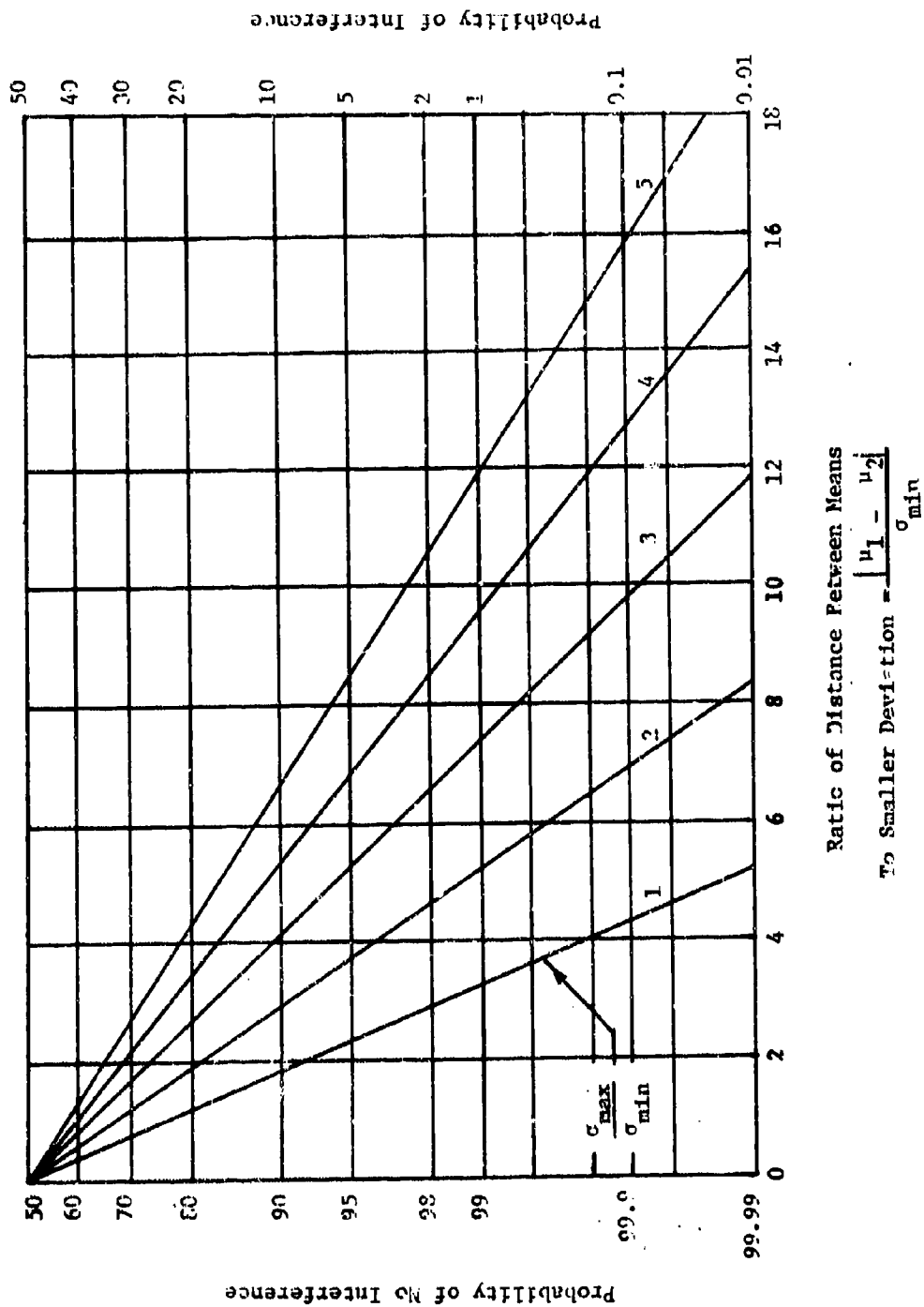


Figure 2.2 Probability of Interference of Two Normal Distributions.<sup>1</sup>

of obtaining a strength distribution from a life distribution is described in Section 6).

It follows then that for a consistent development, stress distribution must be of the same nature as strength distribution. That is, it should represent the plot of the frequency of occurrence versus the applied stress. This is not the same as stress distribution conventionally derived from the spectrum of loading acting on the part. The conversion of one to the other must be accomplished through the use of the equivalent stress ( $S_{equ}$ ) and Miner's or Corten-Dolan's Rules. This is the method used in the present investigation, as described in Section 7.

### 2.3 Non-Normal Distributions

So far the discussion has been limited to the cases when both the stress and strength distributions can be assumed to be normal. In cases when either one or both are not normal the problem is much more involved. For example, the intersection of a normal and a log-normal distribution produces a distribution of an unknown origin.

In the past, problems such as this were solved largely through "brute force" by a method commonly referred to as the Monte-Carlo Technique. Essentially, the Monte-Carlo Technique consists of a sophisticated means of randomly selecting a sample from one distribution and comparing it with a sample taken from a different distribution. This is accomplished with the aid of Tables of Random Numbers. The resultant paired data are plotted as a Cumulative Density Function on a Probability, Weibull, etc. paper and percent interference is read from the graph.

### 2.4 The Integral Method

In the present investigation a Method of Integrals was used in preference to the Monte-Carlo Technique. This method involves determining the expression resulting from the interference of the two distributions under consideration and establishing percent interference from this integral.

The advantages of the Integral Method are:

1. For some distributions the integrals have been already tabulated and percent interference can be read directly from the table.
2. In those cases where the integrals have not been already tabulated, they can be evaluated by Numerical Analysis as done in the present investigation.



3. The major shortcoming of the Monte-Carlo Technique is that it requires a very large sample size for any accuracy. This shortcoming is avoided when the Integrals are used.
4. One of the objectives of the present study is to develop and evaluate an analytical expression for interference of any two distributions. Such an expression is possible when the Method of Integrals is used, but not when the Monte-Carlo Technique is employed.

### SECTION 3 OBJECTIVES OF THE PRESENT STUDY

The objectives of the study described in this report were:

1. To refine and to reduce to practice the stress/strength Interference Theory technique for designing for and predicting the quantitative reliability of mechanical parts and components under mechanical loading. Maximum use was to be made of empirical practical engineering values as well as a sound theory.
2. To study the effect of such factors as type of loading, surface finish, temperature, heat treatment, stress concentration, and surface treatment on the statistical distribution of fatigue strength.
3. To determine from the existing available empirical data the distribution of the fatigue strength under the effect of each of the above factors.
4. To develop the means of synthesizing the strength distribution function when such function is non-time variant, i.e., infinite life design (infinite fatigue strength), and when such function is time variant i.e. finite life design (finite fatigue strength).
5. To develop an analytical expression of the distribution of interference for the general case where the interfering distributions are different.

## SECTION 4 STUDY APPROACH

### 4.1 LITERATURE SEARCH

At the outset of the investigation an exhaustive literature search was made to determine the State of the Art in the field of Reliability Prediction-Mechanical Stress/Strength Interference. Some of the specific topics covered were: Interference Theory, Mathematical Tools as related to the Interference Theory, Monte-Carlo Technique, Reliability Prediction, Fatigue of Metals under different conditions, Statistical Analysis of Fatigue Data, Spectral Loading, Cumulative Damage due to Spectral Loading, etc. The sources where pertinent information was located are listed in the Bibliography. No references, however, were found dealing with the specific work objectives listed in Section 3, thus confirming the basic need for this type of information.

### 4.2 THEORETICAL ANALYSIS

One of the objectives of this investigation was to develop and evaluate an analytical expression for the interference of two distributions. When the two distributions are normal the interference can be simply expressed by a z-distribution, as described in Section 2. From an extensive survey of literature no work was found dealing with the analytical expression when the two interfering distributions are not normal. The purpose of this phase of the investigation, then, was to develop such an expression.

For reasons stated in Section 2, the Method of Integrals was chosen and an analytical expression was developed for the general case of two interfering distributions. This would include cases such as Weibull-Weibull, Weibull-Normal, Normal-Normal, Exponential-Exponential, etc. It was then necessary to find the way of solving the complex integrals expressing such interferences. Numerical analysis was carried out using an IBM 7090 Computer with MAD language to solve these integrals. Tables were then prepared for the interference as a function of the distribution parameters.

These tables included the combinations:

<u>Stress Distribution</u>	<u>Strength Distribution</u>
Normal	Weibull
Weibull	Weibull

because Normal and Weibull are the distributions most frequently found in actual engineering practice. The reason for choosing Weibull as the strength distribution for the two cases was that strength data, particularly

fatigue data, can be more conveniently expressed in terms of Weibull parameters ( $X_0, 0, b$ ) than Normal parameters ( $\mu, \sigma$ ). It was felt that this restriction would not apply to the Stress Distribution.

#### 4.3 EMPIRICAL DATA

The tables of interference are the heart of the Interference Theory as applied to the engineering practice. Once the stress distribution and strength distribution parameters are known, percent interference, and thus the probability of failure, can be read directly from these tables (for a procedure see Section 9 and for the tables see Appendix 2).

Over 250 articles from literature and other sources were examined and practically no data were found concerning the statistical distribution of stress. The only data located referred to spectrum of loads or stresses, which, as pointed out in Section 7, does not represent the Stress Distribution required for the Interference Theory. Interference Tables, therefore, were constructed so that for given dispersion characteristics of stresses a percent interference can be found. The range of these characteristics chosen here, and corresponding to engineering practice, are (if the stress distribution is Normal).

$$.01 \leq \frac{\sigma}{\mu} \leq .10$$

As to the strength distribution it was found necessary to collect a great deal of data in order to arrive at a meaningful distribution. The search for these data turned out to be an involved task. To systematize the effort a format was prepared which included the factors which are known to affect the final distribution of strength. An attempt was made to collect data in different areas in order to determine the effect of such factors as type of loading, size, processes, surface conditions, heat treatment, surface environment, temperature, surface treatment, stress concentration etc.

An initial step was to gather scatter data from presently available published work. Many sources of information were examined, such as: RAND Reports, NASA Technical Notes, NACA Technical Notes, ASTM Transactions, ASM Transactions, SAE Transactions, ASME Transactions, etc.

Most of these data, however, were found to be in a graphical form, in many cases with test points not indicated, whereas statistical analysis requires data in a tabular form, for higher accuracy. The Mechanical Properties Data Center in Traverse City, Michigan was found to be a very useful

source of information for tabular data. . . They have been very cooperative in providing the necessary information. Although it was not possible to find data for every single factor affecting strength, still a great deal of fatigue data was found and these data were systematized, evaluated in terms of Weibull parameters (Section 6), tabulated (Appendix 1) and plotted (Section 6).

While scanning through literature and other sources for possible fatigue data, some useful data for determining the statistical distribution of tensile strength and rupture strength of various ferrous materials was found. The effect of temperature and heat treatment on tensile strength and the effect of time and temperature on rupture strength were studied and the results were tabulated and plotted in the same manner. (see Appendix 1 and Section 6.4).

#### 4.4 FACTORS AFFECTING THE STATISTICAL DISTRIBUTION OF FATIGUE STRENGTH

Since fatigue strength represents the major interest in the engineering applications of the Interference Theory, this problem was studied in some detail. The statistical distribution of the fatigue strength of a mechanical component is a function of a number of factors, such as type of loading, surface finish, stress concentration, heat treatment, temperature, processes, and time. Each shows variability which is characterized by some form of a distribution. The effects of these factors on the statistical distribution of strength were studied in the present investigation.

Fatigue strength can be defined as the maximum stress that can be sustained for a specified number of cycles without failure, the stress being completely reversed within each cycle. In the case of steels a component is said to have finite fatigue strength if it fails between  $10^3$  and  $10^6$  or  $10^7$  cycles due to a given magnitude of cyclic load.

Type of Loading: The three major types of fluctuating load encountered in designing parts are axial, bending and torsion. Experimentally determined values of the ratio of average fatigue strength for axial loading, as compared to bending load were reported in literature as ranging generally from 0.75 to 1.0.<sup>2,3</sup> Although great deal of work has been done to obtain precise values for this ratio, no detailed study has ever been made as to the statistical aspects of these strengths. Investigations have been conducted to find statistical distributions (Normal, Exponential, Weibull, etc.) of fatigue strength tested under a given type of loading, such as bending. No work was done to determine the effect on the distribution if the loads were other than bending. In the present investigation an attempt was made to study the effect of different loads on the statistical distribution of the fatigue strength. The statistical parameters of the distribution for various materials under different loads were determined, tabulated according to materials (Appendix 1) and plotted (Section 6).

**Effect of Surface Finish:** The surface finish of a part does affect its endurance strength. Hence, the condition of finish should be taken into account when the design is based on fatigue. Surfaces which have an effect on the significant strength can be classified into five broad categories: polished, ground, machined, hot-rolled, and as-forged. The worse the surface the lower will be the mean fatigue strength but the higher will be the scatter. As a result, the degree of interference is likely to be pronouncedly affected by the type of surface finish imparted to the member. Different surface effects were studied in the present investigation and the Weibull parameters were tabulated (Appendix 1) and plotted in Section 6.

**Effect of Stress Concentration:** A notch or a stress raiser in a part subjected to fatigue loading can be regarded as a factor causing a local increase in stress or as reduction in strength. For example, a notch with a stress concentration factor of 2 can be thought of as doubling the stress or as halving the strength. In the present investigation this factor was taken as a strength reduction factor.

If all parts were made of materials which are completely homogeneous and have perfectly polished surface finishes, the effect of a notch would be to increase the stress by the factor  $K_t$ . Since actual materials are not perfectly homogeneous and actual surfaces are seldom perfectly polished, there exist internal and surface stress raisers. For this reason, the addition of a notch to a part, already having stress concentration due to geometry, generally produces a smaller effect than would be predicted from the theoretical stress concentration factor,  $K_t$ . The extent to which a notch reduces the endurance limit of a part is referred to as the fatigue stress concentration factor, or the fatigue strength reduction factor, and is designated by the symbol  $K_f$ . This is defined as:

$$K_f = \frac{\text{endurance limit of specimen without the notch}}{\text{endurance limit of specimen with the notch}}$$

In this study an attempt was made to determine the effect of stress concentration on the statistical scatter of the fatigue strength. More specifically, the objective was to find out whether this factor changes the mean strength only, whether it has an effect on the standard deviation or whether it completely changes the nature of the distribution itself. The data were collected for various materials at different temperatures, because, for example, the scatter of fatigue strength at  $10^4$  life cycles with  $K_t = 2.0$  for AISI 1040 steel tested at 70°F may be different than at say 100°F, 200°F, or 500°F. Changes in the parameters of the statistical distribution of the strength due to the effect of stress concentration at various testing temperatures, for different materials are tabulated in Appendix 1 and plotted in Section 6.

The effect of stress concentration was to decrease the values of  $X_0$  and  $\theta$  and in some cases of  $b$ , where  $X_0$  is the lower bound of strength,  $\theta$  is the characteristic strength, where 63.2% of the population have strengths less than or equal to this value, and  $b$  is the Weibull slope.

**Effect of Heat Treatment:** Different heat treatments such as annealing, quenching, tempering, aging etc., can be imparted to materials to change their mechanical properties. Heat treatment may change the average fatigue strength but also the statistical scatter. Pertinent parameters are tabulated in Appendix 1 and plotted in Section 6.

The effect of heat treatment is to increase or to decrease  $X_0$  and  $\theta$ , depending on the design life. In most of the materials which were investigated the slope  $b$  increased with life for a given heat treatment.

**Effect of Temperature:** In a similar manner the effect of temperature on the statistical scatter is shown in Appendix 1 and Section 6.

With few exceptions the effect of temperature is to decrease  $X_0$  and  $\theta$  and increase  $b$  with increased temperature.

#### 4.5 FACTORS AFFECTING THE STATISTICAL DISTRIBUTION OF TENSILE, RUPTURE STRENGTH

In this phase of the study dispersion characteristics of the tensile strength and its statistical distribution were studied for several materials, heat treatments and operating temperatures. The scatter data were plotted in the same manner as the fatigue data, and the Weibull parameters were determined. These parameters were tabulated in Appendix 1. From these tables graphs were prepared with abscissa as temperature and ordinate as Weibull parameters (Section 6.)

Rupture strength can be defined in terms of that static stress which will result in a fracture within a specified time for a specified temperature. Data were collected to determine the statistical distribution of the rupture strength of various materials. The distribution parameters were computed for different operating temperature and for different times such as 100, 1,000 or 10,000 hours. These parameters were then tabulated according to the temperature and time (See Appendix 1).

#### 4.6 ANALYSIS OF STRENGTH DATA

Data collected during this phase of the investigation were organized and systematized according to materials and conditions. In the case of fatigue, these data were plotted on S-N diagrams. Fatigue life data were subsequently converted to fatigue strength data for a given life. (Section 6.) The fatigue strength data thus obtained were plotted on the modified Weibull probability paper to determine Weibull parameters. The same procedure was repeated for different life cycles, for various materials and under different conditions. The Weibull parameters thus found were then tabulated in Appendix 1 and plotted in Section 6.

## SECTION 5 ANALYTICAL EXPRESSIONS FOR THE INTERFERENCE

### 5.1 INTRODUCTION

#### 5.1.1 Interference Probabilities

In interference theory one supposes that the strength of a manufactured part is not known with certainty prior to performing some test on it and that the stress induced by a load is not known with certainty prior to actually loading the part. Thus, for example one does not know with certainty that the strength of a part is exactly 50 ksi. He may know that the part cannot have a strength greater than 58 ksi or less than 40 ksi. Or he may know that the average strength that has been obtained in previous tests on these parts is 49 ksi. He may have some measure of how dispersed the strength measures are around this average strength. The point, of course, is that this type of knowledge is quite different from knowing precisely what the strength is prior to testing. For a multiplicity of reasons strengths of seemingly identical parts are not exactly the same and precisely what strength a part will have cannot be known until some type of strength test is performed. In the theory of probability one says that the strength of a part is a random variable. Certainly the same type of reasoning applies to the stress. Thus for a mathematical theory of interference one starts with the idea that strength is a random variable, say  $X$  and stress is a random variable,  $Y$ .

In describing the properties of random variables, since their values are not known exactly, one supposes that associated with every set of values that the random variable can take there is a real number called the probability that the random variable takes values in the set. These probabilities are non-negative real numbers, they are all less than 1 and in the sense given below they "sum" to 1.

If  $x$  is any real number then there is a probability that the random variable takes some value less than or equal to  $x$ . Symbolically,

$$\Pr(X \leq x)$$

is a number such that  $0 \leq \Pr(X \leq x) \leq 1$ . Surely (i.e. with probability 1)  $X \leq \infty$  so that

$$\Pr(X < \infty) = 1,$$



and

$$\Pr(-\infty > X) = 0$$

Clearly  $\Pr(X \leq x)$  depends on the real number  $x$ . Consequently one defines a probability distribution function  $F(x)$  by the relation

$$F(x) = \Pr(X \leq x).$$

One sees immediately that

$$0 \leq F(x) \leq 1,$$

$$F(-\infty) = 0$$

$$F(\infty) = 1$$

and that  $F(x)$  is a non-decreasing function.

In most engineering applications  $F(x)$  has a derivative for every value of  $x$  and one defines the probability density function,  $f(x)$ , by

$$f(x) = \frac{dF(x)}{dx}.$$

One takes

$$f(x) dx = \Pr(x < X \leq x + dx).$$

(i.e. the probability density function multiplied by  $dx$  is the probability that  $X$  takes values in the neighborhood of  $x$ ). Since

$$f(x)dx = dF(x)$$

one has

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} dF(x) = F(x) \Big|_{-\infty}^{\infty} = 1,$$

the probabilities given by  $f(x)dx$  "sum" to 1.

In the mathematical theory of interference one assumes that the probability density function for the random variables  $X$  (strength) and  $Y$  (stress) are known. Sections 5 and 6 following show how these functions can be found from engineering data. Thus the "givens" of the mathematical theory of interference use the random variables  $X$  and  $Y$ , the set of values that they each can take (usually the non-negative real line), and the probability density or distribution function  $F(x)$  (or  $f(x)$ ) and  $G(y)$  (or  $g(y)$ ).

The problem to which interference theory addresses itself is that of finding the probability of failure. Failure is said to occur whenever the stress exceeds strength. Thus from the known probabilities for the  $X$  and  $Y$  random variables one wishes to find

$$\Pr(Y \geq X),$$

which is the probability that stress exceeds strength or the probability of failure.

#### 5.1.2 Calculation of Probabilities of Failure

There are two useful ways of determining the probability of failure from the known properties of  $X$  and  $Y$ .

(a) Since one wishes to find  $\Pr(Y \geq X)$  it is convenient in some cases (e.g. when stress and strength are normally distributed) to define a new random variable  $Z$  by the relation

$$Z = X - Y,$$

Then if one can find the probability density function of  $Z$ ,  $h(z)$ , the probability of failure will be simply the probability that  $z \leq 0$ . In terms of  $h(z)$  this is found by

$$\Pr(\text{failure}) = \int_{-\infty}^0 h(z) dz.$$

The problem in general is then to find  $h(z)$  from the known probability density functions  $f(x)$ ,  $g(y)$ . A complete discussion of this method and its applications is found in Section A-3.2\*

\* All references to Section A pertain to the Appendix. Thus, Section A-3.2 means Section 3.2 of Appendix 3.

In the important special case in which both stress and strength are normally distributed random variables it is well known that  $Z$  is also normally distributed with parameters. An outline of the proof of these results is given in Section A-3.3.1.

$$\mu_Z = \mu_X - \mu_Y$$

and

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$$

Consequently the probability of failure can be found directly from tables of the normal curve areas. One wants the area from  $-\infty$  to 0 from these tables. A complete discussion of how to do this is given.

A complete example of how this method can be used for non-normally distributed random variables is given in section A-3.3.3 for the case in which stress and strength are each random variables with gamma density functions.

A comparison of this method of finding the probability of failure with the method of part b below is given in section A-3.3.2 for the case of negative exponential probability density functions.

(b) For most applications method (a) above is unnecessarily complex because one must first find the entire density function of the random variable  $Z$  before finding the probability of failure. Since the random variable  $Z$  is of no practical value for  $Z < 0$  the approach in part (a) is unduly long. The methods described in this section are more direct and from our experience more useful in general.

One can derive the probability of failure as follows. Suppose we superimpose the stress and strength density function on the same graph as shown in Figure 5.1.

Although  $Y$  is a random variable let us fix attention on a particular, small interval that  $Y$  can take values in. Let us fix  $y < Y \leq y + dy$ . Then let us find the probability that the random variable  $X$  takes values less than this fixed  $Y$ . One can show that this probability is

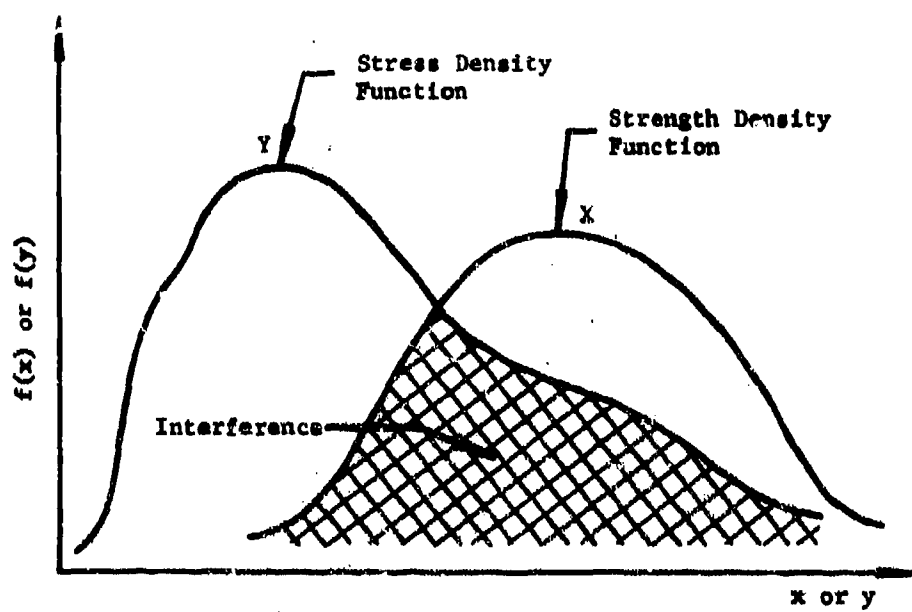


Figure 5.1 Interference of Stress and Strength Distributions

$$\Pr(X \leq y \mid y < Y \leq y + dx) = \int_0^y f(x) dx.$$

The left hand side of this expression is called a conditional probability. It is the probability that the random variable  $X$  takes values less than the real number  $y$  when it is known ("given that") the random variable  $Y$  is "nearly"  $y$ . By the definition of  $f(x)$  this probability is obviously the same as the right hand side of the expression. If now we multiply this conditional probability by

$$\Pr(y < Y \leq y + dy) = g(y)dy$$

we obtain the joint probability that  $X \leq y$  and  $y < Y \leq y + dy$  which symbolically is

$$\begin{aligned} &\Pr(X \leq y \mid y < Y \leq y + dy) \Pr(y < Y \leq y + dy) \\ &= \Pr(X \leq y ; y < Y \leq y + dy) . \end{aligned}$$

This probability is given by the integral

$$\int_0^y f(x) dx \quad g(y) dy .$$

From the joint probability one obtains the probability of failure by:

$$\Pr(\text{failure}) = \int_0^\infty \int_0^y f(x) g(y) dx dy .$$

Thus this double integral gives the probability of failure directly.

If one recalls the relation between  $f(x)$  and  $F(x)$ , it is clear that the double integral is easily reduced to the single integral

$$\Pr(\text{failure}) = \int_0^\infty F(y) g(y) dy .$$

If  $F(x)$  is easily obtainable then this expression is easier to work with than the double integral. For example one knows, that for a strength with a Weibull probability density function the probability distribution

function  $F(x)$  is given by

$$F(x) = 1 - e^{-\left(\frac{x-x_0}{\theta_x-x_0}\right)^b}$$

In these cases the probability of failure is then given by

$$\begin{aligned} \text{Pr(failure)} &= \int_0^{\infty} \left(1 - e^{-\left(\frac{y-x_0}{\theta_x-x_0}\right)^{b_x}}\right) g(y) dy \\ &= 1 - \int_0^{\infty} e^{-\left(\frac{y-x_0}{\theta_x-x_0}\right)^{b_x}} g(y) dy. \end{aligned}$$

The latter expression follows because

$$\int_0^{\infty} g(y) dy = 1$$

if the random variable  $Y$  takes only positive values which is the usual case in interference theory.

Similar expression to those above can be found as shown in Section A-3.3.4. In any event one is free to use whichever expression is easiest to work with.

Examples of the use of this method for non-normally distributed random variables is found in Section A-3.3.

### 5.1.3 Interference Tables, pages 258-396.

In Section 5.1.2 it was shown that the probability of failure could be expressed as an integral involving the known probability density or distribution functions. In certain cases this integral can be evaluated in closed form (e.g. when  $f(x)$  and  $g(y)$  are both exponential functions). In some cases this integral can be evaluated in terms of other well known and tabulated functions (e.g. when  $f(x)$  and  $g(y)$  are both gamma functions or when  $f(x)$  and  $g(y)$  are both normal functions). In general it is not to be expected that the integral for the probability of failure can be evaluated in closed form or in a form involving other well known functions. (e.g.

when  $f(x)$  and  $g(y)$  are both Weibull functions or when  $f(x)$  is a Weibull function and  $g(y)$  is a normal function). In those cases one must resort to numerical evaluation of the integral.

From the discussion in Section 2 it is apparent that two cases are of importance to interference theory. They are

(a)  $f(x)$  and  $g(y)$  are each Weibull functions

(b)  $f(x)$  is a Weibull function and  $g(y)$  is a normal function.

Since the integrals giving the probability of failure cannot be expressed in terms of well known functions, in general, we have evaluated the integral numerically. Tables of the probability of failure are given in Section A-2. A full discussion of the numerical methods used and the errors of approximation appropriate to the tables are given in Section A-4.

## 5.2 USE OF INTERFERENCE TABLES, pages 258 - 396.

### 5.2.1 Parameters for the Weibull-Weibull Case

The form of the integral evaluated for finding the probability of failure when both the strength and stress are Weibull distributed random variables is given in Section A-3.3.4. Tables of this probability of failure are given in Section A-2.2. A discussion of the numerical analysis, errors and accuracy of the tables is given in Section A-4.

For each of the random variables  $X$  and  $Y$  the probability density function is of the form

$$f(x) = \frac{b_x}{\theta_x - x_0} \left( \frac{x - x_0}{\theta_x - x_0} \right)^{b_x - 1} e^{-\left( \frac{x - x_0}{\theta_x - x_0} \right)^{b_x}}, \quad x_0 \leq x < \infty.$$

Each density function is completely characterized by three parameters  $b_x$  (or  $b_y$ ),  $\theta_x$  (or  $\theta_y$ ),  $x_0$  (or  $y_0$ ). These parameters are called the slope, the characteristic value and the truncation parameter respectively. These names follow from the facts that

$$(1) \quad f(x) = 0 \quad \text{if } x < x_0.$$

Hence the strength (or stress) has zero probability of taking values less than  $x_0$  - the probability density function is "truncated" at  $x_0$ .

(2) If one plots  $1/(1-F(x))$  vs  $(x-x_0)$  on  $\ln$  vs  $\ln$  paper the graph will be a straight line with slope  $b_x$ .

(3) If  $(x-x_0) = \theta'_x$ , 63.2% of the area under  $f(x)$  falls below  $(x-x_0)$ . Hence  $\theta_x$ , the "characteristic" of  $x$ , is equal to  $(\theta'_x + x_0)$ .

It is to be noted that since  $f(x)$  is characterized by three parameters one expects the probability of failure to be characterized by six parameters (3 for strength and 3 for stress). Fortunately, this is not the case. As is shown in section A-3.3.4 the integral for the probability of failure is determined by four parameters. These are used in the tables as:

(1)  $b_x$  - the slope of the strength distribution. In the tables this is called  $B_1$ .

(2)  $b_x/b_y$  - the ratio of the slopes of the strength distribution ( $b_x$ ) and the stress distribution ( $b_y$ ). In the tables this is called  $B_1/B_2$ .

(3)  $(x_0-y_0)/(\theta_x-x_0)$  - the difference of the truncation parameters divided by the difference of characteristic value and the truncation parameter of the strength distribution. In all of the tables it is assumed that  $x_0 \geq y_0$ . This appears to be the most useful case for interference theory. In the tables this is called  $(X_0-Y_0)/\text{THETA } 1$ .

(4)  $\theta_y-y_0/\theta_x-x_0$  - the ratio of the difference of the characteristic values and the truncation parameters. In the tables this is called  $\text{Theta } 2/\text{Theta } 1$ .

The following values of these parameters are used in the table. They are considered to be the most useful values for interference theory in mechanical problems.

$$B_1, B_2 = 1, 1.5, 2, \dots 10.$$

$$B_1/B_2 = .1, .2, \dots 1 \text{ and } 1, 2, \dots 10.$$

$$(X_0-Y_0)/\text{THETA } 1 = .000, .250, .500, .750, 1.000$$



THETA2/THETA 1 = 1/1 , 1/1.25 , 1/1.50 , 1/1.75 , ... , 1/3 .

or

THETA2/THETA 1 = 1, .800, .667, .571, ... .333 .

The values in the body of each table are the probability of failure for the parameters given at the heading of the tables. From the discussion given in Section A-4.1.4 our estimate is that these tables are correct to  $\pm 1 \times 10^{-4}$  and most of the values are correct to  $\pm 5 \times 10^{-5}$ .

### 5.2.2 Parameters for the Weibull Distributed Strength, Normal Distributed Stress Case

The form for the integral involved in finding the probability of failure when the strength is Weibull distributed and the stress is normally distributed is given in Section A-3.3.5. Tables of these probabilities are given in Section A-2.1. A discussion of the numerical analysis, error and accuracy of the tables is given in Section A-4.1.7.

The form of the distribution of the strength has been given in section 5.2.1. The parameters were discussed in that section. If the stress is normally distributed the probability density function is given by

$$g(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}, \quad -\infty < y < \infty.$$

Each density function of this form is completely characterized by two parameters,  $\mu$  and  $\sigma$ . These parameters are called the mean stress (average stress, expected stress are also used) and standard deviation of the stress (the square  $\sigma^2$  is called the variance of the stress.) If one plots  $f(x)$  on rectangular coordinates  $\mu$  is the value of  $x$  at which the peak of  $f(x)$  occurs. It is also the point of symmetry of  $f(x)$  and mathematically is the value of the integral

$$\mu = \int_{-\infty}^{\infty} x f(x) dx.$$

$\sigma$  can be interpreted as the value for which the following probability statement is true.

$$\Pr(\mu - \sigma < Y < \mu + \sigma) \approx .68.$$

Since the strength distribution is characterized by three parameters ( $b_x$ ,  $\theta_x$ ,  $x_0$ ) and the stress distribution is characterized by two parameters  $\mu$ ,  $\sigma$  one expects that the integral for the probability of failure in this case is characterized by five parameters. Fortunately this is not the case. As shown in Section A-3.3.5 the integral for the probability of failure is characterized by three parameters. These three parameters as used in the tables are:

(1)  $b_x$  - the slope of the strength distribution. In the tables this is called  $B(x)$ .

(2)  $\theta_x - x_0 / \sigma$  - the ratio of the difference of the characteristic strength and the truncation parameter to the standard deviation of the stress. In the table this is called  $C$  for typographical simplicity.

(3)  $(x_0 - \mu) / \sigma$  - the difference between the strength truncation parameter and the mean stress divided by the standard deviation of the stress. In the tables this is called  $A$  for typographical simplicity.

The following values of these parameters are used in the table. They are considered to be the most useful values for interference theory in mechanical problems.

$B(x) = 1, 1.2, 1.3, \dots 3.2$

$C = 10, 15, 20 \dots 100$

$A = 0, 0.2, 0.4, \dots 2.8$ , and from  $-0.2$  to  $-10.0$ .

The values in the body of each table are the probabilities of failure for the parameters given at the heading of the tables. From the discussion given in Section A-4.1.7 these probabilities are correct to  $\pm 5 \times 10^{-5}$ .

### 5.2.3 Use of the Tables, Explanation of Missing Values and Interpolation

Numerical examples of the use of the tables are given in Section 5.3. In general the user will enter the table with known parameters ( $b_x$ ,  $\theta_x$ ,  $x_0$  and the appropriate parameters for the stress distribution) and wish to find the probability of failure. This is a direct table look-up. In some design problems the user will have a given probability of failure to achieve and will know the general

shape of the distribution of stress and strength appropriate to the material that he is using. The table will then give him the relative parameters (there may be many of these) to design for. It would be expected that a cost analysis would give the acceptable parameter values for each distribution. As long as the relative values are as given in the table the probability of failure will be the same no matter what the values for each distribution are.

In the tables there are some values not tabulated. For example in the Tables A-2.2 for the Weibull strength and Weibull stress there is a row of non-tabulated values for  $B1/B2 = .1$  and  $B1 > 1$  for every value of  $(X_0 - Y_0)/\text{THETA } 1$  and  $\text{THETA } 2/\text{THETA } 1$ . These values were not tabulated because they require values of the parameters (i.e.  $B1$  and  $B2$ ) that are outside the limits considered useful for mechanical problems in interference theory. For example the value in the table  $(X_0 - Y_0)/\text{THETA } 1 = .000$ ,  $\text{THETA } 2/\text{THETA } 1 = .571$  at  $B1 = 5$ ,  $B1/B2 = .1$  is missing. To include this value in the table would have required determining the probability of failure for the case  $B2 = 50$ . But  $B2 = 50$  is a value seldom found in mechanical interference theory. Hence this probability of failure was not computed.

Some of the non-tabulated values are nearly zero and hence have not been tabulated. This occurs only in the Weibull Strength-Normal Stress tables. For example when  $A > 3.5$  one knows that  $(x_0 - \mu)/\sigma > 3.5$ . From the theory of the normal curve one knows that the area under the normal curve from 3.5 to  $\infty$  is less than  $3 \times 10^{-4}$  and the probability of failure is less than  $1 \times 10^{-4}$ . We take these values to be too small to be significant for the mechanical interference theory.

It can be seen that the tables are non-linear for almost all values of the parameters. This can cause inaccuracies when the tables are interpolated. The absolute value of the interpolation error depends on which tables are interpolated. For precise values the user should use a higher order interpolation formula rather than linear interpolation. We have not explored the relative errors of interpolation closely. In those cases checked, the relative errors are small.

### 5.3 EXAMPLES OF USE OF THE TABLES

For an example illustrating the use of these tables in an application of the Interference Theory see Section 9.

## SECTION 6 STATISTICAL DISTRIBUTION OF STRENGTH

### 6.1 ANALYSIS OF STRENGTH DATA

Most fatigue testing involves subjecting a number of specimens or parts to the same stress and repeating this process at various stress levels. The data thus obtained, known as life data, are used to construct the conventional S-N diagram. In this case, the scatter obtained is the scatter in life at a given stress. In the present investigation the attention was focused on the nature of the scatter in the fatigue strength at a given life. To obtain such data it is necessary to fatigue test all the specimens with different stresses imposed on them in such a manner that all would fail at a predetermined life. Practically, this is impossible and, therefore, in the present investigation, two alternate methods, described below, were considered.

#### 6.1.1 Conversion of Life Data to Strength Data

The fatigue life data were obtained for various materials under various conditions. These data were then plotted on the conventional S-N diagram. Here, it is assumed that to each specimen of the population can be attributed an individual S-N curve, and that there exists for any population of specimens (at fixed test conditions) a family of non-intersecting S-N curves, which can be determined with any desired accuracy, each curve corresponding to a given probability.

The average S-N curve is then fitted to all the test points on the S-N diagram using the least square method. Passing through each test point draw an S-N curve parallel to the average S-N curve. These will make a family of S-N curves. (see Figure 6.1). Now if the fatigue strength distribution at  $N = N_1$  life is required, draw a vertical line at  $N = N_1$  intersecting the family of S-N curves. These points of intersection  $S_1, S_2, \dots$  represent a sample from the strength distribution at a desired life. These data are then plotted on the probability paper as a cumulative distribution function to determine the strength distribution. (see Figure 6.3).

#### 6.1.2. Strength Response Test

As an alternative method, the strength response test was considered. The cumulative percentage point of fatigue strength distributions can be determined at any stress level  $S$  by testing a large number of specimens at this level and counting the fraction of specimens failing at the preassigned life  $N$ . If this procedure is repeated at different

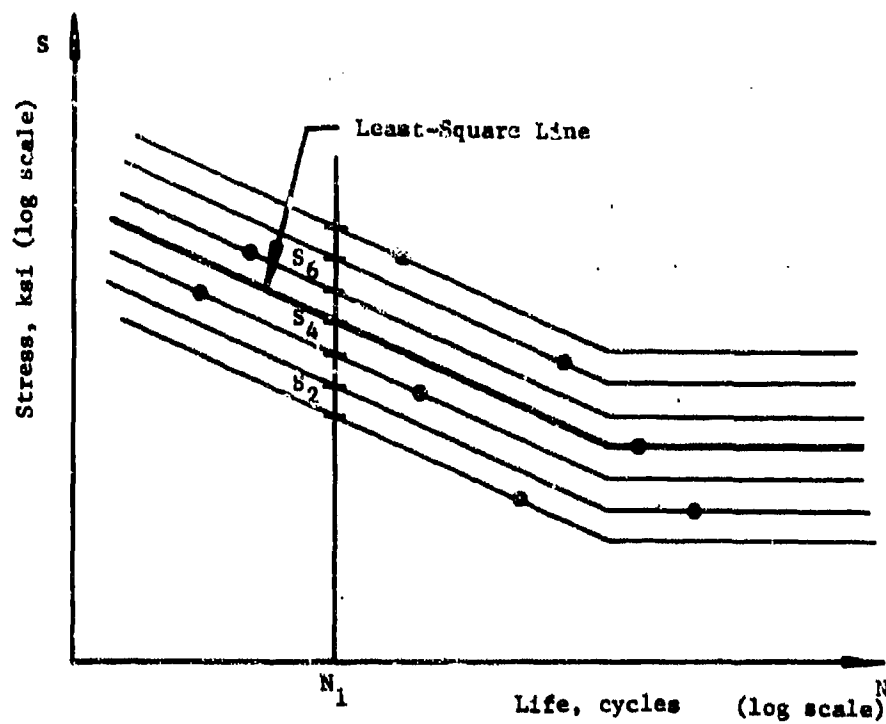


Figure 6.1 S-N Diagram for converting Life Data to Strength Data

levels, several points of the strength distribution are obtained and can be used for the analysis of strength distribution.

For example, suppose, the fatigue strength distribution at life  $N_1$  is desired. (See Figure 6.2). A large number of specimens, say 50, are tested at stress level  $S_1$  and if only one out of these 50 fails before or at the preassigned life  $N_1$  then it can be said that on an average only  $F_1 = 2\%$  from the lot of specimens have fatigue strength less than or equal to  $S_1$ . The same procedure can be repeated for several other stress levels  $S_2, S_3, \dots, S_i$ , and corresponding percentage points ( $F_2, F_3, \dots, F_i$ ) can be determined. These points represent the cumulative behavior of strength, and can be plotted on the several probability papers (such as Weibull, Normal, Logistic, Extreme value, etc.) with  $S$  as its abscissa and  $XF$  as its ordinates, (Figure 6.3).

The percentage points of the strength distribution measured by this method are independent of each other and accordingly the method of least squares can be applied.

As this method requires testing of a large number of specimens at any one stress level, very limited data of this type are available, although recently a method was proposed for generating such data.<sup>6</sup> Hence, in the present study the fatigue strength distributions were analyzed by converting the directly observed scatter in fatigue life into a scatter in fatigue strength, as discussed in Section 6.1.1, rather than by evaluating the data from response tests.

## 6.2 PLOT OF STRENGTH DATA

In order to determine the distribution of strength at a given life, it was necessary first to obtain empirical data from literature and other sources (see Section 4.3) and then to plot these data so that the parameters of the distributions could be determined. The type of information desired is illustrated in Figure 6.4 where it is shown that the strength distribution may be different at the different lives.

The usual method of determining this statistical distribution is to construct a histogram. This was tried in the present investigation for a number of cases, one of which is shown in Figure 6.5. This refers to strength data obtained from Mechanical Properties Data Center for D<sub>6AC</sub> Steel under conditions as follows:

Type of load - completely reversed; Surface finish - Mechanically Polished; Stress Concentration factor -  $K_t = 1.0$ ; Test Temperature 80°F. Fatigue Strength distribution data at  $10^6$  cycles are: 55.3, 57.3, 59.2, 61.4, 62.5 ksi.

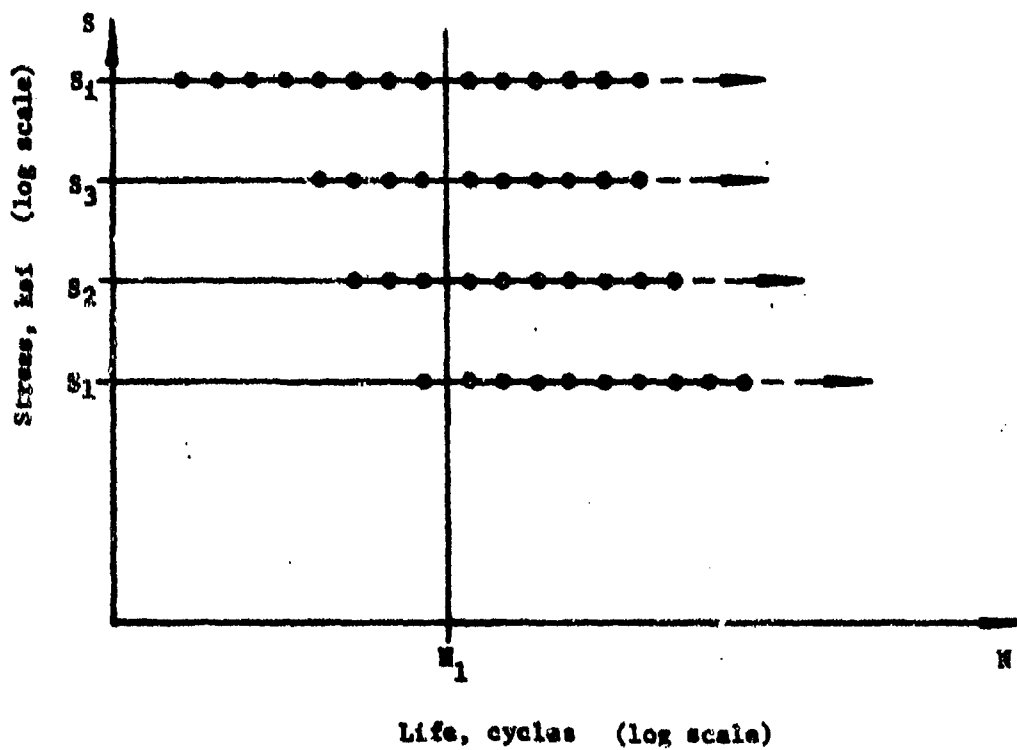


Figure 6.2 S-N Diagram for Strength Response

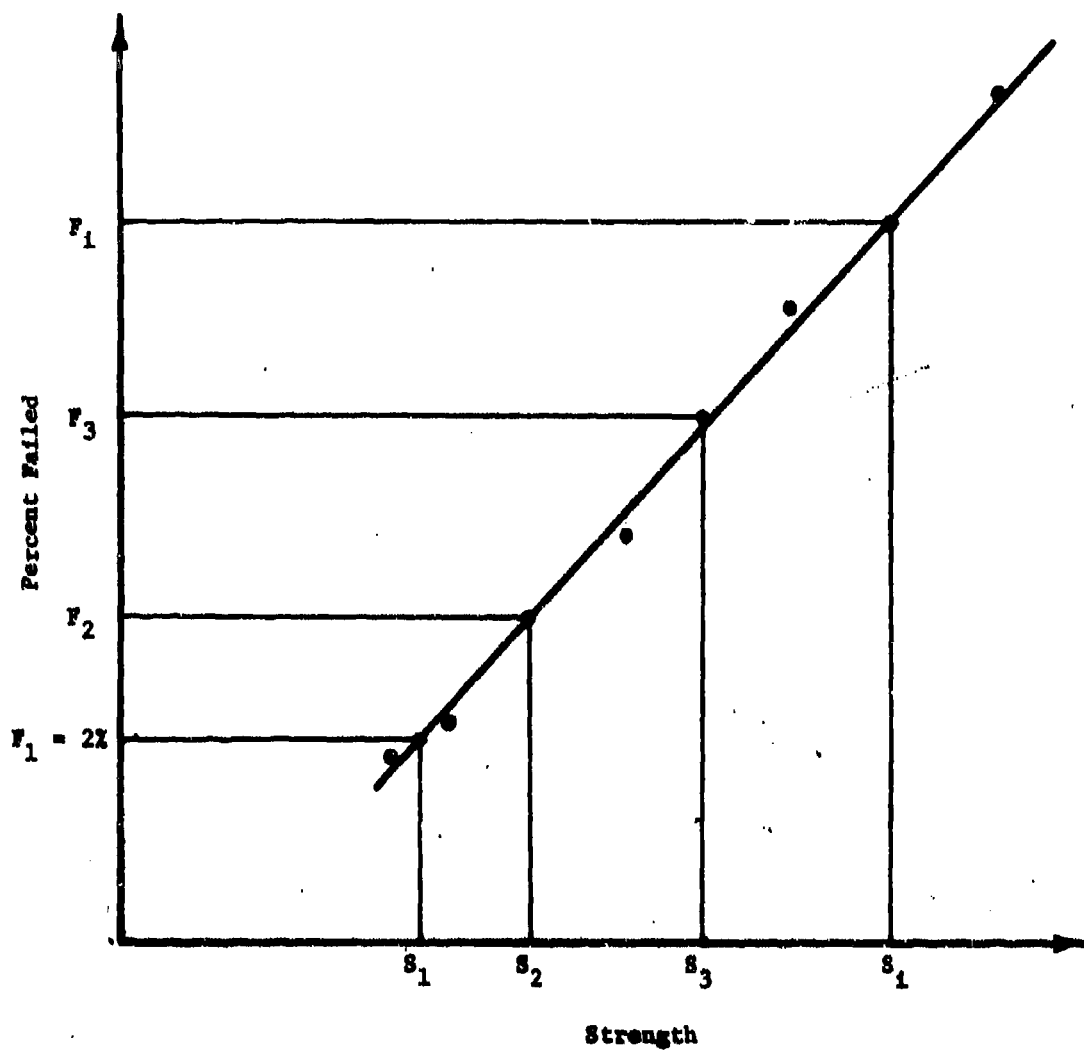


Figure 6.3 Plot of Strength Response Data on Probability Paper



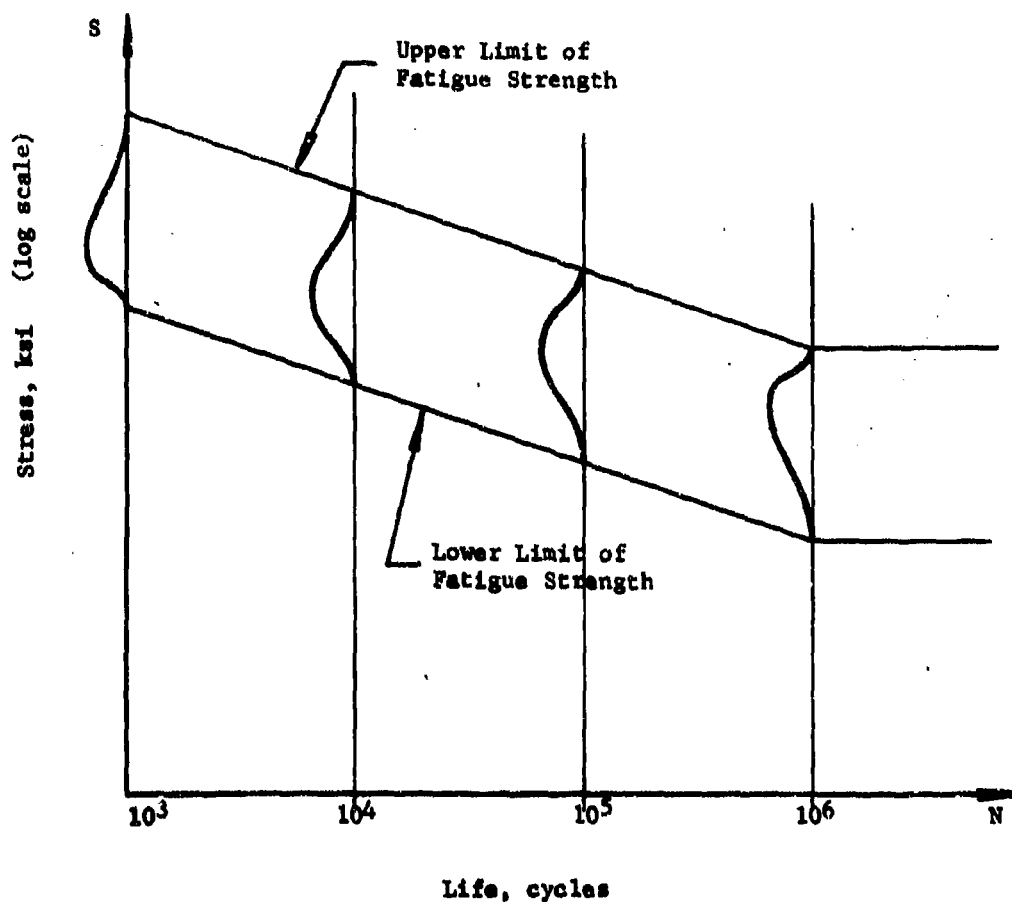


Figure 6.4 Change in the Fatigue Strength Distribution with Life

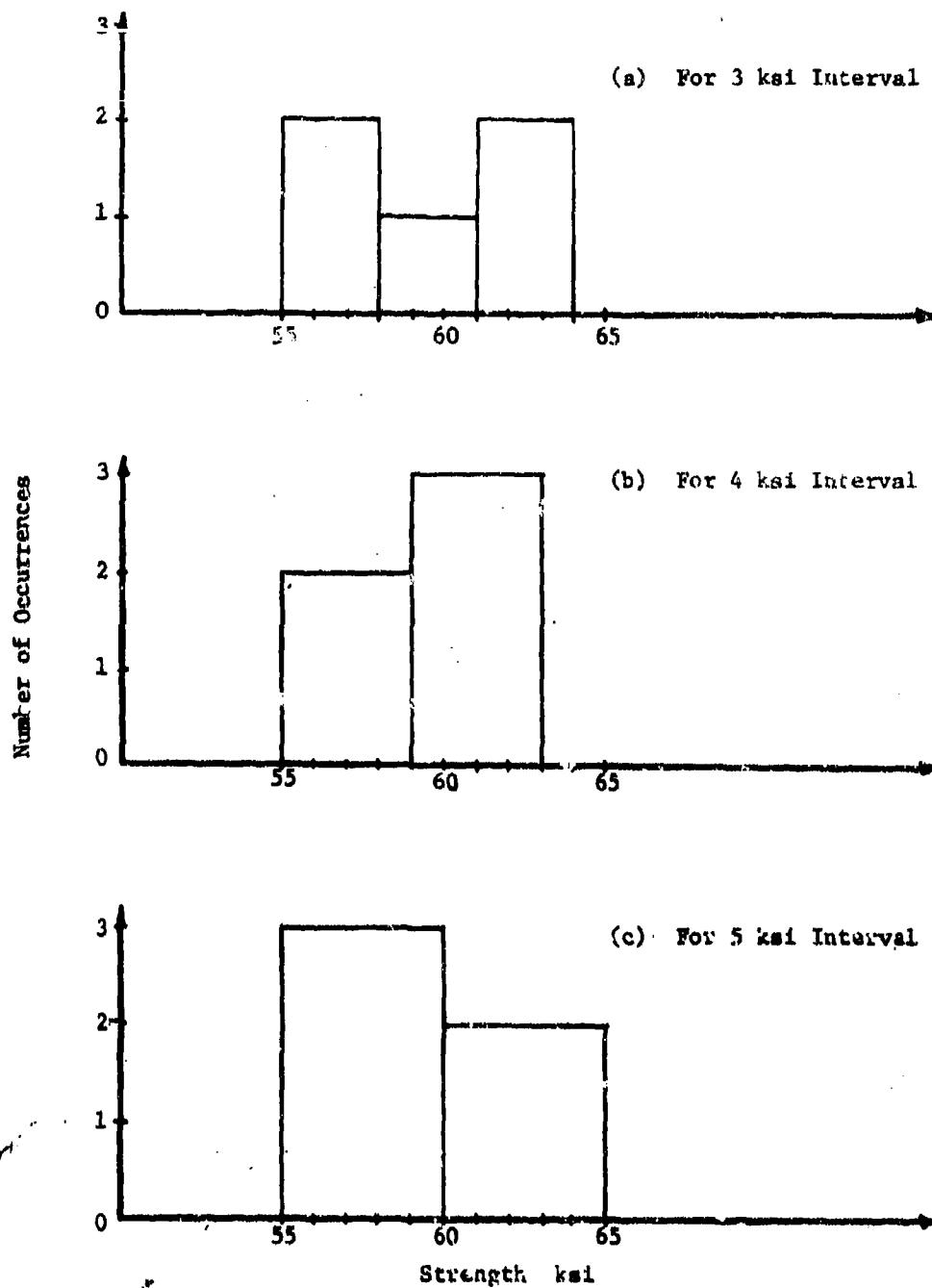


Figure 6.5 Histograms for Fatigue Strength of  $D_{6Ac}$  Steel at 800F for  $10^6$  Life Cycles

In Figure 6.5 this data are plotted for 3, 4, and 5 ksi intervals. It can readily be seen that each interval suggests a different form of distribution. Furthermore, for the histogram method to be effective, a large amount of data, well in excess of the data available in the present investigation, is required.

For this reason, the histogram method was not used here, and, instead, the Weibull distribution was adopted.

The Weibull distribution is of great usefulness in the analysis of fatigue data. The utility and value of the Weibull distribution results from the fact that it covers a considerable variety of distribution patterns, and data which fit any of these patterns plots as a straight line on special graph paper, known as Weibull probability paper. (For explanation see page 44). Although the Weibull distribution provides a versatile means for describing the life characteristics, it can also be used for describing the mechanical properties, such as, fatigue, tensile and rupture strengths studied in the present investigation.

The Weibull equation is a three parametric mathematical function having  $x$  as a variable. The general expression for the Weibull density function is:

$$f(x) = \frac{b}{\theta - X_0} \left( \frac{x - X_0}{\theta - X_0} \right)^{b-1} e^{-\left( \frac{x - X_0}{\theta - X_0} \right)^b}, \quad (6.1)$$

$$X_0 \leq x \leq \infty$$

and the general expression for the cumulative distribution function is:

$$F(x) = 1 - e^{-\left( \frac{x - X_0}{\theta - X_0} \right)^b}, \quad (6.2)$$

$$X_0 \leq x \leq \infty$$

where, as used in this study,

$X_0$  is the lower bound of strength

$\theta$  is the characteristic strength, where 63.2% of the population have strengths less than or equal to this value.

$b$  is the Weibull slope.

Versatility of the Weibull distribution is illustrated in Figure 6.6 and Figure 6.7 which show different forms of the distribution for various values of  $b$ . The Weibull slope  $b$  defines the shape of the curve, whereas  $\theta$ , the characteristic strength, defines the scale of the curve (see definition on page 36). It is therefore possible to have several forms of a particular distribution depending on:

1. The value of  $b$  (where  $\theta$  and  $X_0$  are constant)
2. The value of  $\theta$  (where  $b$  and  $X_0$  are constant)
3. The value of  $X_0$  (where  $\theta$  and  $b$  are constant).

As to special cases of Weibull distribution, it reduces to the truncated normal distribution when  $b$  is approximately equal to 3.5 and to the truncated exponential distribution when  $b$  is equal to 1.0.

### 6.3 DETERMINATION OF THE WEIBULL PARAMETERS

In order to determine the Weibull parameters for the strength data the following steps are required:

1. The scatter of fatigue life at a given stress level, as obtained from the literature or other sources, is converted to the scatter of fatigue strengths at a given life in the manner discussed in Section 6.1.1.
2. The fatigue strengths obtained from above are then arranged in the increasing order of value and median rank is assigned to each value as described in the example that follows.
3. The strengths are then plotted on the modified Weibull probability paper on the abscissa against the median ranks on the ordinate.
4. A correction is then made to the resultant curve by determining the probable value of the lower bound of strength  $X_0$ .
5. From the curve thus modified the three parameters of Weibull are then determined.

This method is illustrated by the following example.

Material: D6AC Steel,  $S_u = 270$  Ksi

Conditions: Type of Load - Completely Reversed Bending

Surface Finish - Mechanically Polished

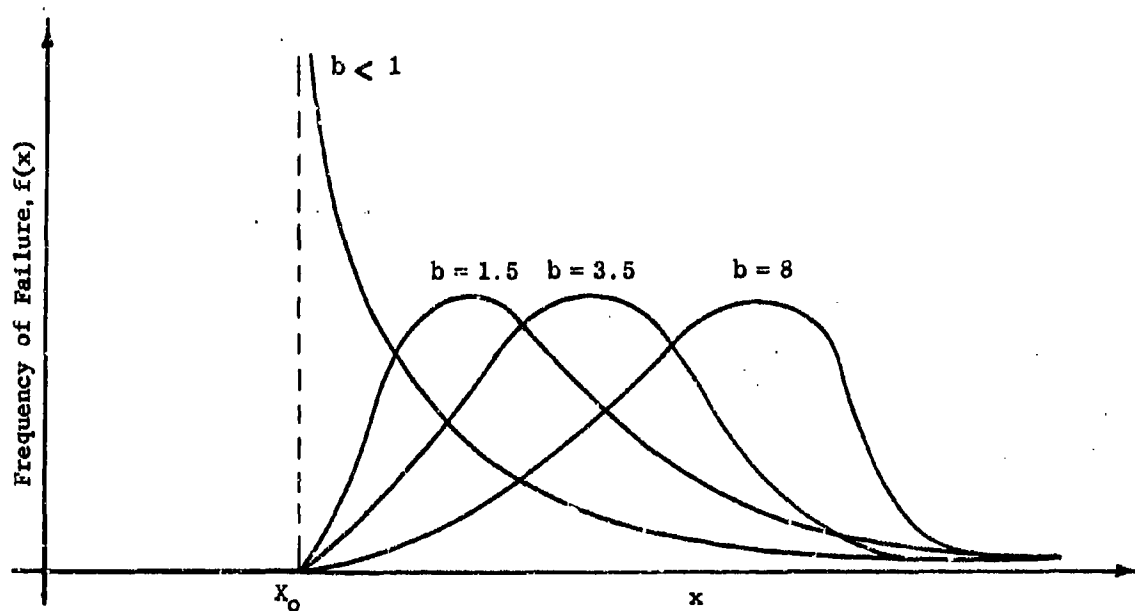


Figure 6.6 Plot of  $x$  vs  $f(x)$  in a Weibull Distribution

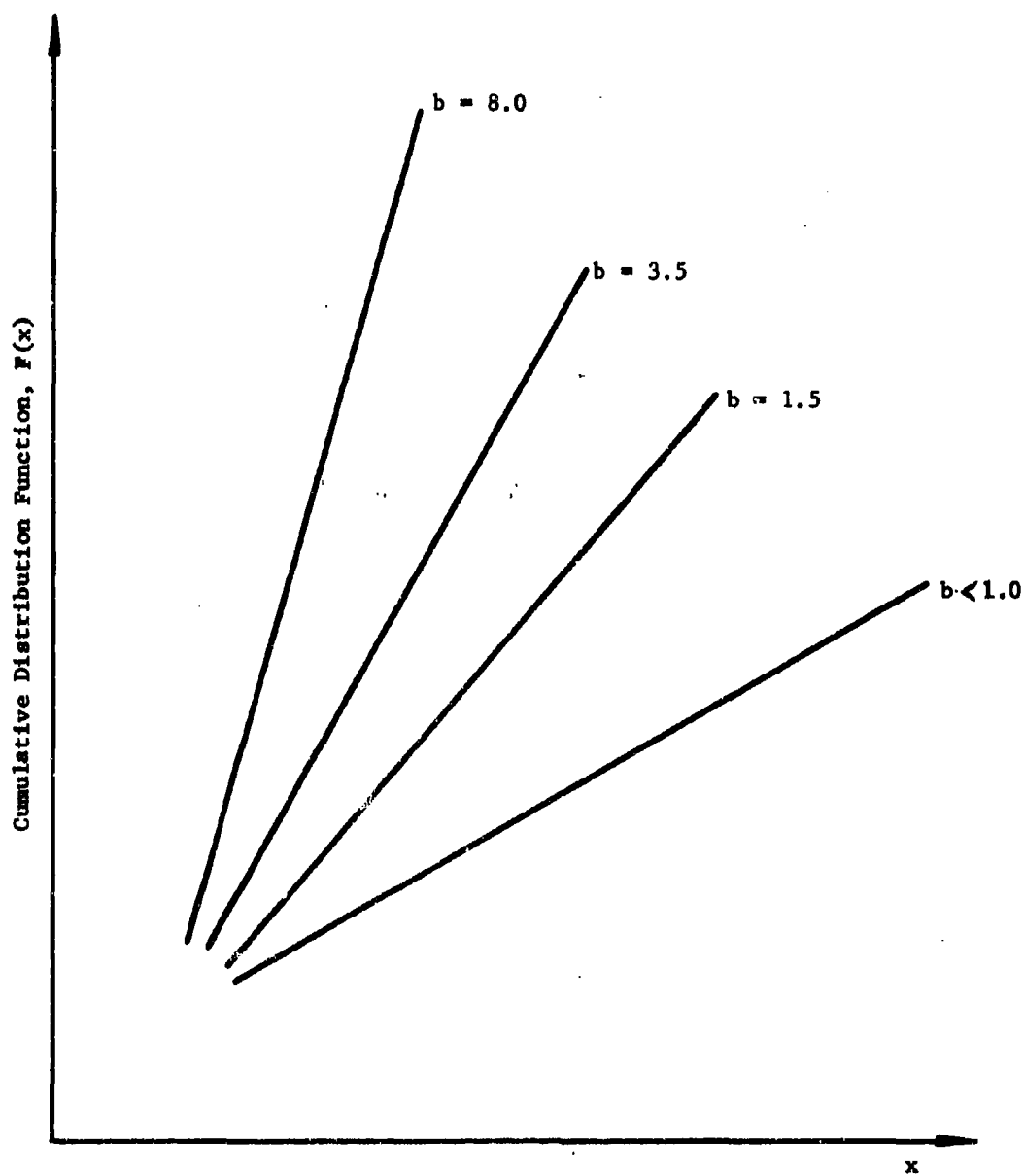


Figure 6.7 Weibull Plots for Various Slopes on Weibull Probability Paper

Stress Concentration Factor,  $K_t = 1.0$

Test Temperature, 800°F

Fatigue Strength distribution data at  $10^5$  cycles are (in ksi):

57.3, 59.2, 62.5, 55.3, 61.4

In order to make the Weibull cumulative plot, it becomes necessary to decide what rank is to be assigned to each particular strength value. The lowest strength in a group tested will have a definite percentage of the total population having strengths lower than this, if the entire population were tested. If we knew exactly the percentage of the population having strengths lower than the lowest in the sample, that percentage would be the true rank of the lowest strength in the sample. However, since we do not know the true rank, we make an estimate of it. We use an estimate such that in the long run the positive and negative errors of the estimate cancel each other. That is, half the time we would give the lowest strength a rank that is too high and the other half of the time a rank too low. A rank with this property is called median rank. A table of median ranks is given in Table 6.1. The test data are then arranged in an increasing order of value and the appropriate median ranks for sample size  $n = 5$  are read from Table 6.1 as follows:

x, Ksi	Median Ranks, %
55.3	12.94
57.3	31.47
59.2	50.00
61.4	68.53
62.5	87.06

These data are then plotted on the modified Weibull probability paper as shown in Figure 6.8, curve A.

In plotting these data an assumption was made that the lower-bound of strength  $X_0$  (i.e. the minimum strength that can be expected in the whole population) is zero. This is obviously not the case, as mechanical parts must have a strength greater than zero. Therefore the next step was to determine the probable value of  $X_0$ . This value should be somewhere between the lowest value of the sample (55.3 ksi) and zero. As the first trial therefore assume that  $X_0$  is 35 ksi.

By subtracting  $X_0$  from the original set of data, the following is obtained:

Sample Size = n										
1	2	3	4	5	6	8	10	12	14	16
1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Sample Size = n										
1	2	3	4	5	6	8	10	12	14	16
1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 6.1 Table of Median Ranks<sup>1</sup>



D<sub>6AC</sub> Steel, Life of  $10^6$  Cycles, Fatigue

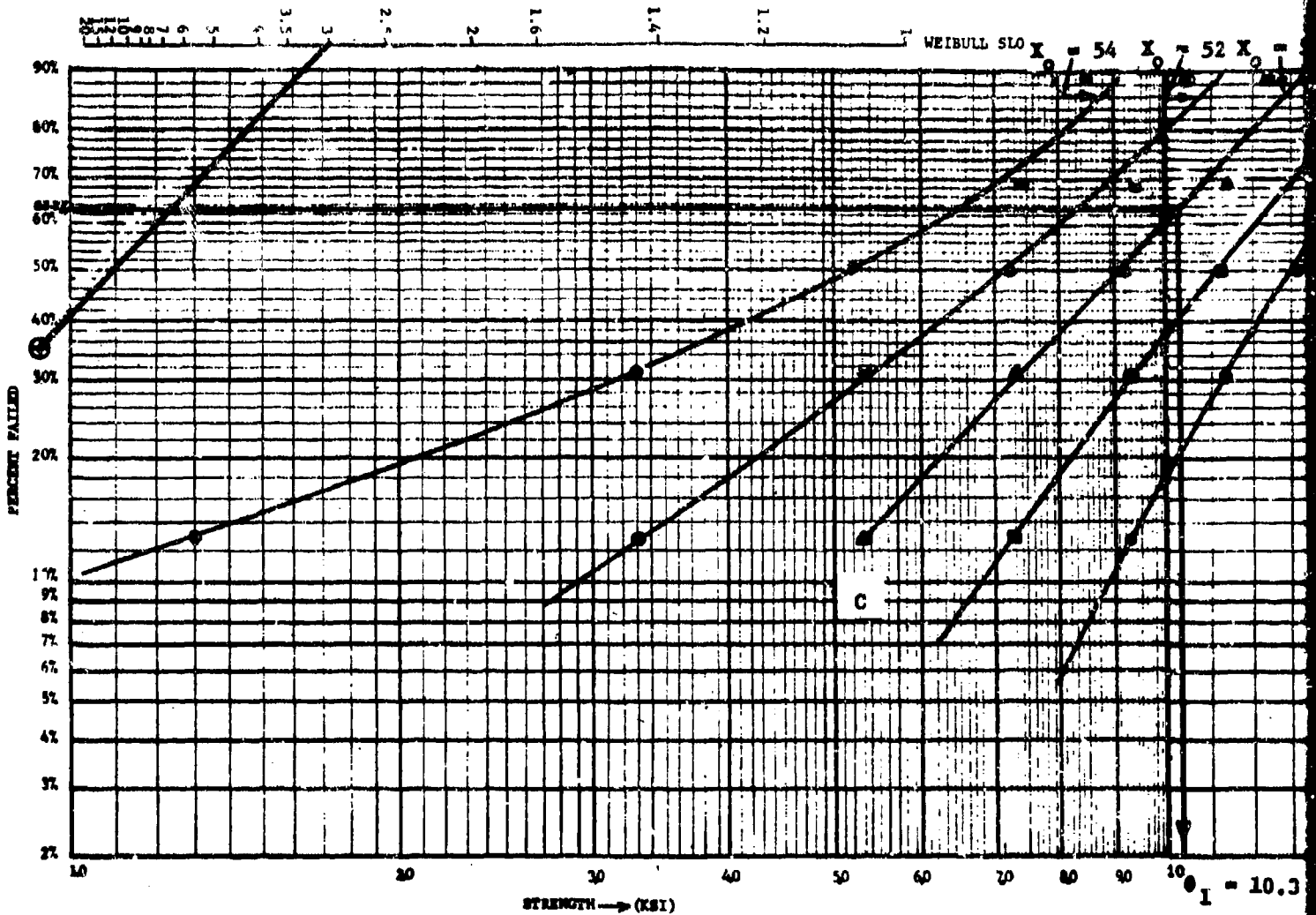
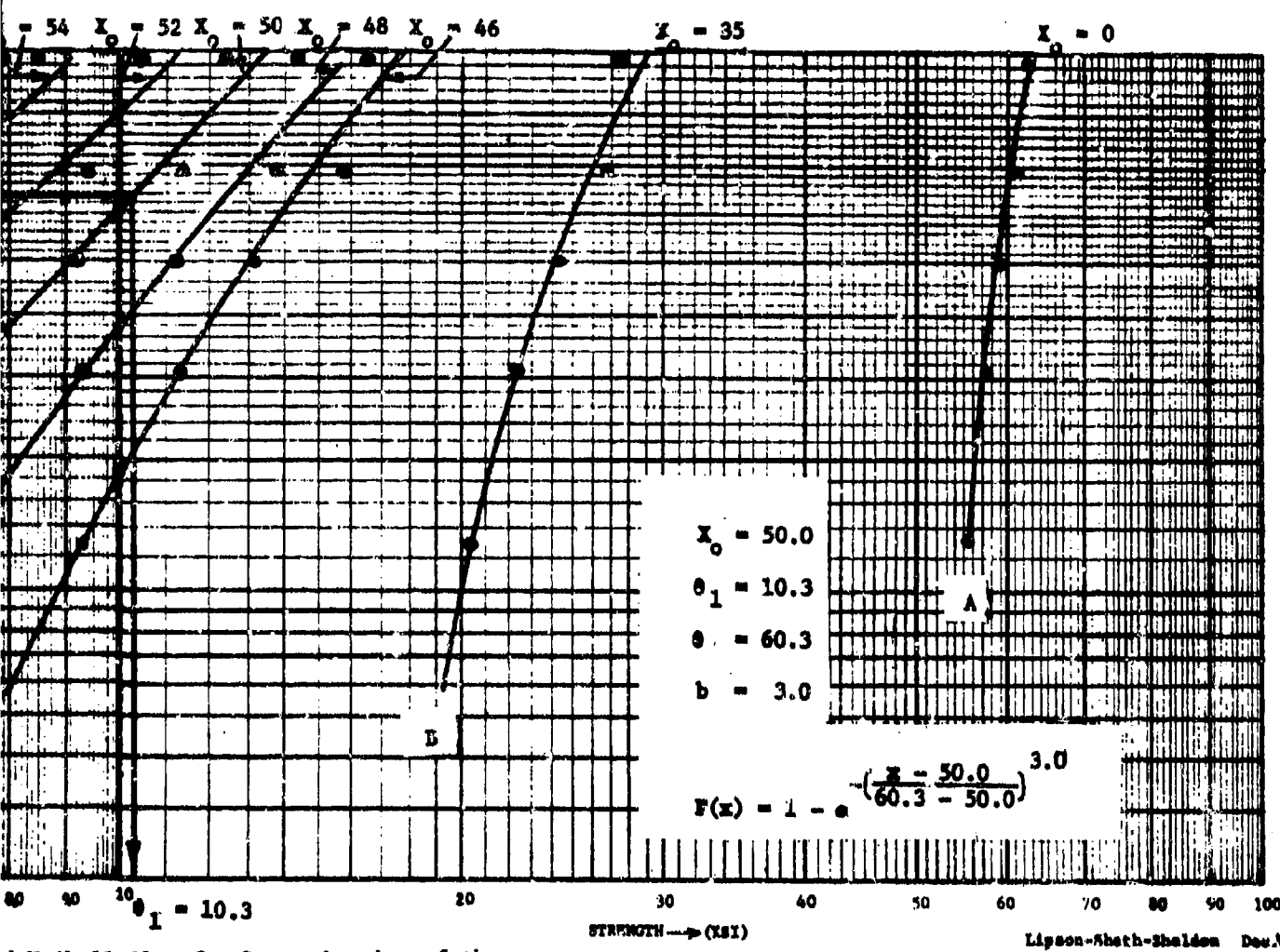


Figure 6.8 Modified Weibull Plot for D<sub>6AC</sub> Steel  
Weibull Parameters for the Above

10<sup>6</sup> Cycles, Fatigue Strength at 80°F



and Weibull Plot for Determination of the Parameters for the Above Conditions

2

$(x - X_0)$ Ksi	Median Ranks, %
20.3	12.94
22.3	31.47
24.2	50.00
26.4	68.53
27.5	87.06

When these are plotted (Figure 6.8, curve B) the resultant curve is not a straight line. Therefore, other values of  $X_0$  are assumed,<sup>5</sup> and the same procedure is repeated until, for a certain assumed  $X_0$ , one can best linearize all the test points. In this case the best line nearest to a straight line is for  $X_0 = 50$  Ksi, curve C. Through these points, then, a straight line is fitted using the Least Square Method.

The value of  $(x - X_0)$  at 63.2% is read off to determine the characteristic strength  $\theta$ :

$$\theta = x \text{ at } 63.2\%$$

$$(x - x_0)_{63.2\%} = \theta_1 = 10.3 \text{ Ksi}$$

$$\begin{aligned} \theta &= (x)_{63.2\%} = \theta_1 + x_0 = 10.3 + 50 \\ &= 60.3 \text{ Ksi} \end{aligned}$$

The Weibull slope  $b$  is determined by drawing a line parallel to the straight line of  $X_0 = 50$  and passing it through the pivot point. The point where this line intersects the Weibull slope scale is the value of the Weibull slope. In this case,  $b = 3.0$ . Hence, the Weibull parameters for the given set of fatigue strength data are:

$$X_0 = 50 \text{ Ksi}$$

$$\theta = 60.3 \text{ Ksi}$$

$$b = 3.0$$

The analytical form for the corresponding Weibull equation is:

$$F(x) = 1 - e^{-\left(\frac{x-50}{80.3-50}\right)^{3.0}}$$

These parameters were tabulated for various materials under various conditions, (see Tables Appendix 1) on the basis of all the available test data obtained. The most representative parameters were then plotted, as shown in Figure 6.9 to Figure 6.115.

As stated on page 36, one of the advantages of the Weibull distribution is that it plots as a straight line on a Weibull probability paper. This is shown below:

Equation 6.2 gives:

$$F(x) = 1 - e^{-\left(\frac{x - X_0}{\theta - X_0}\right)^b}$$

$$\text{or } \frac{1}{1 - F(x)} = e^{\left(\frac{x - X_0}{\theta - X_0}\right)^b}$$

$$\ln \ln \left[ \frac{1}{1 - F(x)} \right] = b \ln (x - X_0) - b \ln (\theta - X_0)$$

This equation has a form  $Y = b(X) + C$  which represents a straight line with a slope  $b$  and intercept  $C$  on the Cartesian  $X, Y$  co-ordinates. Hence, a plot of  $\ln \ln 1/(1-F(x))$  against  $\ln (x - X_0)$  will also be a straight line with a slope  $b$ .

#### 6.4 GRAPHS OF WEIBULL PARAMETERS

Weibull parameters  $\theta$ ,  $b$  and  $X_0$  for fatigue strength determined, as shown in Section 6.3, were then plotted against life on log-log scale for various materials including the effect of heat treatment, stress concentration, temperature, type of loading, surface finish, etc. Weibull parameters for tensile strength determined in the same manner were then plotted against temperature on Cartesian coordinates for various materials.

For ease of locating specific information the following Table of Contents is offered.

# MEANING OF SYMBOLS

$S_u$	Ultimate Tensile Strength of Specimen
$S_y$	Yield Strength of Specimen
$\sigma$	Characteristic Strength, ksi
$X_o$	Lower Bound of Strength, ksi
$b$	Weibull Slope
T of L	Type of Loading
R	Rotary Bending
P	Plate Bending
A	Axial Bending
Spec	Type of Specimen
V-N	Vee Notched, Flank Angle = 60°
H-N	Hole Notch
No-N	Unnotched
$S_m$	Mean Stress, ksi
$K_t$	Theoretical Stress Concentration Factor
Melt.	Type of Melt Practice
Sec. Op.	Secondary Operation Applied to Test Specimen
T.I.G.	Tungsten Inert Gas Welded
Surf.Cond.	Test Surface Condition
S.P.	Shot Peened
C.P.	Chrome Plated
C.B.	Chromed and Baked
M.P.	Mechanical Polish
G.	Ground
Scr.	Scratched Mechanically
N.P.	No Preparation to Surface
H.T.	Heat Treatment Applied to Specimen
W.Q.	Water Quenched
A.C.	Air Cooled
O.Q.	Oil Quenched
Sol.Tr.	Solution Treated
Temp.	Tempered
Aust.	Austinitized
Norm.	Normalized
Cond.	Conditioned

INDEX TO GRAPHS OF  
WEIBULL PARAMETERS

Material:

Fatigue Strength

<u>Material</u>	Weibull Parameters can be found:	
	<u>Page No.</u>	<u>Figure No.</u>
1. AISI 3140 Steel		
Effect of Stress Concentration, H.T. = $K_3$	51	6.9
Effect of Stress Concentration, H.T. = $K_4$	52	6.10
Effect of Heat Treatment, Unnotched	53	6.11
Effect of Heat Treatment, V-Notched	54	6.12
2. AISI 1045 Steel		
Effect of Heat Treatment, V-Notched	55	6.13
Effect of Stress Concentration, H.T. = $K_1$	56	6.14
3. AISI 2340		
Effect of Heat Treatment, V-Notched	57	6.15
Effect of Heat Treatment, Unnotched	58	6.16
Effect of Stress Concentration, Oil Quench	59	6.17
Effect of Stress Concentration, Air Blast Quench Temp.	60	6.18
Effect of Stress Concentration, Air Blast Quench, no Temp.	61	6.19
4. 4140 Steel		
Effect of Stress Concentration	62	6.20
5. D5AC Steel		
Effect of Temperature, $K_t = 1.0$	63	6.21
Effect of Temperature, $K_t = 3.0$	64	6.22
Effect of Temperature, $K_t = 3.0$ $S_m = 30-50$ ksi	65	6.23
Effect of Temperature, $K_t = 1.0$ $S_m = 70-80$ ksi	66	6.24
Effect of Stress Concentration, $T = 80^\circ\text{F}$	67	6.25
Effect of Stress Concentration, $T = 450^\circ\text{F}$	68	6.26
Effect of Stress Concentration, $T = 550^\circ\text{F}$	69	6.27
6. H-11 Steel		
Effect of Surface Treatment, N.P.	70	6.28
Effect of Surface Treatment, N.P.	71	6.29
Effect of Surface Treatment, Pretest Exposure at $375^\circ\text{F}$	72	6.30
Effect of Surface Treatment, Pretest Exposure at $500^\circ\text{F}$	73	6.31
Effect of Surface Treatment, Pretest Exposure at $750^\circ\text{F}$	74	6.32
Effect of Surface Treatment, Pretest Exposure at $1000^\circ\text{F}$	75	6.33
Effect of Heat Treatment, N.P., M.P.	76	6.34

Effect of Heat Treatment, S.P., M.P.	77	6.35
Effect of Heat Treatment, C.P., C.B.	78	6.36
Effect of Heat Treatment, S.P., C.B.	79	6.37
7. 4340 Steel		
Effect of Heat Treatment, V-Notched, Air Melt	80	6.38
Effect of Heat Treatment, Unnotched, Air Melt	81	6.39
Effect of Heat Treatment, V-Notched, Vac. Melt	82	6.40
Effect of Heat Treatment, Unnotched, Vac. Melt	83	6.41
Effect of Heat Treatment, V-Notched, Vac. Melt	84	6.42
Effect of Melt Practice, V-Notched, H.T.A.	85	6.43
Effect of Melt Practice, V-Notched, H.T.B.	86	6.44
Effect of Melt Practice, Unnotched, H.T.A.	87	6.45
Effect of Melt Practice, Unnotched, H.T.B.	88	6.46
Effect of Stress Concentration, H.T.A. Air Melt	89	6.47
Effect of Stress Concentration, H.T.B. Air Melt	90	6.48
Effect of Stress Concentration, H.T.A. Vac.Melt	91	6.49
Effect of Stress Concentration, H.T.B. Vac.Melt	92	6.50
Effect of Stress Concentration, H.T.A. Vac.Melt	93	6.51
Effect of Stress Concentration, H.T.B. Vac.Melt	94	6.52
8. AISI 4340 Steel		
Effect of Heat Treatment, Hot Rolled and Lathed	95	6.53
Effect of Heat Treatment, Forged and Ground	96	6.54
9. Thermold J.		
Effect of Stress Concentration	97	6.55
10. Fe, 5.5%, Mo, 2.5% Cr., 5% C		
Effect of Stress Concentration	98	6.56
11. M10 Tool Steel		
Effect of Heat Treatment	99	6.57
12. 321 Stainless Steel		
Effect of Stress Concentration at 80°F	100	6.58
Effect of Stress Concentration at -320°F	101	6.59
Effect of Stress Concentration at -423°F	102	6.60
Effect of Process at 80°F, $K_t = 1.0$	103	6.61
Effect of Process at -320°F, $K_t = 1.0$	104	6.62
Effect of Process at -423°F, $K_t = 1.0$	105	6.63
Effect of Temperature, M.P., $K_t = 1.0$	106	6.64
Effect of Temperature, M.P., $K_t = 3.5$	107	6.65
Effect of Temperature, T.I.G. Welded, $K_t = 1.0$	108	6.66
13. A-286 Stainless Steel		
Effect of Temperature, M.P., $K_t = 1.0$	109	6.67
Effect of Temperature, M.P., $K_t = 3.5$	110	6.68
Effect of Temperature, T.I.G. Welded, $K_t = 1.0$	111	6.69
Effect of Stress Concentration at 80°F	112	6.70



Effect of Stress Concentration at -320°F	113	6.71
Effect of Stress Concentration at -423°F	114	6.72
Effect of Process at 80°F	115	6.73
Effect of Process at -320°F	116	6.74
Effect of Process at -423°F	117	6.75
14. Multiment N-155		
Effect of Temperature, Axial Load	118	6.76
Effect of Temperature, Rotary Bending	119	6.77
Effect of Surface Treatment, Axial Load	120	6.78
Effect of Type of Loading, 1200°F	121	6.79
Effect of Type of Loading, 1350°F	122	6.80
Effect of Type of Loading, 1500°F	123	6.81
15. Multiment N1-155		
Effect of Surface Finish	124	6.82
Effect of Surface Finish	125	6.83
Effect of Surface Finish	126	6.84
16. 17-7 PH		
Effect of Stress Concentration	127	6.85
17. Duralumin		
Effect of Salt Water Corrosion	128	6.86
18. Ti-6Al-4V		
Effect of Temperature, H.T.A.	129	6.87
Effect of Temperature, H.T.B.	130	6.88
Effect of Heat Treatment, 80°F	131	6.89
Effect of Heat Treatment, 400°F	132	6.90
Effect of Heat Treatment, 600°F	133	6.91
Effect of Heat Treatment, 800°F	134	6.92
Effect of Heat Treatment, 80°F, Sm = 82-107 ksi	135	6.93
Effect of Heat Treatment, 400°F, Sm = 30-42 ksi	136	6.94
Effect of Heat Treatment, 400°F, Sm = 77-100 ksi	137	6.95
Effect of Heat Treatment, 800°F, Sm = 40-64 ksi	138	6.96
Miscellaneous-Effect of Mean Stress	139	6.97
Miscellaneous-Effect of Mean Stress	140	6.98
Miscellaneous-Effect of Mean Stress	141	6.99
Miscellaneous-Effect of Mean Stress	142	6.100
Miscellaneous-Effect of Mean Stress	143	6.101

<u>Tensile Strength</u>	<u>Page</u>
1. Low Carbon, Low Alloy Steel, (Killed)	144
2. Low-Medium Carbon, Low Alloy Steel,	145
3. Killed, Low Carbon, Low Alloy Steel,	146
4. Low-Medium Carbon, Low Alloy Steel,	147
5. Low Carbon, High Alloy Steel	148
6. Low Carbon, High Alloy Steel,	149
7. Stainless Steel (17% Cr., 12%)	150
8. Low Carbon, High Alloy Steel	151
9. Stainless Steel (25% Cr., 12% Ni, 2% C)	152
10. Stainless Steel (18% Cr., 8% Ni, .8%Ti, (.04-.09)%C)	153
11. Stainless Steel (18% Cr., 8% Ni., .8% Cb, .06%C)	154
12. Stainless Steel (18% Cr., 8% Ni, (.02-.09)%C)	155
13. Stainless Steel (18% Cr., 12% Ni, 2% Mo, .08%C)	156
14. Stainless Steel (25% Cr., 20% Ni., 2% Mn, 2 Si, .25% C)	157

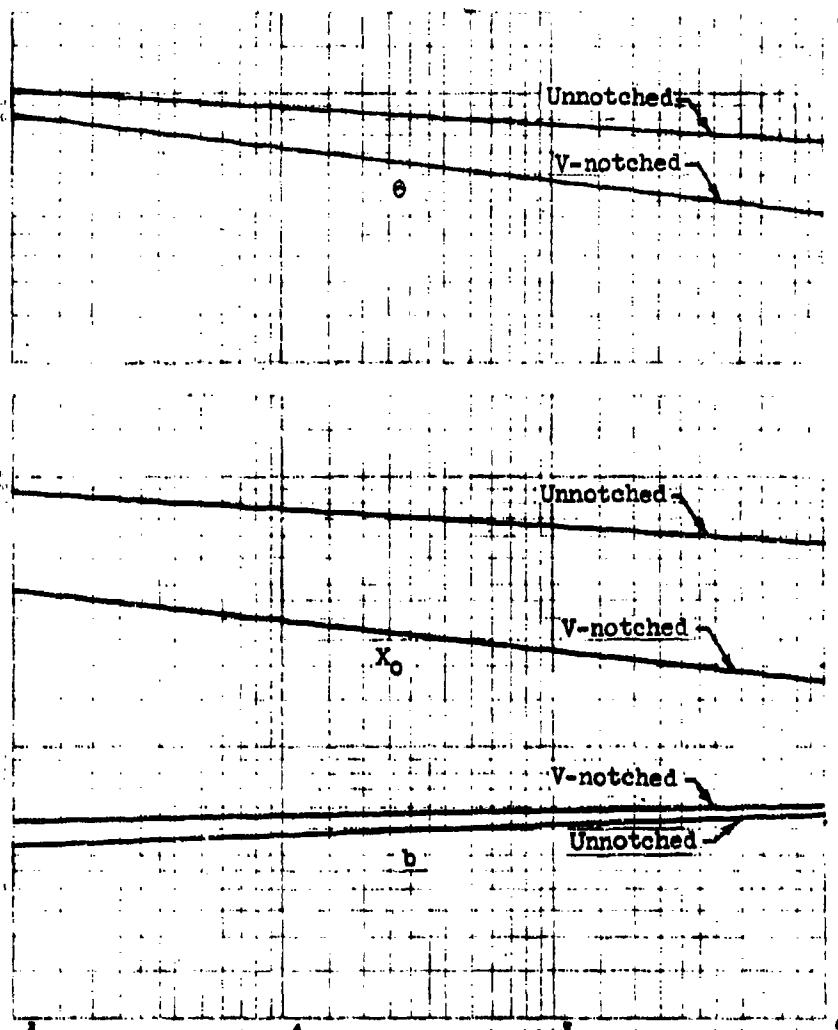
# FATIGUE STRENGTH

AISI 3140

$S_u = 108 \text{ ksi}$

$S_y = 87 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Room Temperature

Composition:

.4% C, .8% Mn, .3% Si,

1.2% Ni, .6% Cr

Hot Rolled, Lathe Turned,

Hand Polished

Mean Stress = 0

Heat Treatment:

$K_f$ : See Page 198, Item 1

Figure: 6.9

(For Tabulated Data See Page 207)

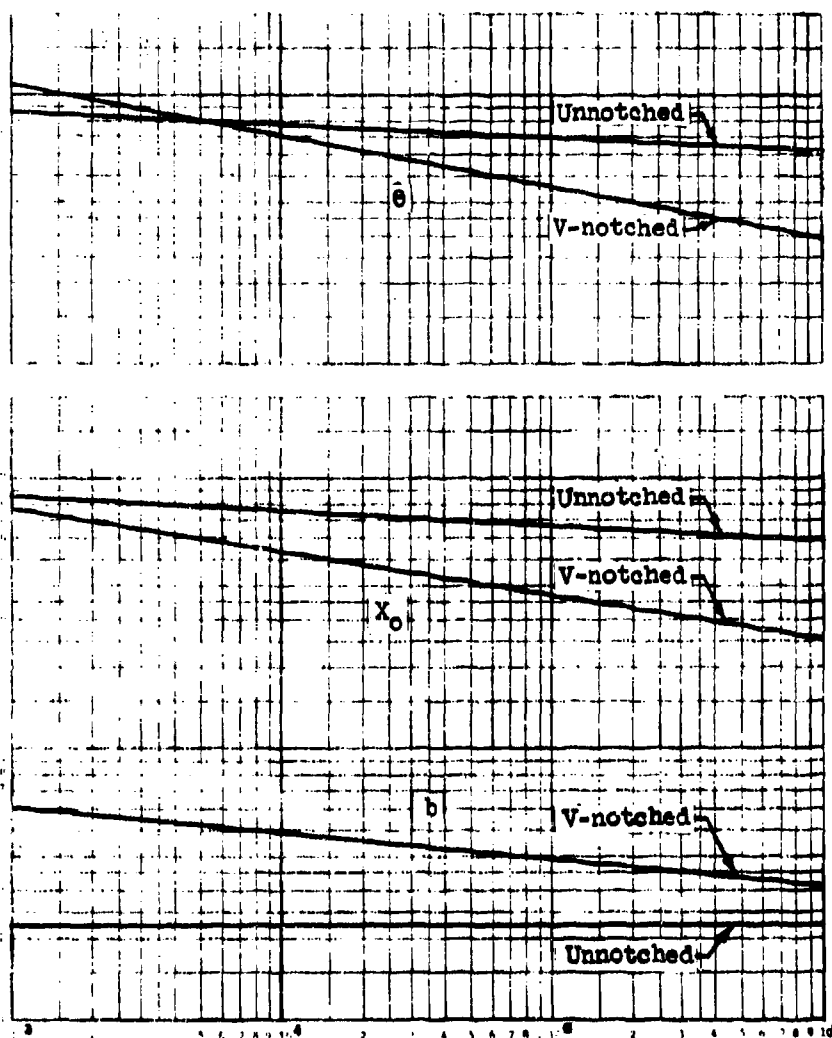
FATIGUE STRENGTH

AISI 3140

$S_u = 109 \text{ ksi}$

$S_y = 75 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Room Temperature

Composition:

.4% C, .8% Mn, .3% Si,

1.2% Ni, .65% Cr

Hot Rolled, Lathe Turned,

Hand Polished

Mean Stress = 0

Heat Treatment:

$K_{11}$ : See Page 198, Item 1

Figure: 6.10 (For Tabulated Data See Page 207)

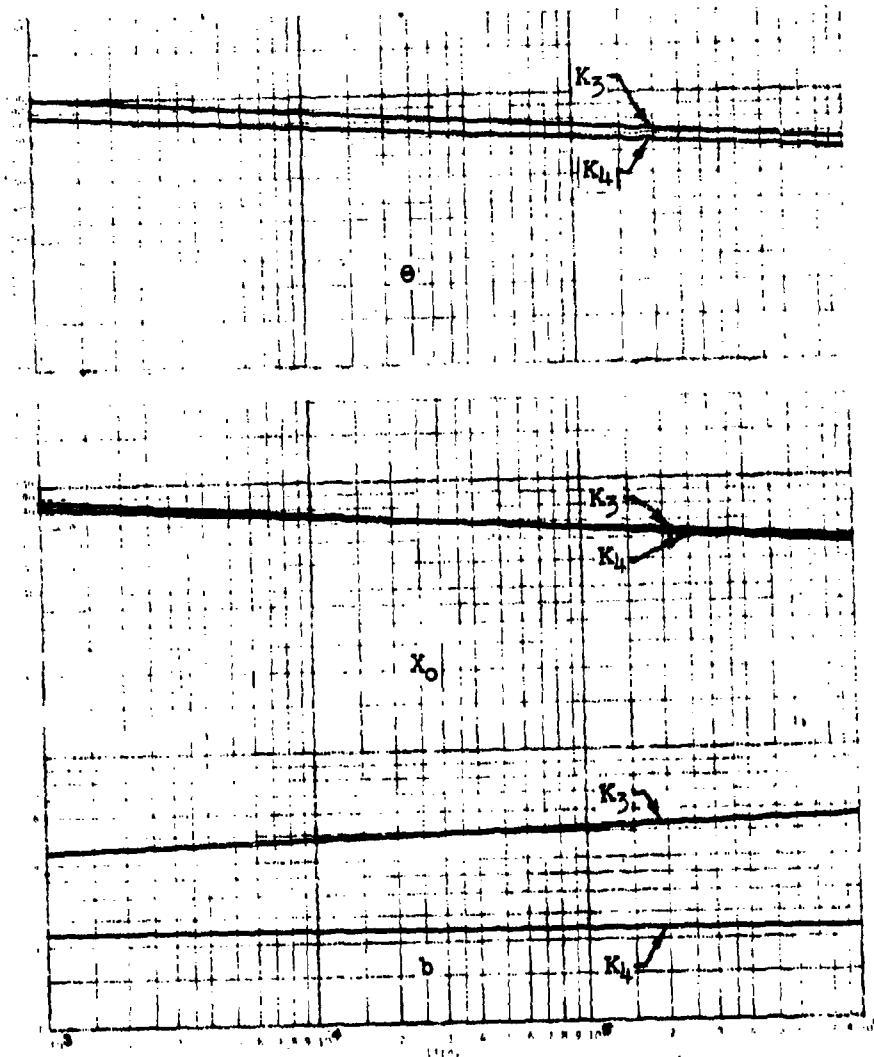
# FATIGUE STRENGTH

AISI 3140

$S_u = 108, 109 \text{ ksi}$

$S_y = 87, 75 \text{ ksi}$

Effect of Heat Treatment



Rotary Beam Bending  
Unnotched

Composition:

.4% C, .8% Mn, .3% Si

1.2% Ni, .5% Cr

Hot Rolled, Lathe Turned,  
Hand Polished

Mean Stress = 0

Heat Treatment:

K3 & K4: See Page 198, Item 1

Figure: 6.11 (For Tabulated Data See Page 207)

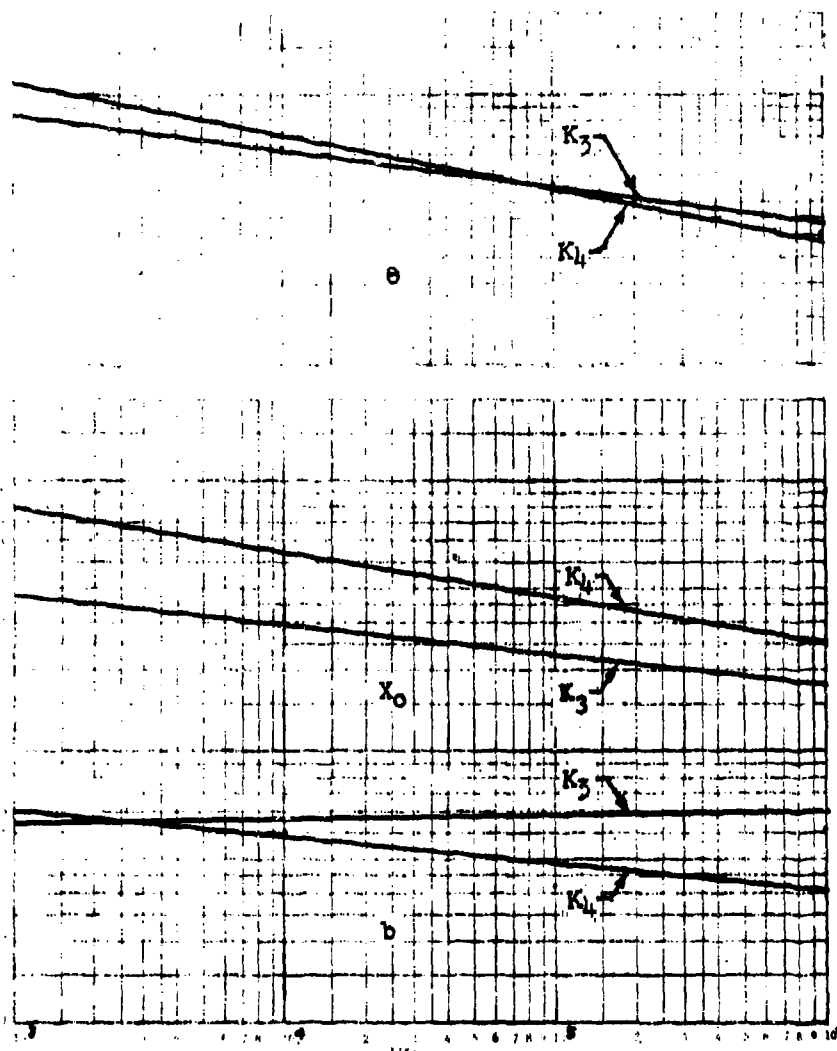
# FATIGUE STRENGTH

ASTM A3140

$\sigma_u = 108, 109 \text{ ksi}$

$\sigma_y = 97, 75 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending  
V-Notched

Composition:

.4% C, .8% Mn, .3% Si  
1.2% Ni, .65% Cr

Figure: 6.12

Hot Rolled, Lathe Turned,  
Hand Polished

Mean Stress = 0

Heat Treatment:

K3 & K4: See Page 198, Item 1

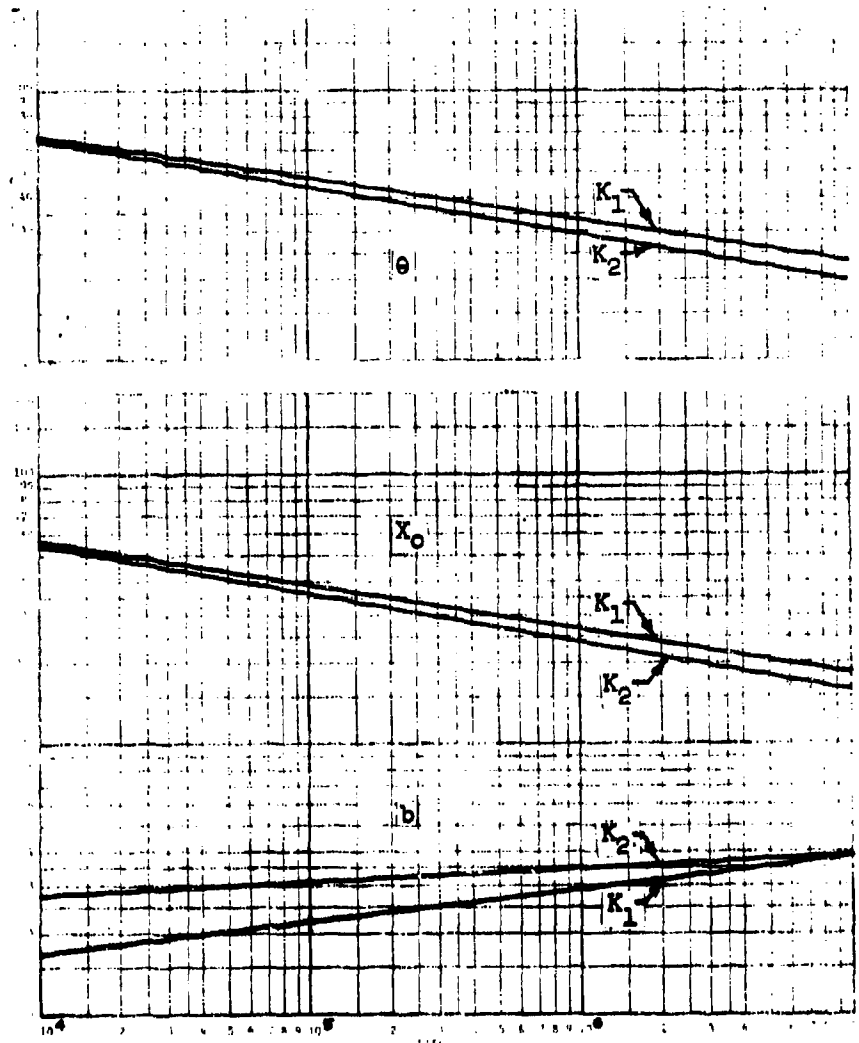
(For Tabulated Data See Page 207)

STRESS-STRAIN  
RELATIONS

AISI 1045 Steel

$S_u$ :  $K_1 = 105 \text{ ksi}$ ,  $K_2 = 120 \text{ ksi}$

Effect of Heat Treatment



Rotary Beam Bending  
Vee Notch, Flank Angle =  $60^\circ$   
Composition:  
.43-.50% C, .60-.90% Mn,  
.040% max. P, .05% max. S

Hot Rolled, Lathed,  
Hand Polished  
Mean Stress = 0  
Heat Treatment:

Figure: 6.13

See Page 198, Item 2  
(For Tabulated Data See Page 208)

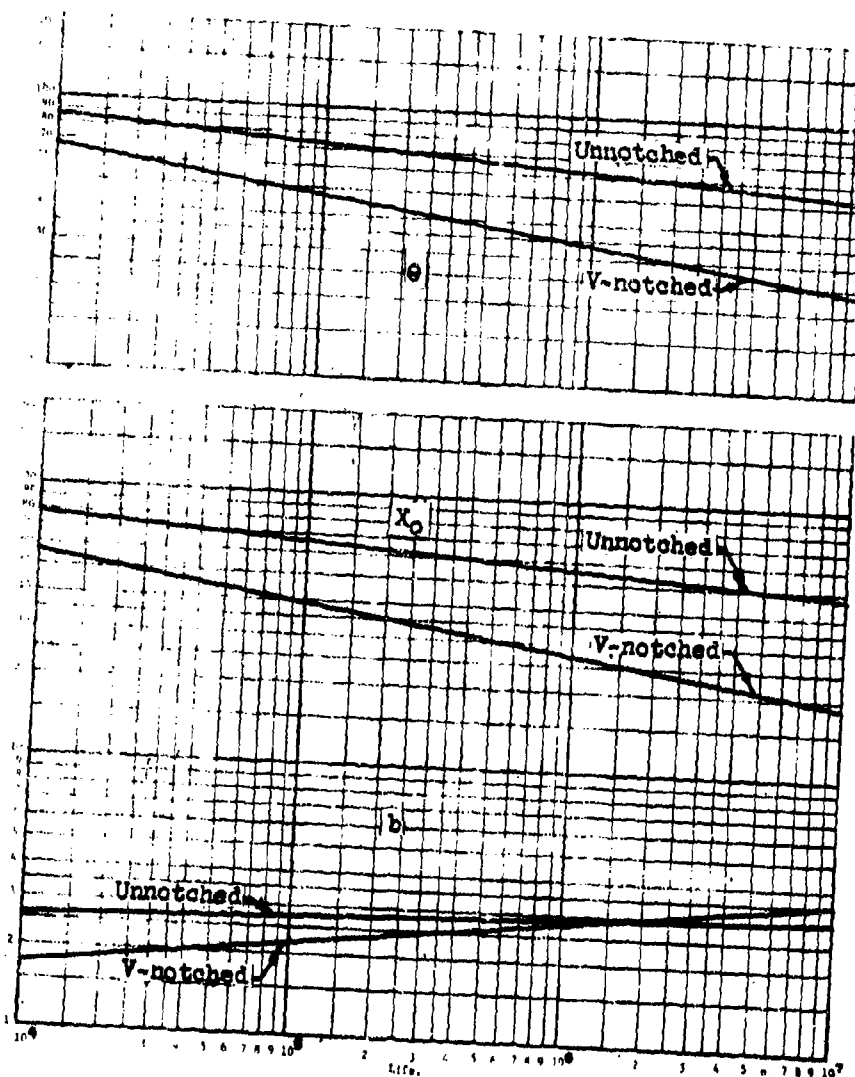
AISI 1045 Steel

FATIGUE STRENGTH

$S_u = 105 \text{ ksi}$

$S_y = 84 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Room Temperature

Composition:

.43-.50% C, .60-.90% Mn

.040 max. P, .05% max. S

Figure: 6.14

Hot Rolled, Lathed

Polished

Mean Stress = 0

Heat Treatment:

$K_1$ : See Page 198, Item 2

(For Tabulated Data See Page 208)

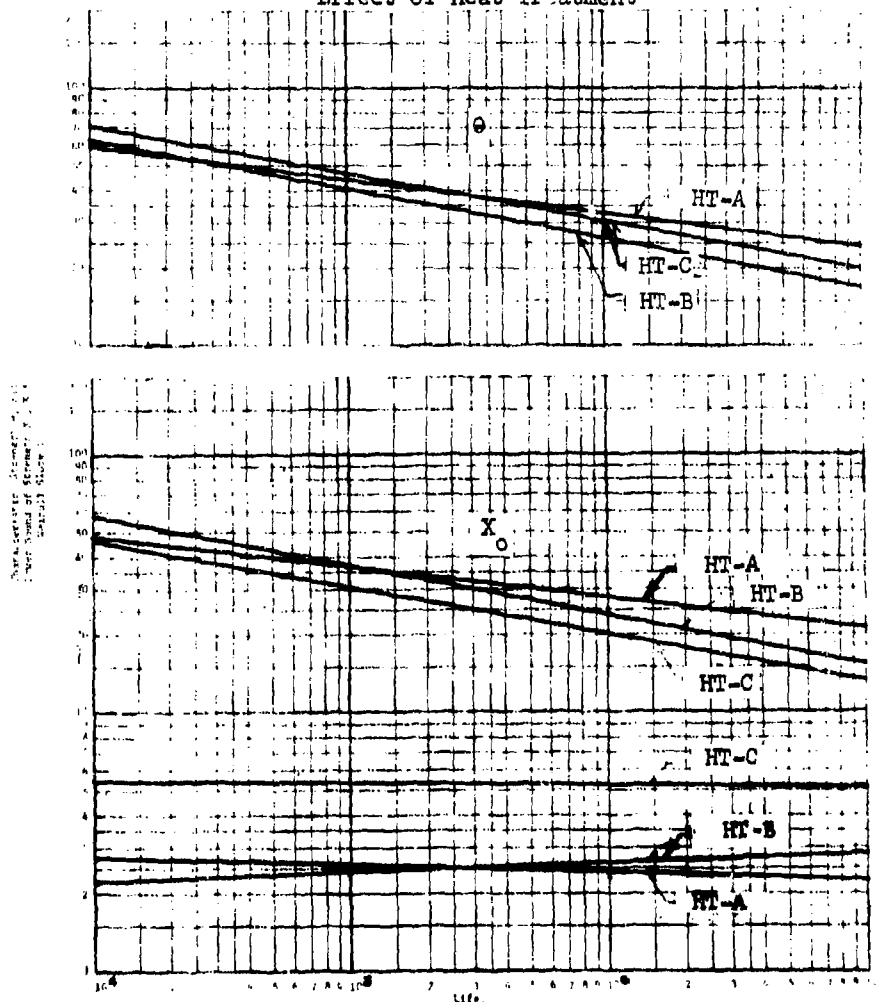


ALSI 2340 Steel

# FATIGUE STRENGTH

$S_u = 116 - 122 \text{ ksi}$ ,  $S_y = 76 - 96 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Hot Rolled, Lathe Turned

V-notched

Hand Polished

Composition:  
Unknown

Mean Stress = 0  
Heat Treatment: See Page 198,  
Item 4

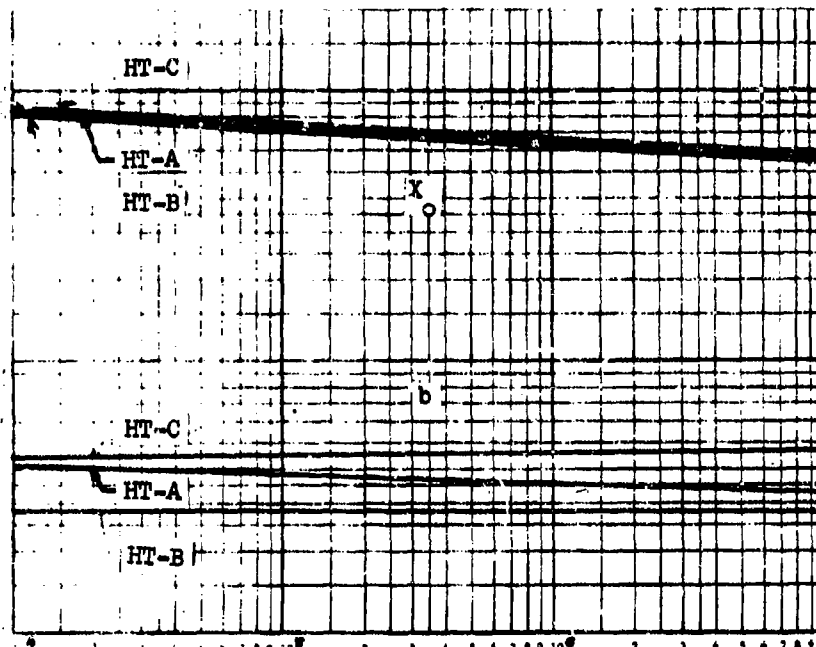
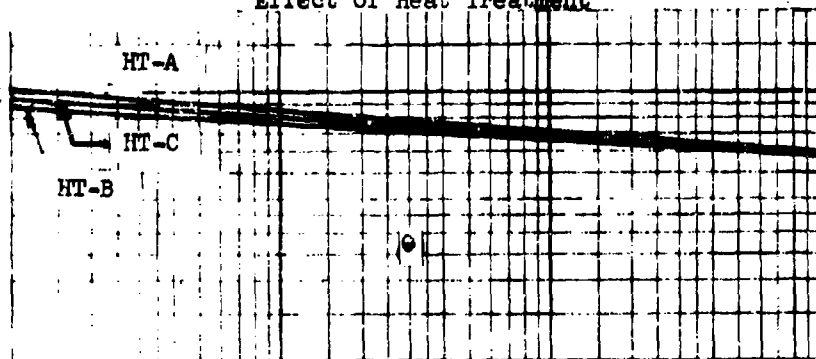
Figure: 6.15 (For Tabulated Data See Page 210 )

AlSi 23<sup>4</sup>, Steel

# FATIGUE STRENGTH

$S_u = 116 - 122 \text{ ksi}$ ,  $S_y = 76 - 96 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Hot Rolled, Lathe Turned

Unnotched

Hand Polished

Composition:  
Unknown

Mean Stress = 0

Heat Treatment: See Page 198,  
Item 4

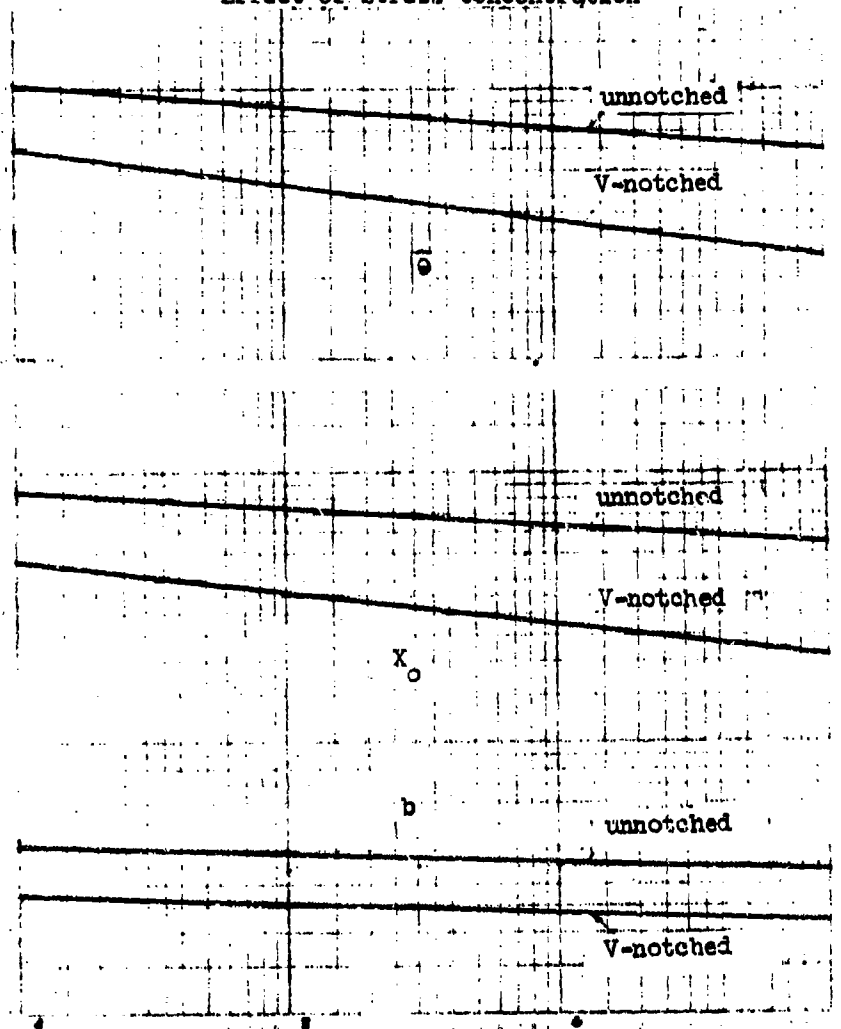
Figure: 6.16 (For Tabulated Data See Page 210)

AlSi 2340 Steel

FATIGUE STRENGTH

$S_u = 116 \text{ ksi}$ ,  $S_y = 96 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Hot Rolled, Lathe Turned

Composition:  
Unknown

Hand Polished  
Mean Stress = 0  
Heat Treatment:  
A = oil quenched from 1450°F,  
tempered at 1200°F

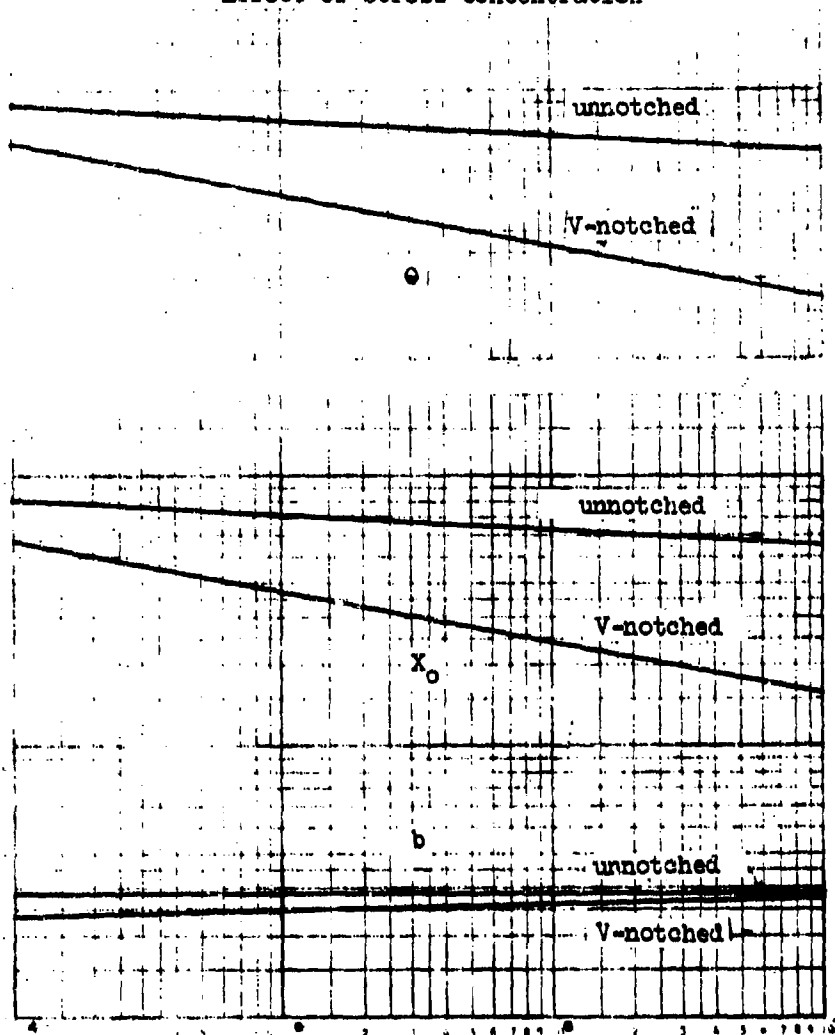
Figure:6.17 (For Tabulated Data See Page 210)

ALSI 2340 Steel

# FATIGUE STRENGTH

$S_u = 119 \text{ ksi}$ ,  $S_y = 79 \text{ ksi}$

## Effect of Stress Concentration



Rotary Beam Bending

Composition:  
Unknown

Hot Rolled, Lathe Turned,  
Hand Polished

Mean Stress = 0

Heat Treatment: B: air blast  
quenched from 1450°F, tempered  
at 700°F

Figure: 6.18 (For Tabulated Data See Page 210.)

AlSi 2340 Steel

# FATIGUE STRENGTH

$S_u = 122 \text{ ksi}$ ,  $S_y = 76 \text{ ksi}$

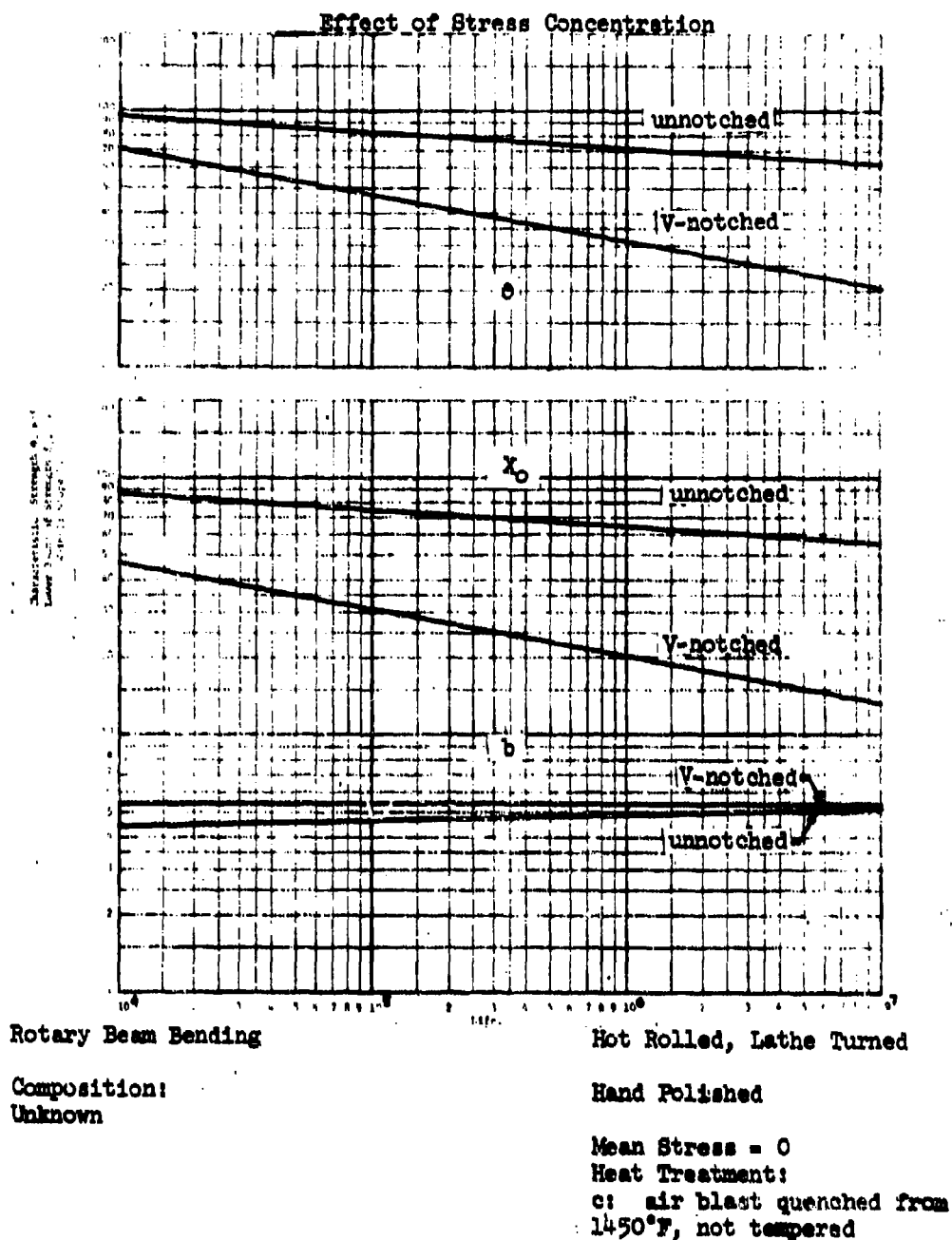


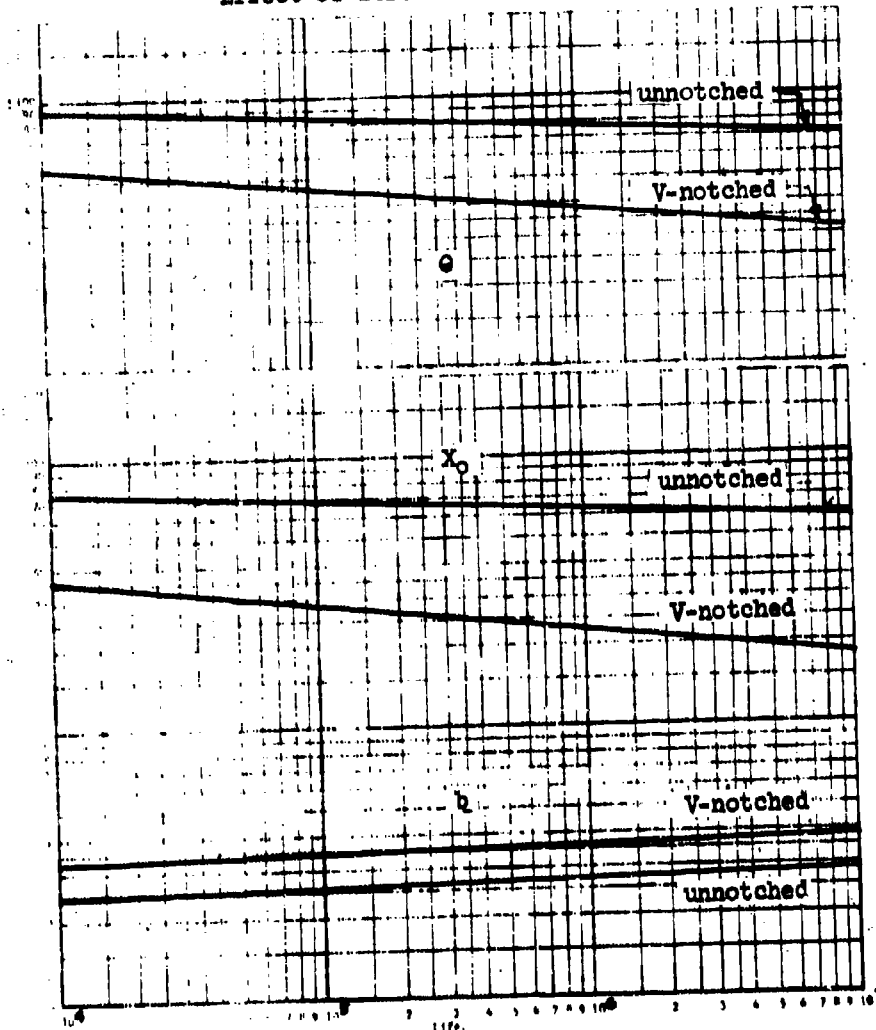
Figure: 6.19 (For Tabulated Data See Page 210)

4140 Steel

FATIGUE STRENGTH

$S_u = 135 \text{ ksi}$ ,  $S_y = 122 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Mean Stress = 0

Stress Conc. Factor  $K_t = 1.0$  for unnotched  
 $K_t = \text{unknown}$  for V-notched

Heat Treatment:  
 Aust.  $1550^\circ\text{F}$

Composition:  
 Standard 4140

1 hr, OQ, Temp.  $1230^\circ\text{F}$  1 hr.

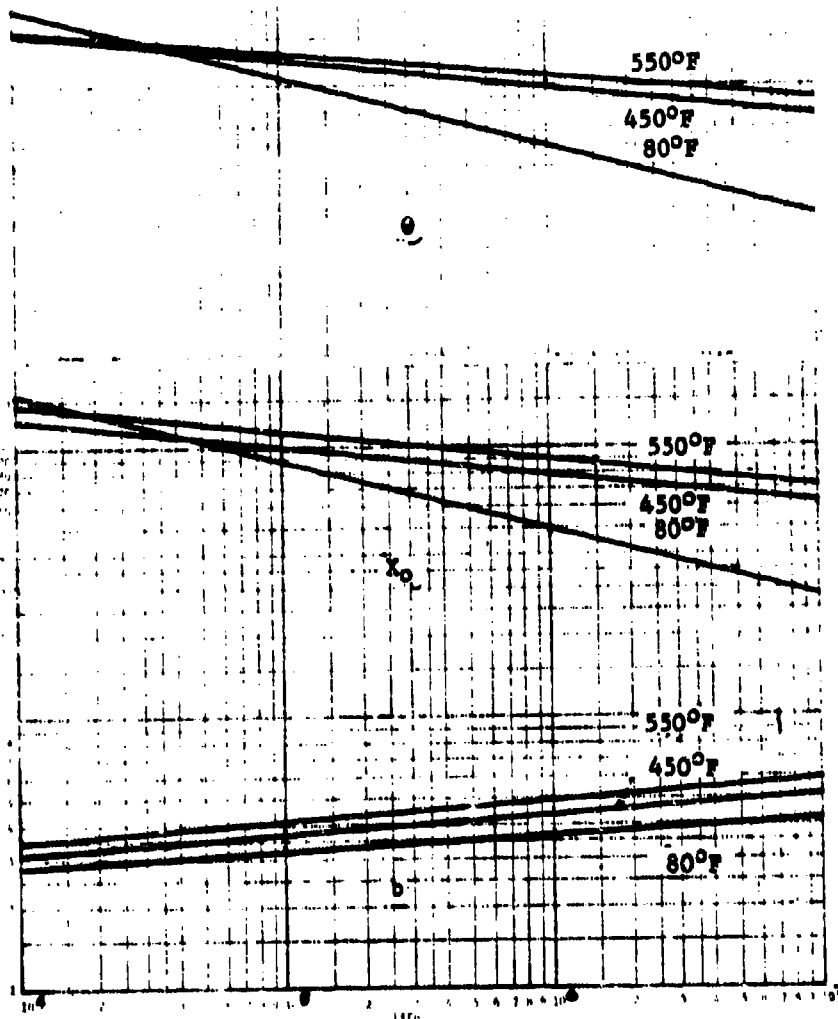
Figure: 6.20 (For Tabulated Data See Page 210)

D6AC Steel

# FATIGUE STRENGTH

$S_u = 270 \text{ ksi}$ ,  $S_y = 237 \text{ ksi}$

## Effect of Temperature



Axial Load, Completely Reversed

Hot Rolled, Polished

Stress Conc. Factor  $K_t = 1.0$

Mean Stress = 0

Composition: See Page 198, Item 6

Heat Treatment:

See Page 198, Item 6

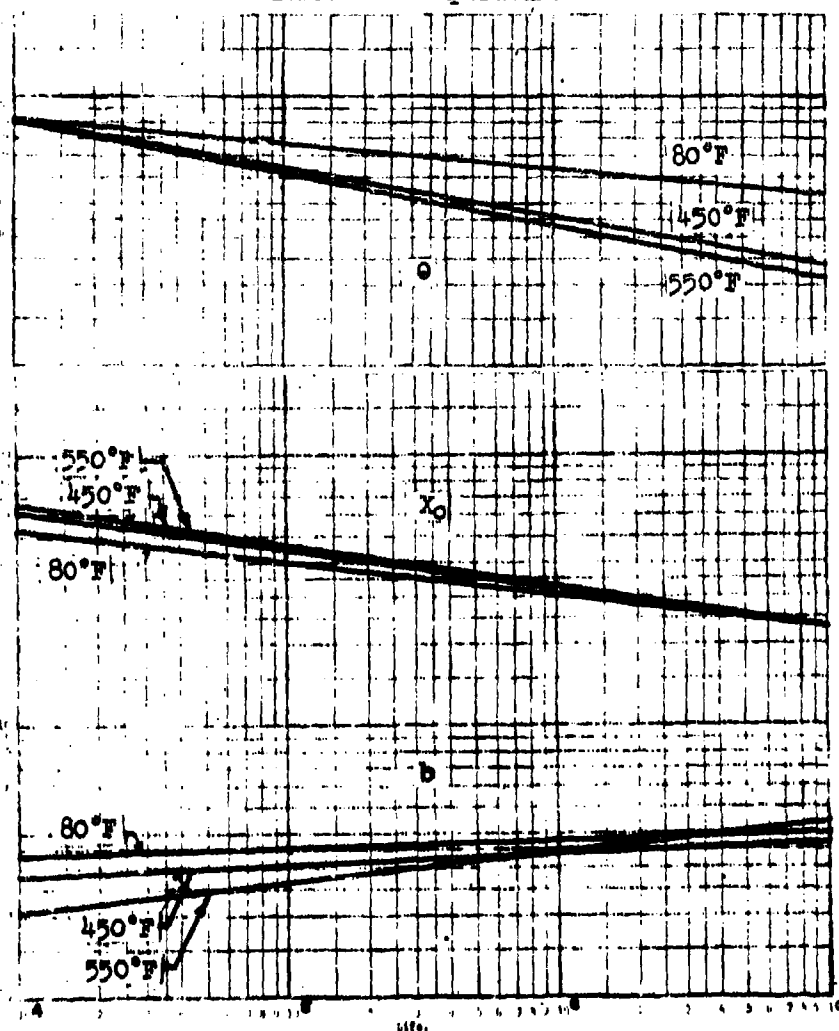
Figure: 6.21 (For Tabulated Data See Page 213)

D6AC Ladish Steel

# FATIGUE STRENGTH

$S_u = 270 \text{ ksi}$ ,  $S_y = 237 \text{ ksi}$

## Effect of Temperature



Axial Load, Completely Reversed

Hot Rolled, Polished

Stress Conc. Factor  $K_t = 3.0$

Mean Stress = 0

Composition, See Page 198, Item 6.

Heat Treatment:  
See Page 198, Item 6.

Figure 6.22 (For Tabulated Data See Page 213)

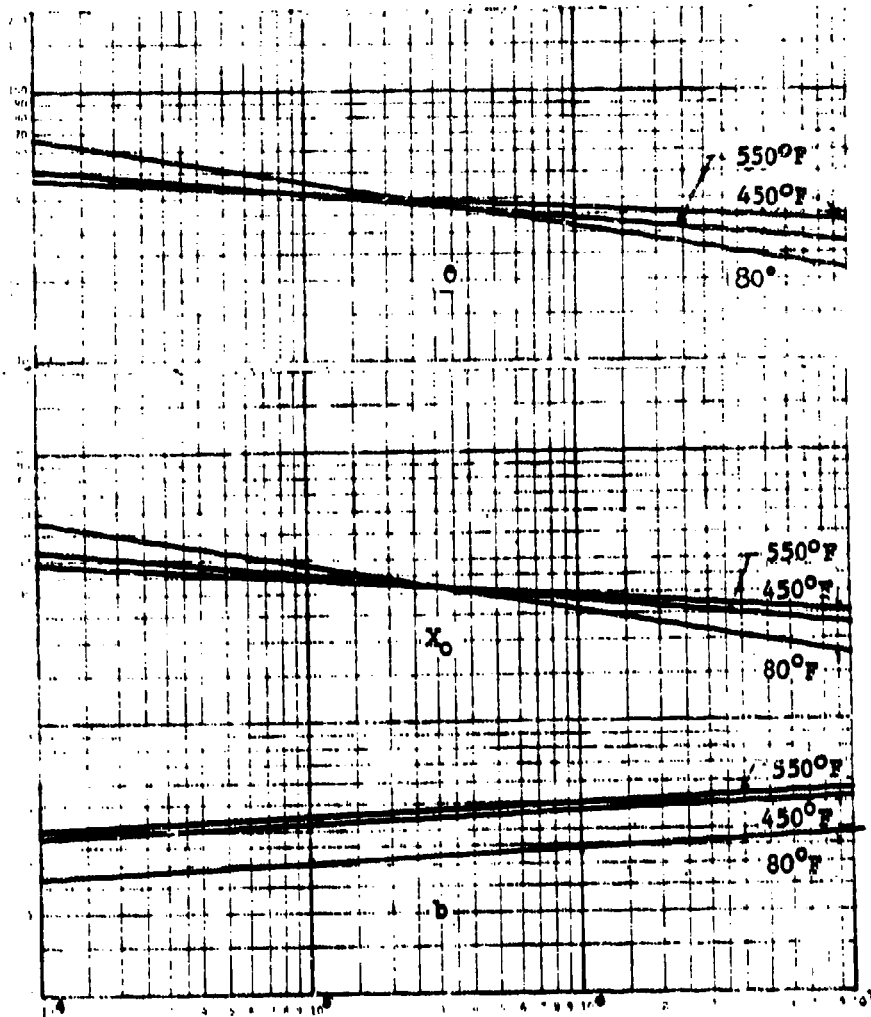


D6AC Ladish Steel

FATIGUE STRENGTH

$S_u = 270$  ksi,  $S_y = 237$  ksi

Effect of Temperature



Axial Load

Hot Rolled, Polished

Stress Conc. Factor  $K_t = 3.0$

Mean Stress = 30-50 ksi

Composition\ See Page 198, Item 6

Heat Treatment:  
See Page 198, Item 6

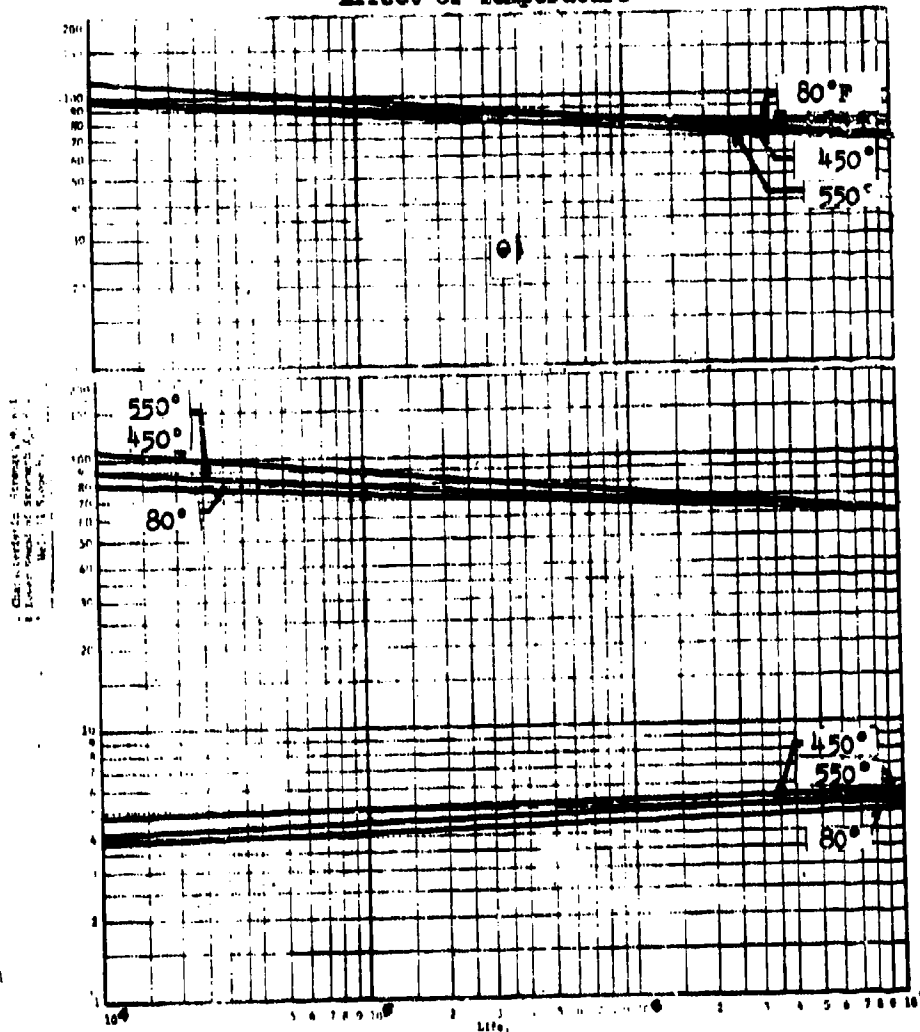
Figure:6.23 (For Tabulated Data See Page 213)

D6AC Ladish Steel

# FATIGUE STRENGTH

$S_u = 270 \text{ ksi}$ ,  $S_y = 237 \text{ ksi}$

## Effect of Temperature



Axial Loaded

Stress Conc. Factor  $K_t = 1.0$

Composition: See Page 198, Item 6

Hot Rolled, Polished

Mean Stress = 70-80 ksi

Heat Treatment:  
See Page 198, Item 6

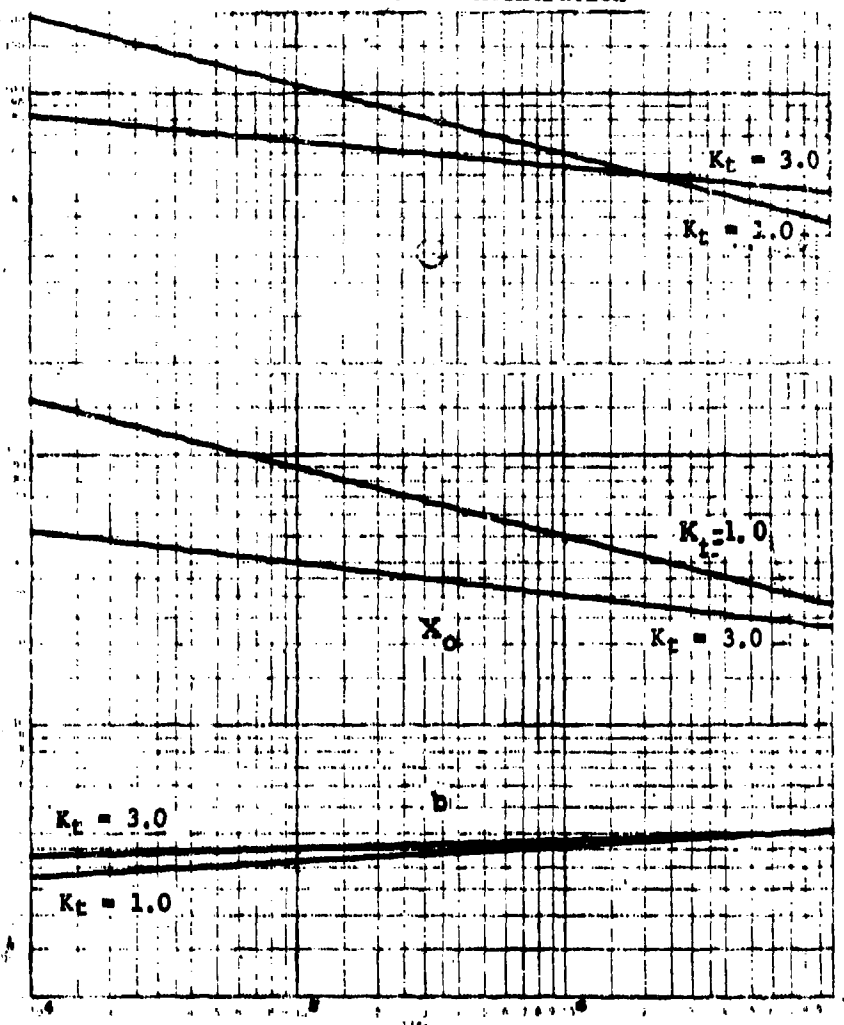
Figure: 6.24 (For Tabulated Data See Page 213)

D6AC Ladish Steel

FATIGUE STRENGTH

$S_u = 270 \text{ ksi}$ ,  $S_y = 237 \text{ ksi}$

Effect of Stress Concentration



Axial Load, Completely Reversed

Hot Rolled, Polished

Temperature = 80°F

Mean Stress = 0

Compositior See Page 198, Item 6

Heat Treatment:  
See Page 198, Item 6

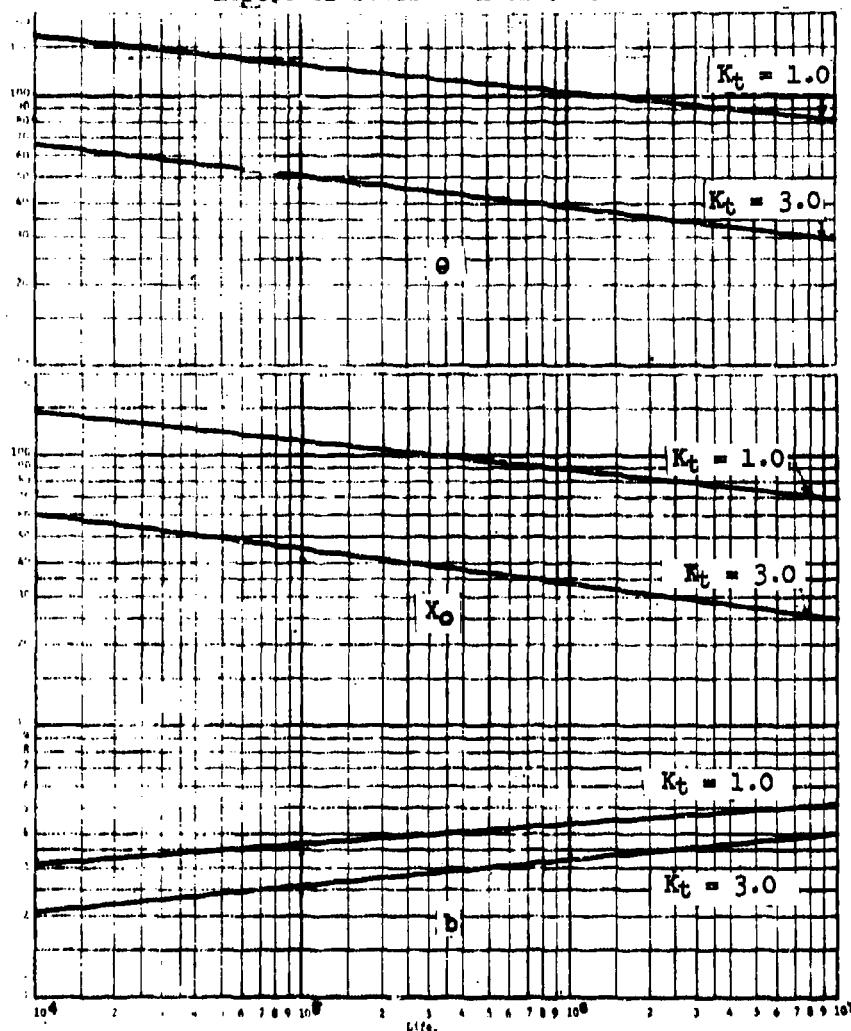
Figure: 6.25 (For Tabulated Data See Page 213)

D6AC Ladish Steel

# FATIGUE STRENGTH

$S_u = 270 \text{ ksi}$ ,  $S_y = 237 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Hot Rolled, Polished

Temperature = 450°F

Mean Stress = 0

Composition: See Page 198, Item 6

Heat Treatment:  
See Page 198, Item 6

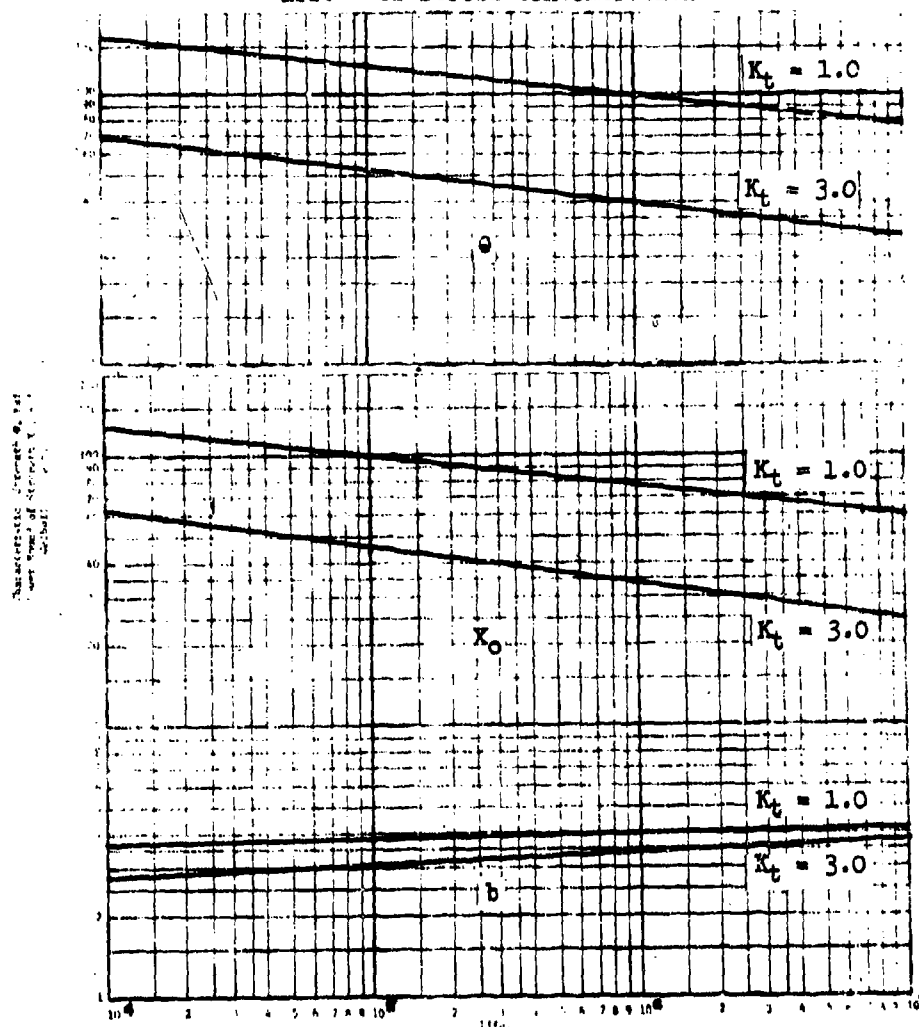
Figure: 6.26 (For Tabulated Data See Page 213)

D6AC Ladish Steel

# FATIGUE STRENGTH

$s_u = 270$  ksi.  $s_y = 237$  ksi

## Effect of Stress Concentration



Axial Load, Completely Reversed

Hot Rolled, Polished

Temperature = 550°F

Mean Stress = 0

Composition: See Page 198, Item 6

Heat Treatment:  
See Page 198, Item 6

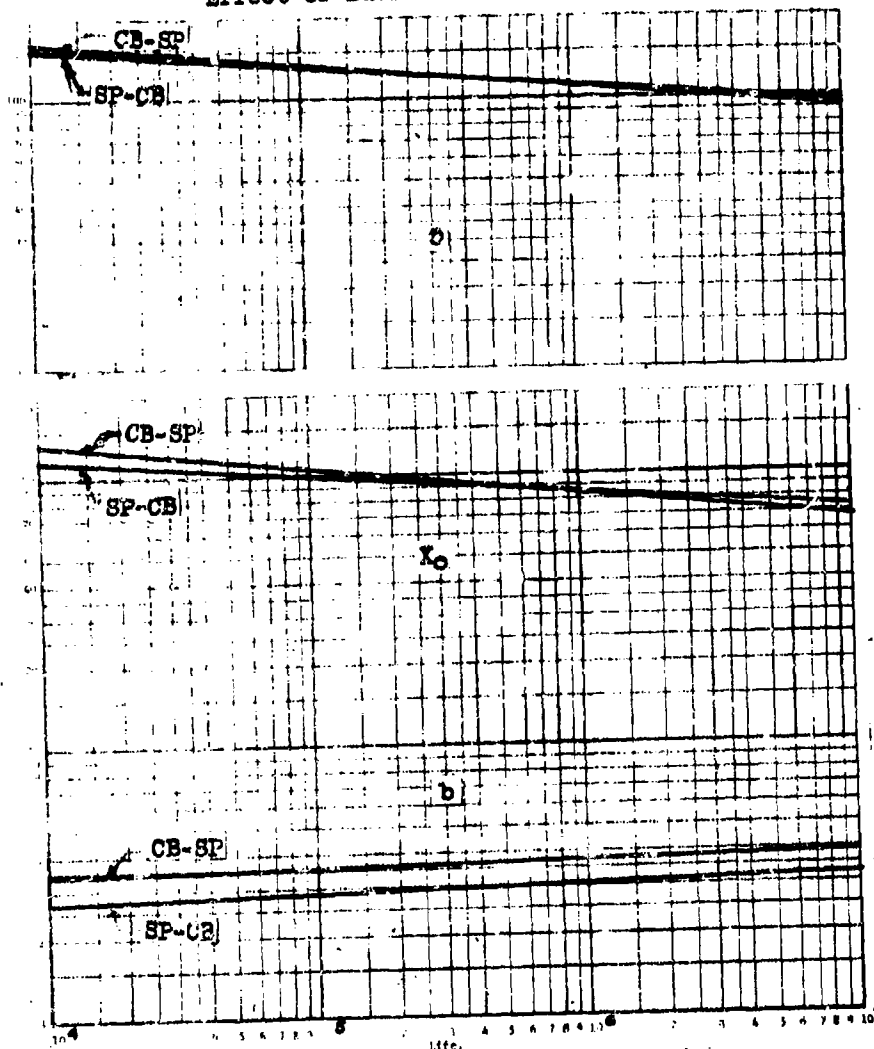
Figure: 6.27 (For Tabulated Data See Page 213)

# FATIGUE STRENGTH

A-11 Steel

$S_u = 272 \text{ ksi}$      $S_y = 226 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction is Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46,

No (Pretest) Conditioning

Initial Heat Treatment:

see page 199, Item 7

Figure: 6.28 (For Tabulated Data See Page 214)

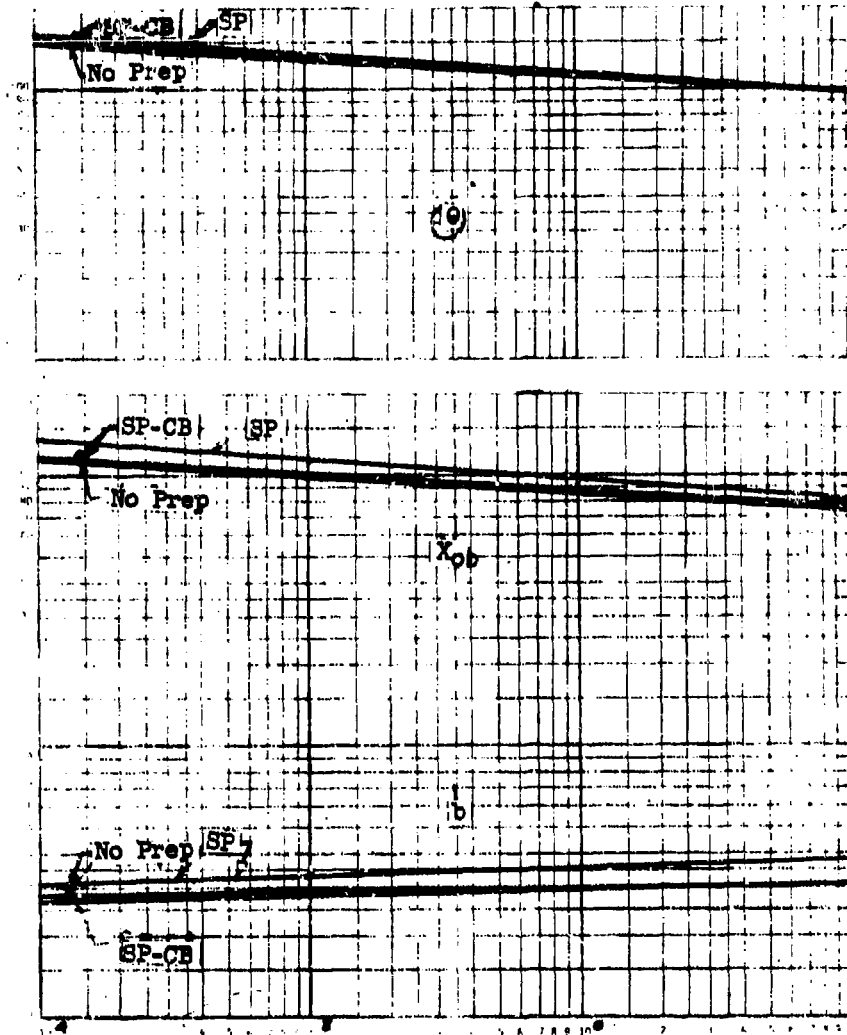
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.50% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46

No Pre-Test Conditioning

Initial Heat Treatment:

See Page 199, Item 7

Figure: 6.29 (For Tabulated Data See Page 214 )

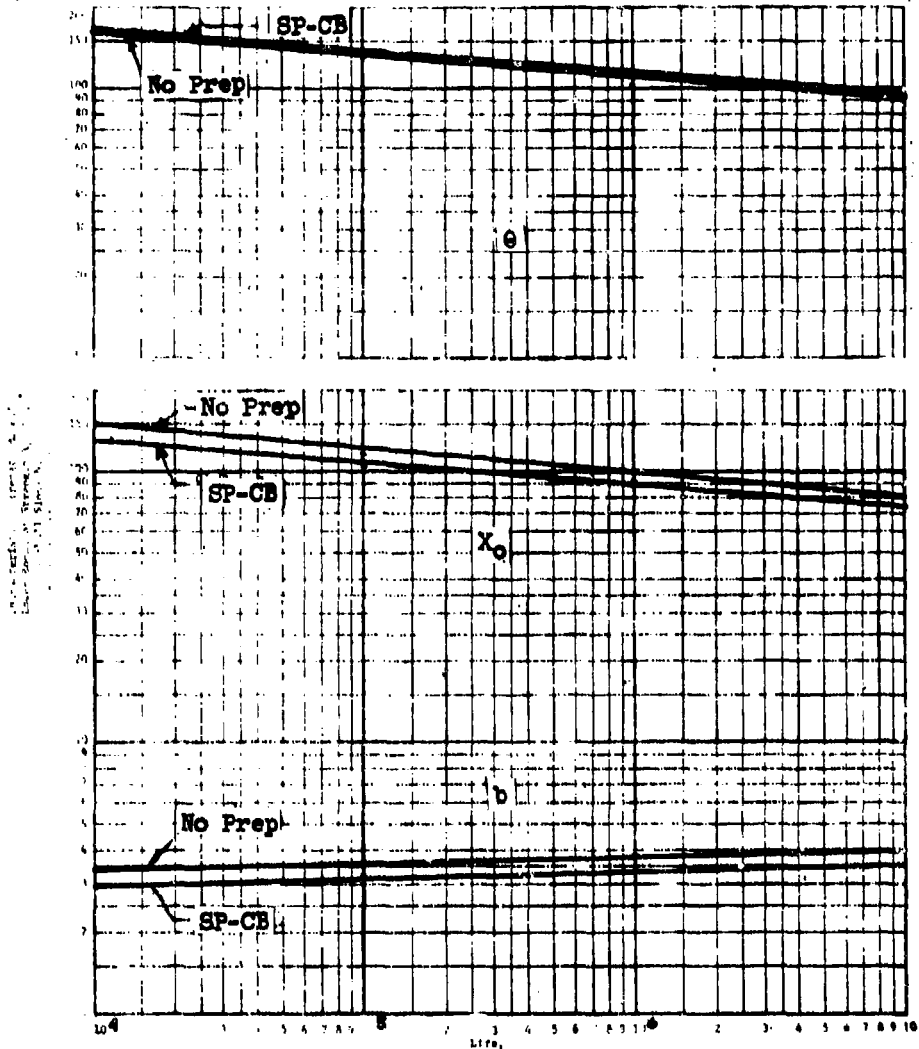
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46

Exposed 4 hr. at 375°F

Initial Heat Treatment:

See Page 199, Item 7

Figure: 6430 (For Tabulated Data See Page 214)



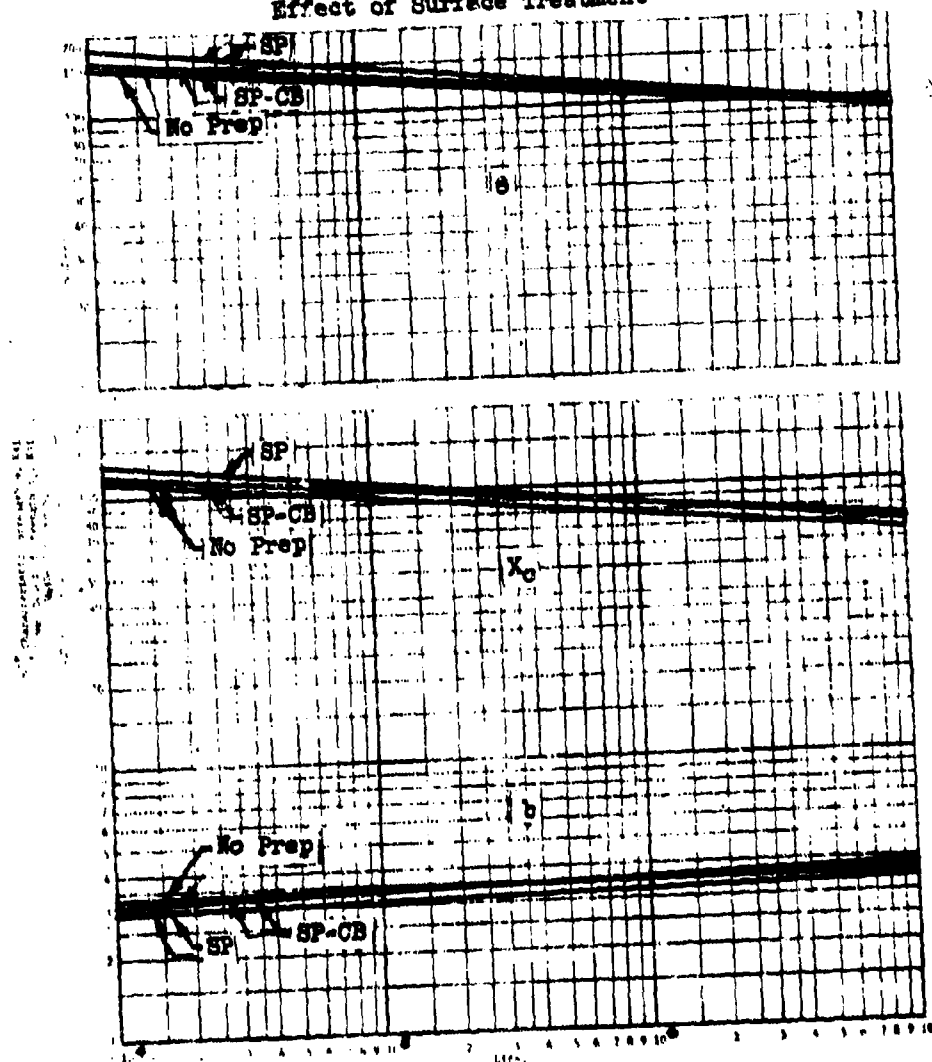
# FATIGUE STRENGTH

R-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46

Exposed 4 hr. at 500°F

Initial Heat Treatment:

See Page 199, Item 7

Figure: 6.31 (For Tabulated Data See Page 214)

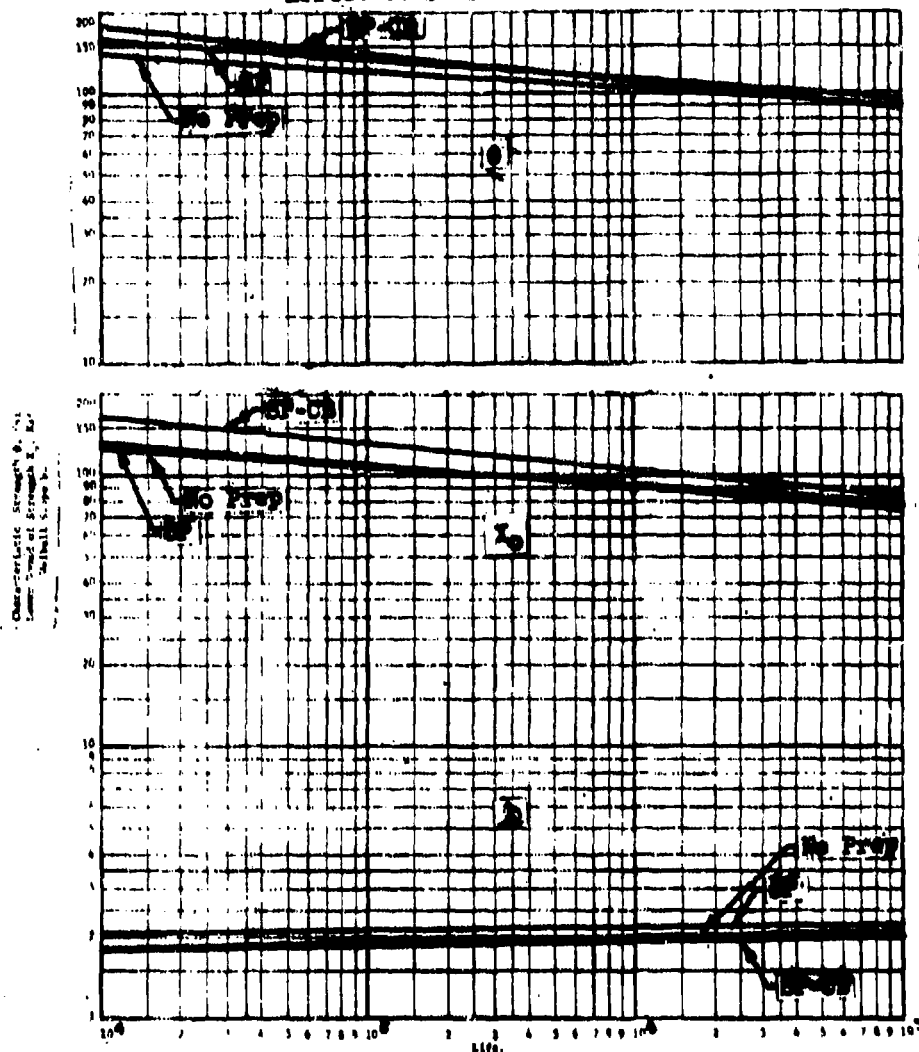
# FATIGUE STRENGTH

R-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

7% Cr, 1.7% Mn, .4% V, .33% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46

Exposed 4 hr. at 750°F

Initial Heat Treatment:

See Page 199, Item 7

Figure 8.12 (For Tabulated Data See Page 214)

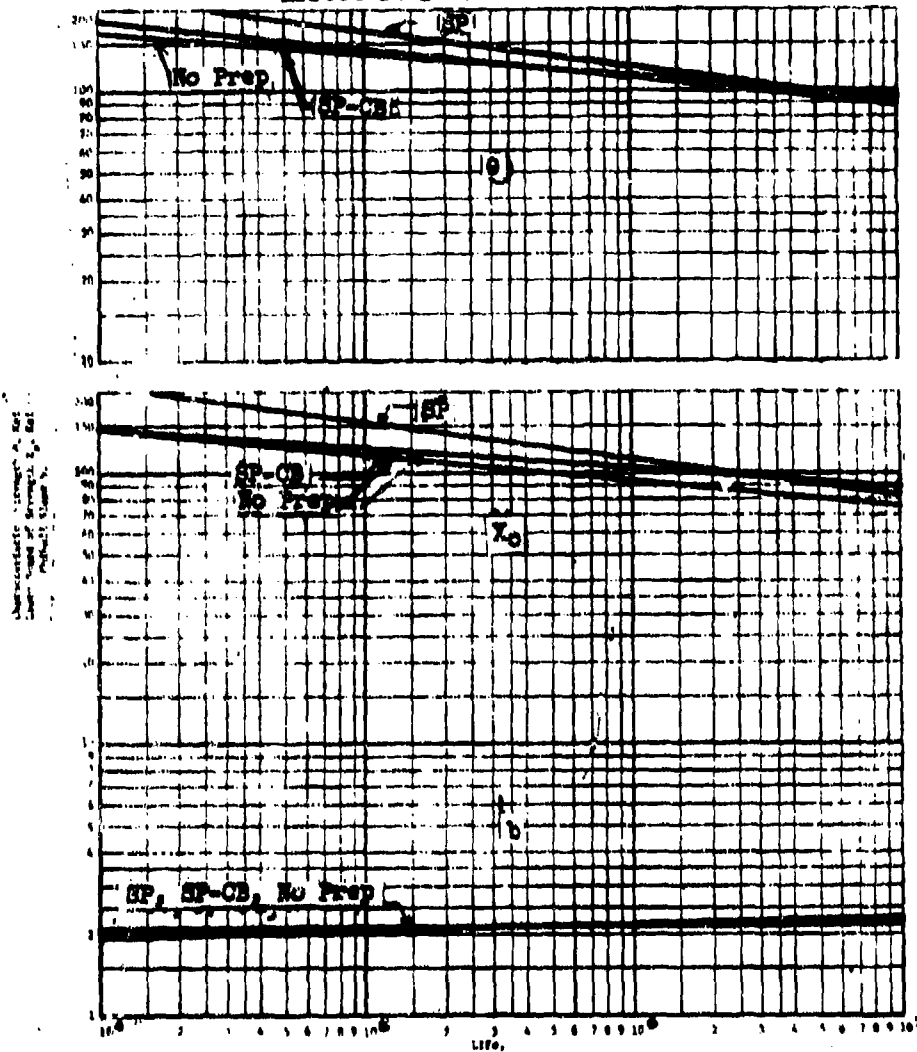
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

## Effect of Surface Treatment



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mn, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment Code:

See Page 46

Exposed 4 hr. at 1000°F

Initial Heat Treatment:

See Page 199, Item 77

Figure: 6.33 (For Tabulated Data See Page 215)

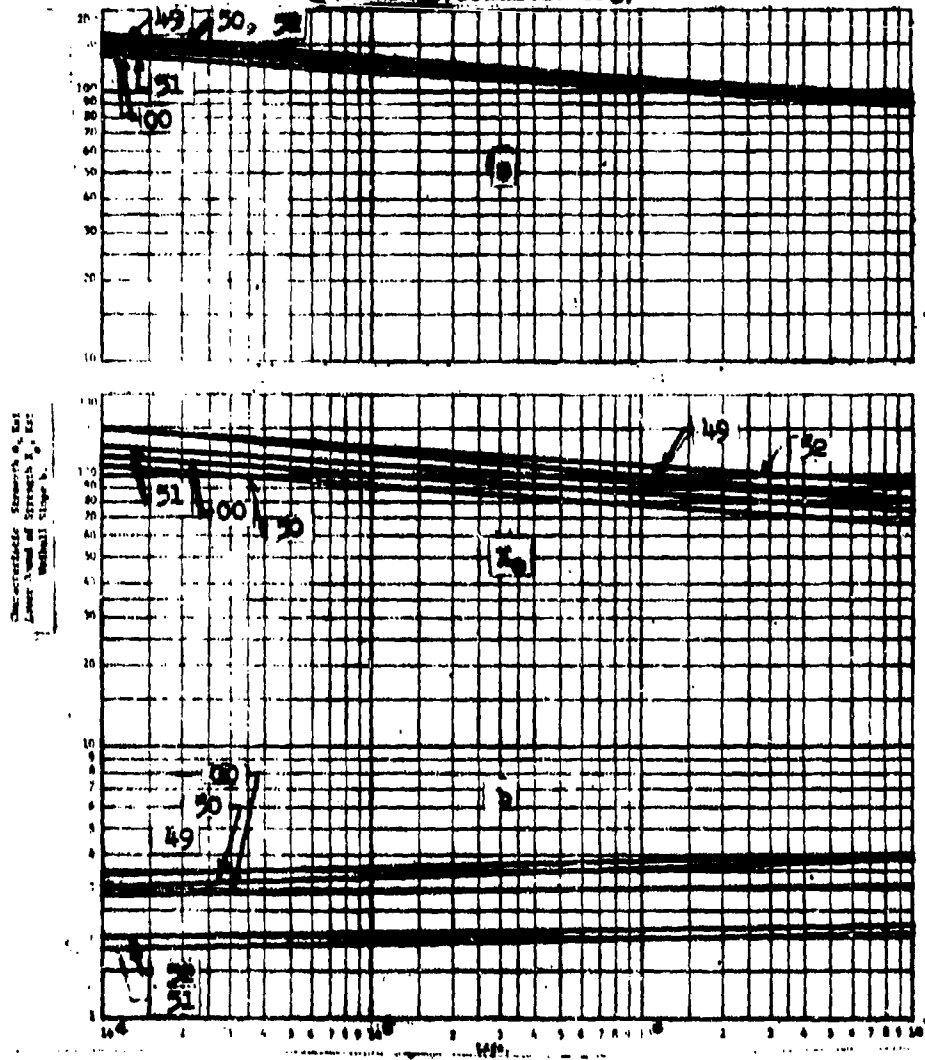
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

Effect of Heat Treatment  
(Pretest Conditioning)



Rotary Beam Bending

Mean Stress = 0

Composition:

9% Cr, 1.5% Mo, .4% V, .33% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Longitudinal Axis

Surface Treatment:

No Preparation—Mechanical Polish

Pretest Conditioning Codes:

See Page 46

Initial Heat Treatment:

See Page 199, Item 7

Figure 6.38 (For Tabulated Data See Page 213)

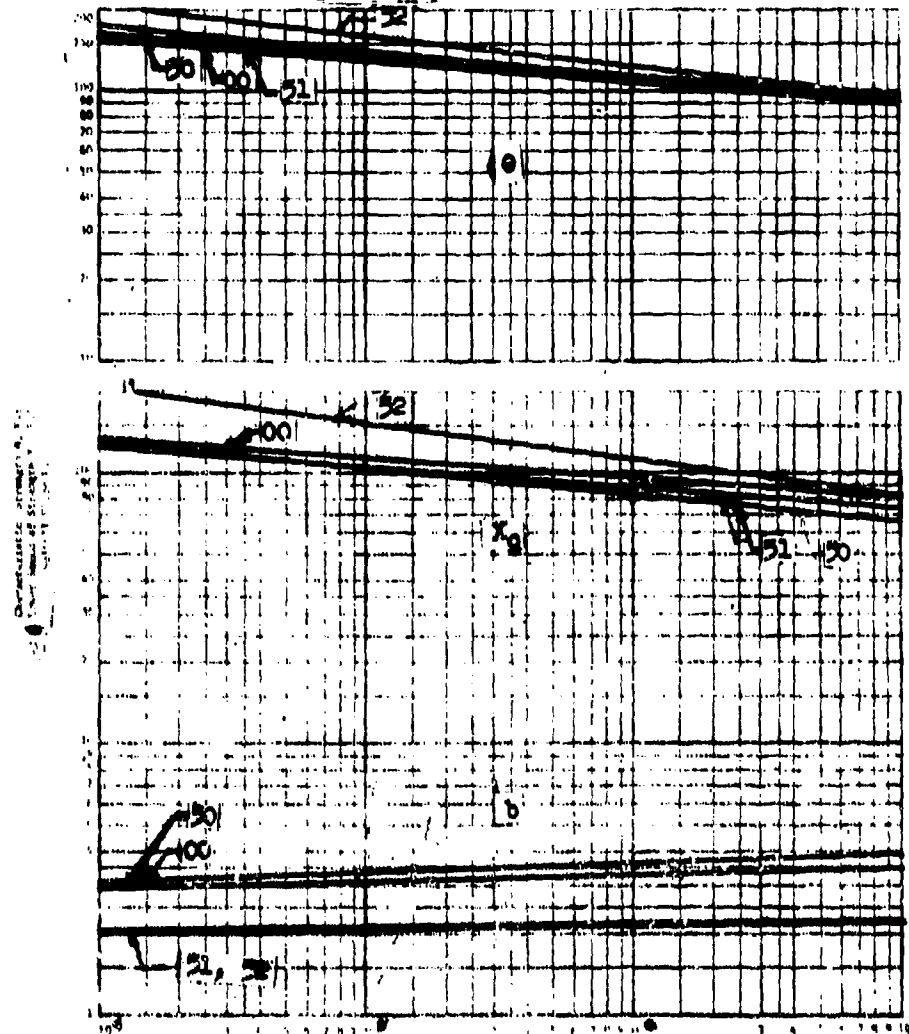
# FATIGUE STRENGTH

M-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

Effect of Heat Treatment  
(Pretest Conditioning)



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned

Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment:

Shot-Peened—Mechanical Polish

Pretest Conditioning Code:

See Page 46

Initial Heat Treatment:

See Page 199, Item 7

See Page 215

Figure: 6.35 (For Tabulated Data See Page 215)

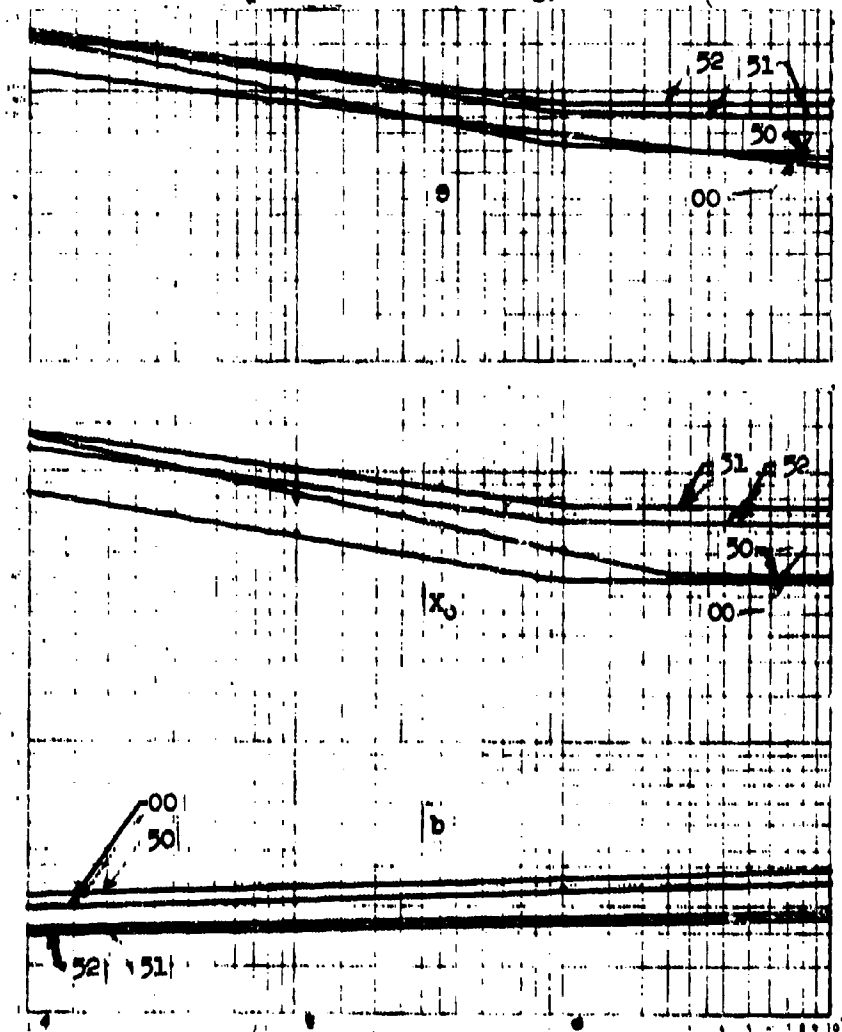
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

Effect of Heat Treatment  
(Pretest Conditioning)



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned

Grain Direction Transverse to

Lengthwise Axis

Surface Treatment:

Chrome Plated--Chrome Baked

Pretest Conditioning Code:

See Page 46

Initial Heat Treatment

See Page 199, Item 7

Figure: 6.38

(For Tabulated Data See Page 215)

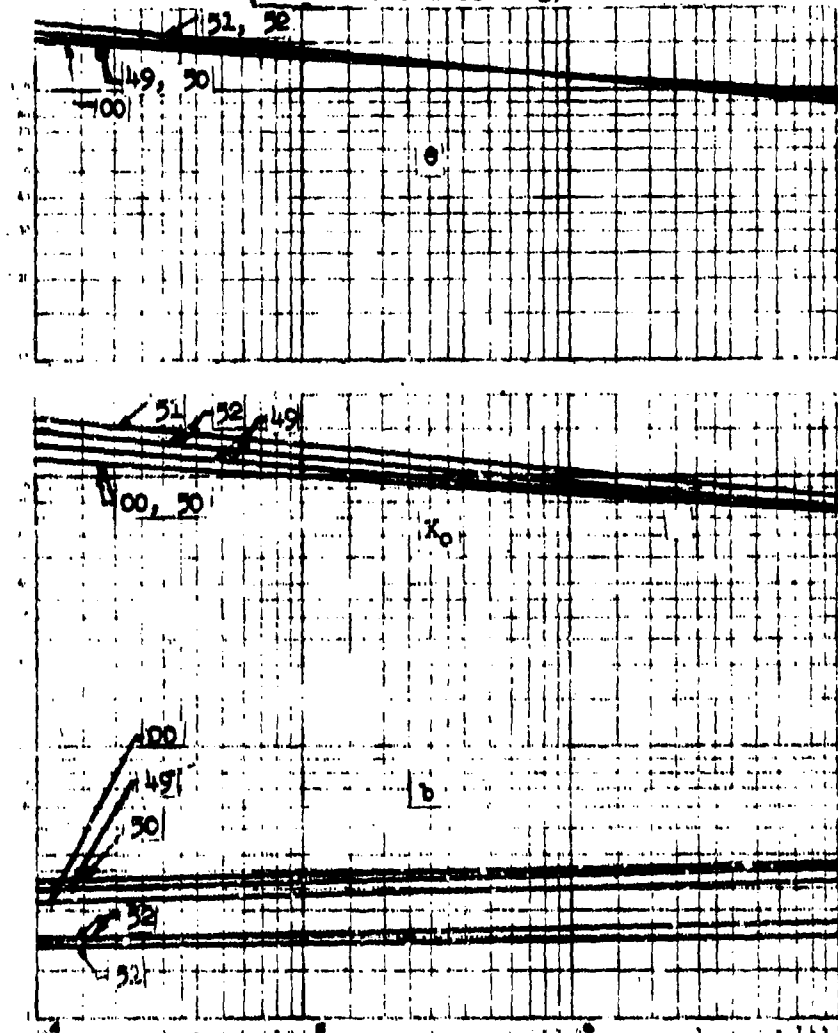
# FATIGUE STRENGTH

H-11 Steel

$S_u = 272 \text{ ksi}$

$S_y = 228 \text{ ksi}$

Effect of Heat Treatment  
(Pretest) Conditioning



Rotary Beam Bending

Mean Stress = 0

Composition:

5% Cr, 1.5% Mo, .4% V, .35% C

Hot Rolled, Lathe Turned  
Grain Direction Transverse  
to Lengthwise Axis

Surface Treatment:

Shot-Peened—Chrome Baked

Pretest Conditioning Code:

See Page 46

Initial Heat Treatment:

See Page 199, Item 7

Figure: 6.37 (For Tabulated Data See Page 215)

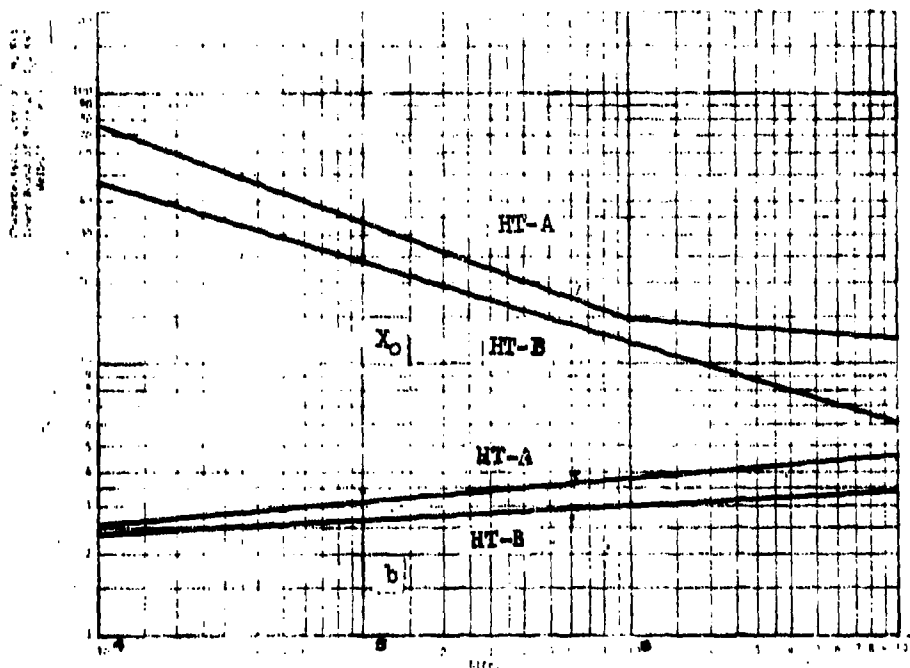
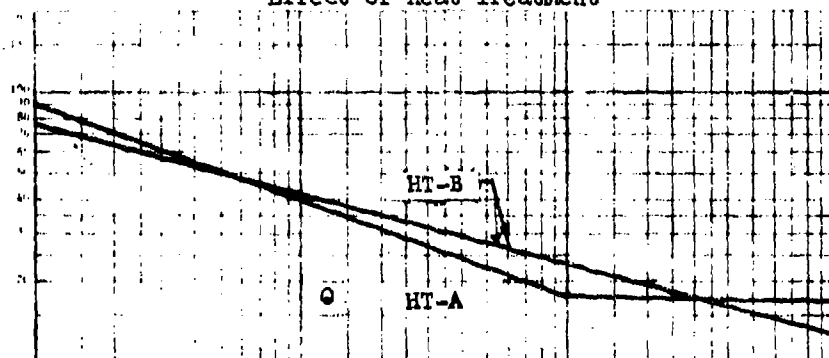
4340 Steel

# FATIGUE STRENGTH

$S_{uA} = 246 \text{ ksi}$

$S_{uB} = 222 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Lathe Turned

Melt Practice: Air Melt, Vacuum ARC remelt

Mechanically Polished  
V-notched, Flank Angle =  $60^\circ$

Composition:

Mean stress = 0

(.37-.44)% C, (.55-.90)%  $M_n$   
(.20-.35)%  $S_i$ , (1.55-2.0)%  $N_i$   
(.65-.95)%  $C_r$ , (.20-.30)%  $M_o$

Heat Treatment:  
See Page 199, Item 8A

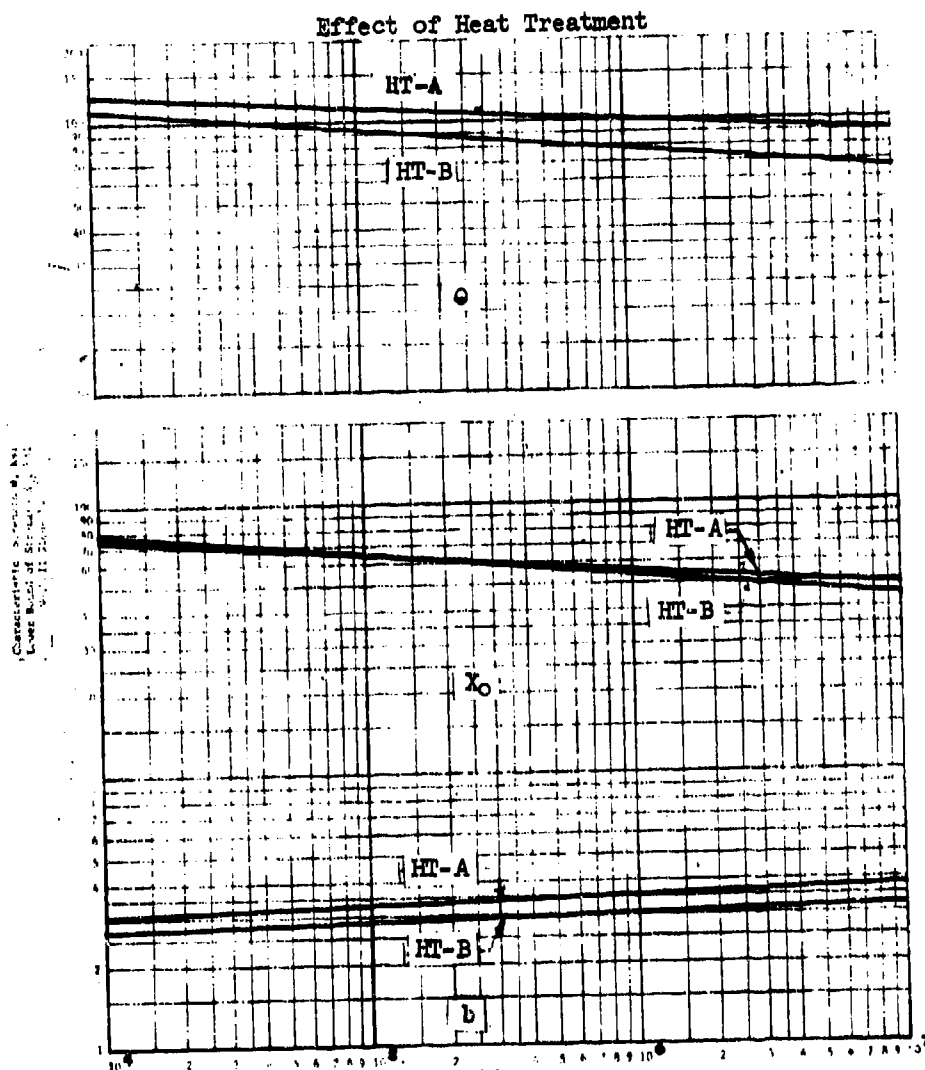
Figure: 6.38 (For Tabulated Data See Page 217).



4340 Steel

# FATIGUE STRENGTH

$S_{uA} = 246 \text{ ksi}$   
 $S_{uB} = 222 \text{ ksi}$



Rotary Beam Bending

Lathe Turned  
 Mechanically Polished  
 Unnotched

Melt Practice:

Air Melt, Vacuum Arc Remelt

Composition:

(.37-.44)% C, (.55-.90)%  $M_n$   
 (.20-.35)% S<sub>1</sub>, (1.55-2.0)% N<sub>1</sub>  
 (.65-.95)% Cr, (.20-.30)%  $M_o$

Heat Treatment:

See Page 199, Item 8A

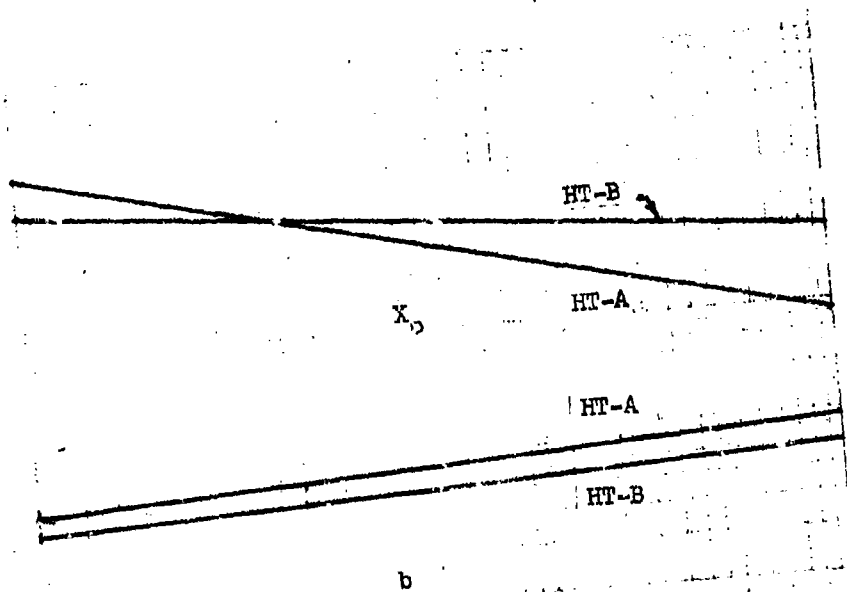
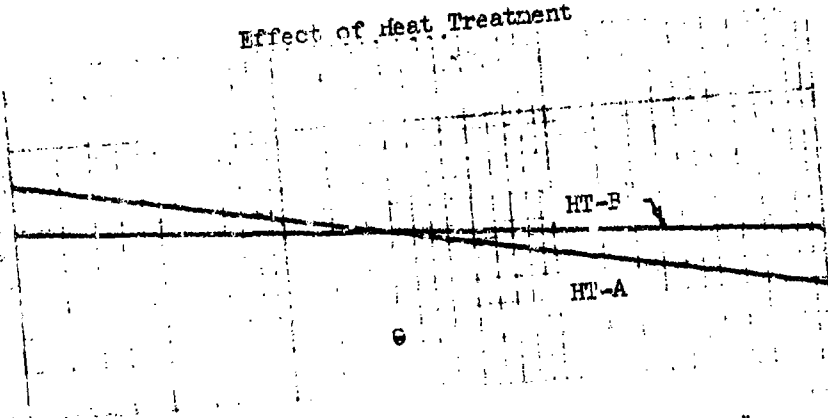
Figure 6.39 (For Tabulated Data See Page 217)

4340 Steel

# FATIGUE STRENGTH

$S_{uA} = 264 \text{ ksi}$   
 $S_{uB} = 206 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Melt Practice:  
 Vacuum Induction Melt

Composition:  
 (.37-.44)% C, (.55-.90)% Mn  
 (.20-.35)% Si, (1.55-2.0)% Ni  
 (.65-.95)% Cr, (.20-.30)% Mo

Lathe Turned  
 Mechanically Polished  
 Mean Stress = 0  
 V-notched Flank Angle = 60°

Heat Treatment:  
 See Page 199, Item 8A

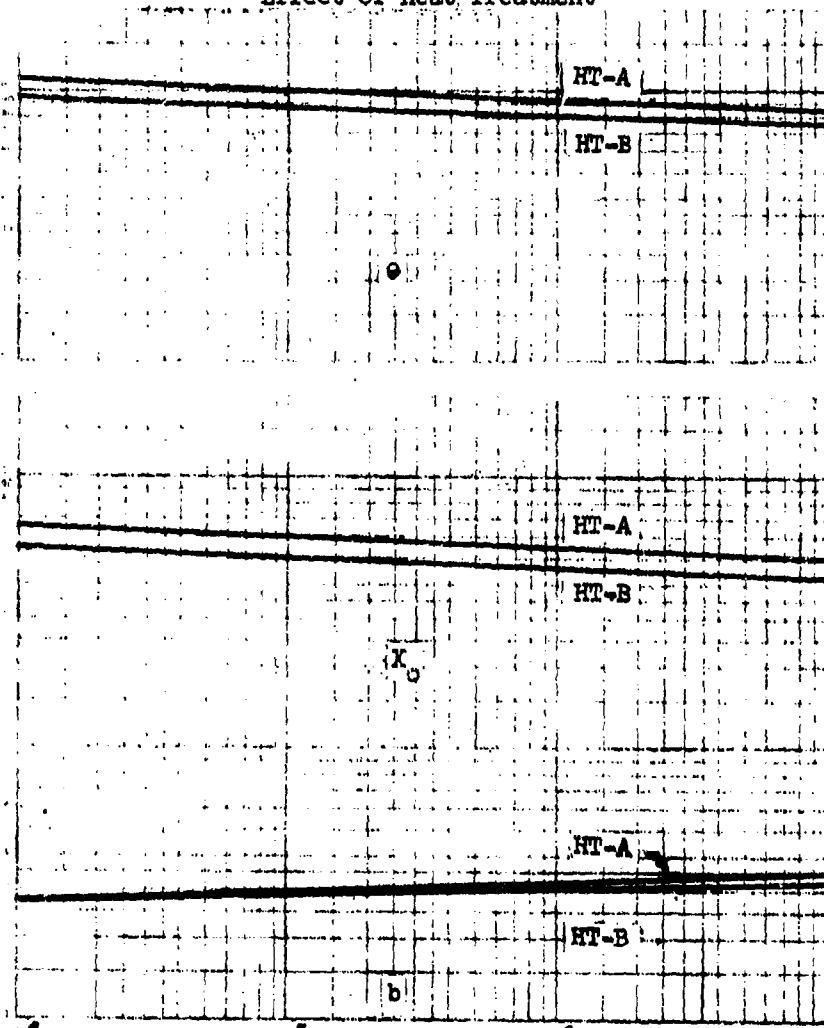
Figure: 6.40 (For Tabulated Data See Page 217)

4340 Steel

# FATIGUE STRENGTH

$S_{uA} = 264 \text{ ksi}$   
 $S_{uB} = 206 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending  
Melt Practice:  
Vacuum Induction melt

Lathe Turned Mech. Polished  
Unnotched  
Mean Stress = 0

Composition:  
(.37-.44)% C, (.55-.90)%  $M_n$   
(.20-.35)% Si, (1.55-2.0)%  $N_i$   
(.65-.95)% Cr, (.20-.30)%  $M_o$

Heat Treatment:  
See Page 199, Item 8A

Figure: 6.41 (For Tabulated Data See Page 217)

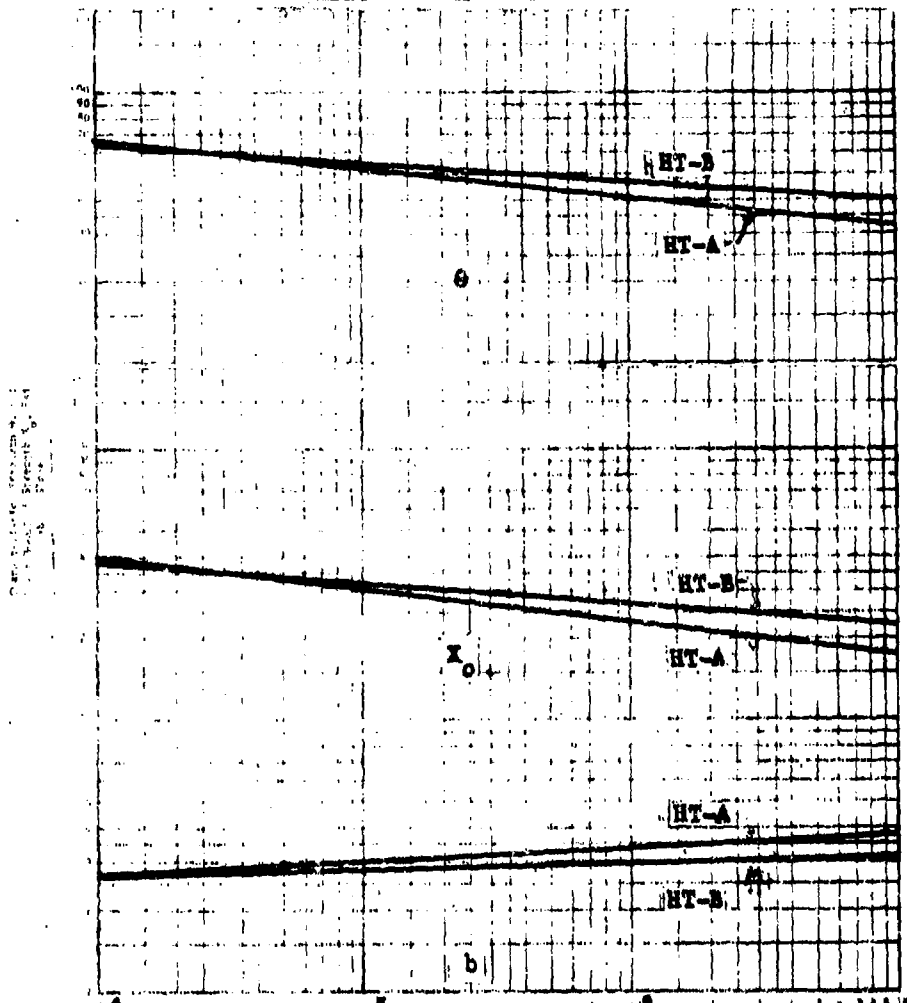
4340 Steel

# FATIGUE STRENGTH

$S_{uA} = 280 \text{ ksi}$

$S_{up} = 206 \text{ ksi}$

## Effect of Heat Treatment



### Rotary Beam Bending

Melt Practice: (Vacuum Induction Melt)

Composition: (Vacuum ARC Remelt)

(.37-.44)%C, (.55-.90)%Mn

(.20-.35)%Si, (.15-.20)%N

(.65-.95)%Cr, (.20-.30)%Mo

### Lathe Turned

Mechanically Polished

V-Notched Fillet angle = 60°

Mean Stress = 0

Heat Treatment: See Page 199, Item 8A

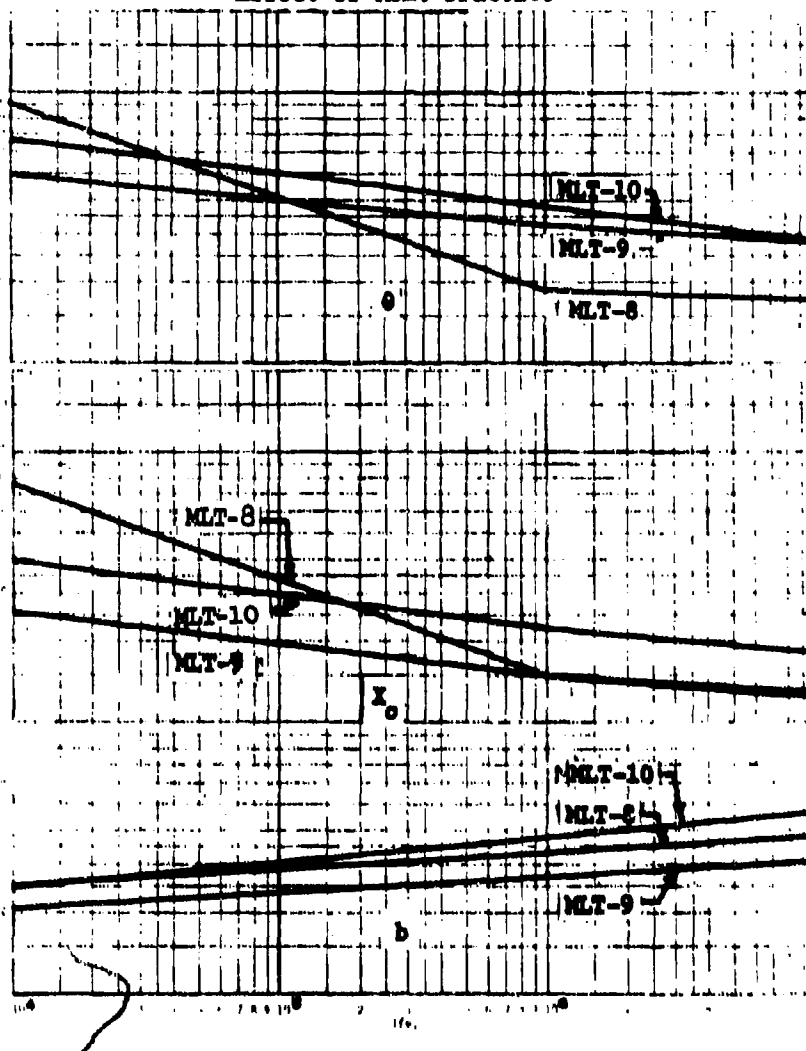
Figure:6.42 (For Tabulated Data See Page 217)

4340 Steel

# FATIGUE STRENGTH

$S_{u8} = 246 \text{ ksi}$ ,  $S_{u9} = 268 \text{ ksi}$ ,  $S_{u10} = 280 \text{ ksi}$

## Effect of Melt Practice



Rotary Beam Bending

Melt Practice:

See Page 199, Item 5A

Composition:

(.37-.44)% C, (.55-.90)% Mn

(.20-.35)% Si, (1.55-2.0)% Ni

(.65-.95)% Cr, (.20-.30)% Mo

Lathe Turned Mach. Polished

Mean Stress = 0

V-notched

Heat Treatment: A

See Page 199, Item 8A

Figure: 6.43

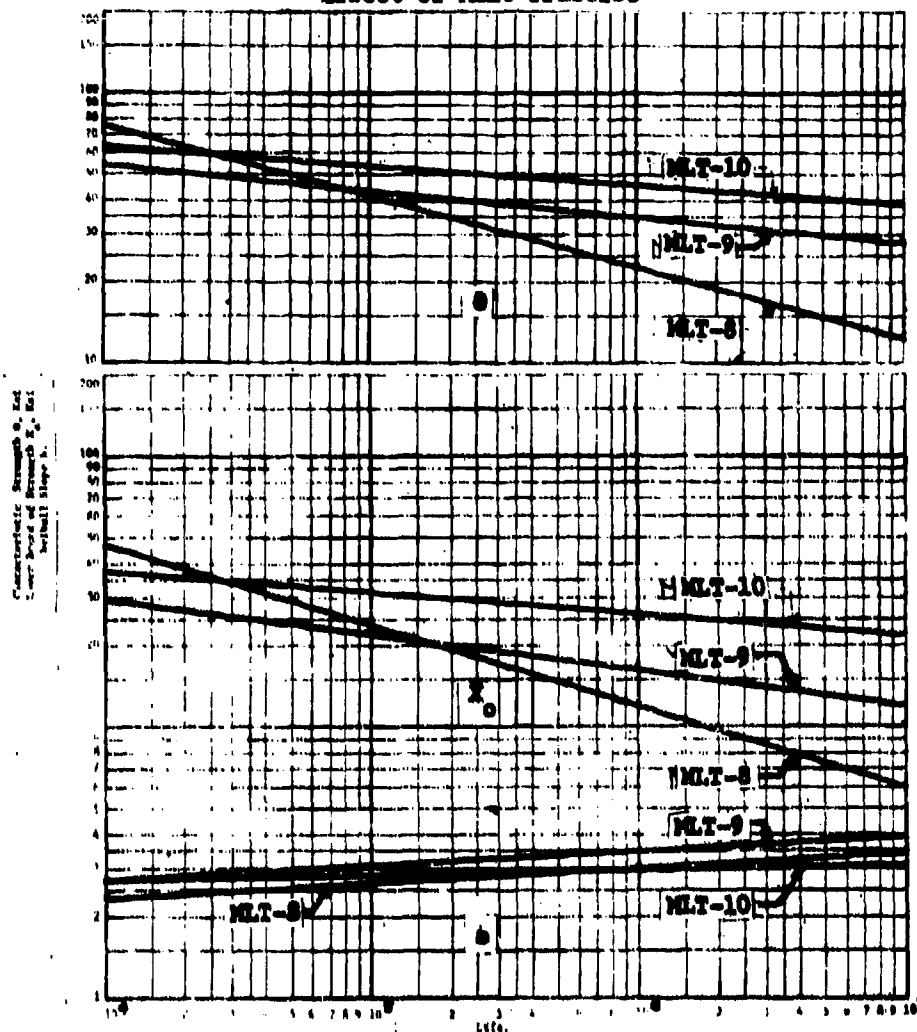
(For Tabulated Data See Page 216)

4340 Steel

# FATIGUE STRENGTH

$S_{ug} = 222 \text{ ksi}$ ,  $S_{u9} = 207 \text{ ksi}$ ,  $S_{u10} = 200 \text{ ksi}$

## Effect of Melt Practice



Rotary Beam Bending

Melt Practice: See Page 199, Item 8A

Composition:

(.37-.44)% C, (.55-.90)% Mn  
(.20-.35)% Si, (1.55-2.0)% Ni  
(.65-.95)% Cr, (.20-.30)% Mo

Lathe Turned

Mechanically Polished

Mean Stress = 0

V-notched

Heat Treatment: See Page 199, Item 8A

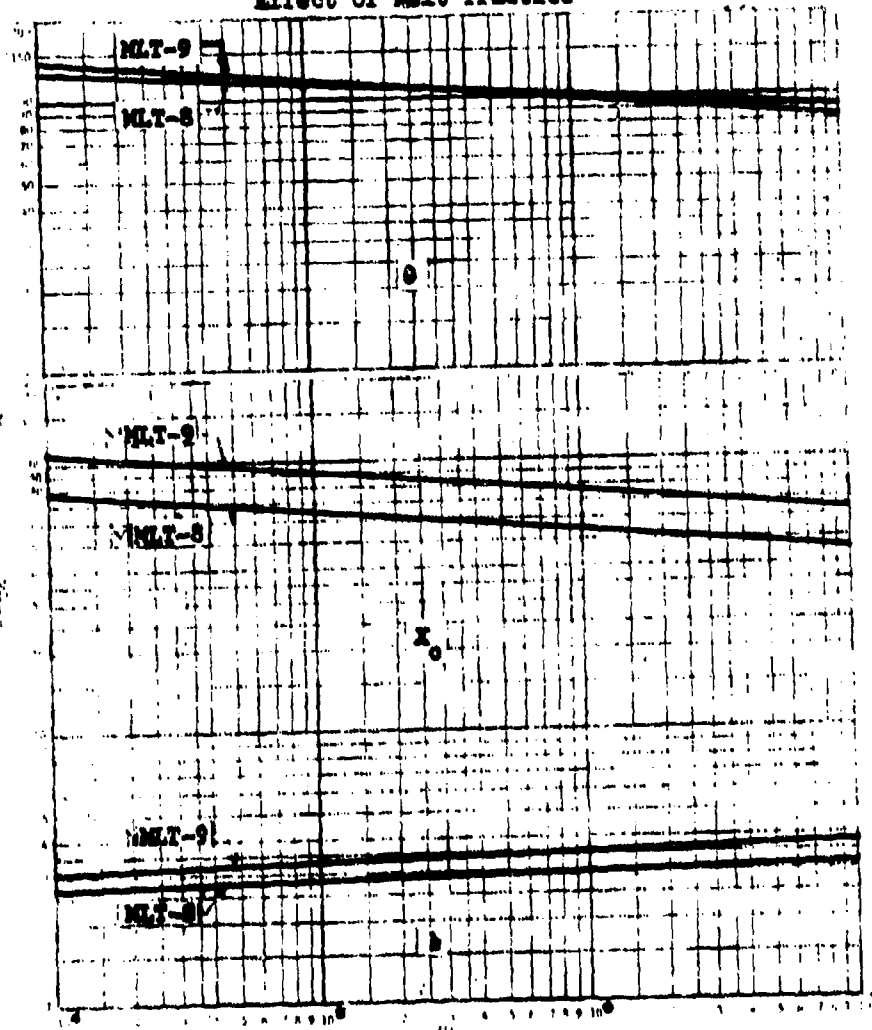
Figure: 6.44 (For Tabulated Data See Page 218 )

4340 Steel

# FATIGUE STRENGTH

$S_{ug} = 246 \text{ ksi}$ ,  $S_{u9} = 268 \text{ ksi}$

## Effect of Melt Practice



Rotary Beam Bending

Lathe Turned

Melt Practice: See Page 199, Item 8A

Mechanically Polished  
Unnotched

Composition:

Melt Treatment: See Page  
199, Item 8A

(.37-.44)% C, (.55-.90)%  $M_n$   
(.20-.35)%  $S_i$ , (1.55-2.1)%  $M_n$   
(.05-.95)%  $C_r$ , (.20-.30)%  $M$

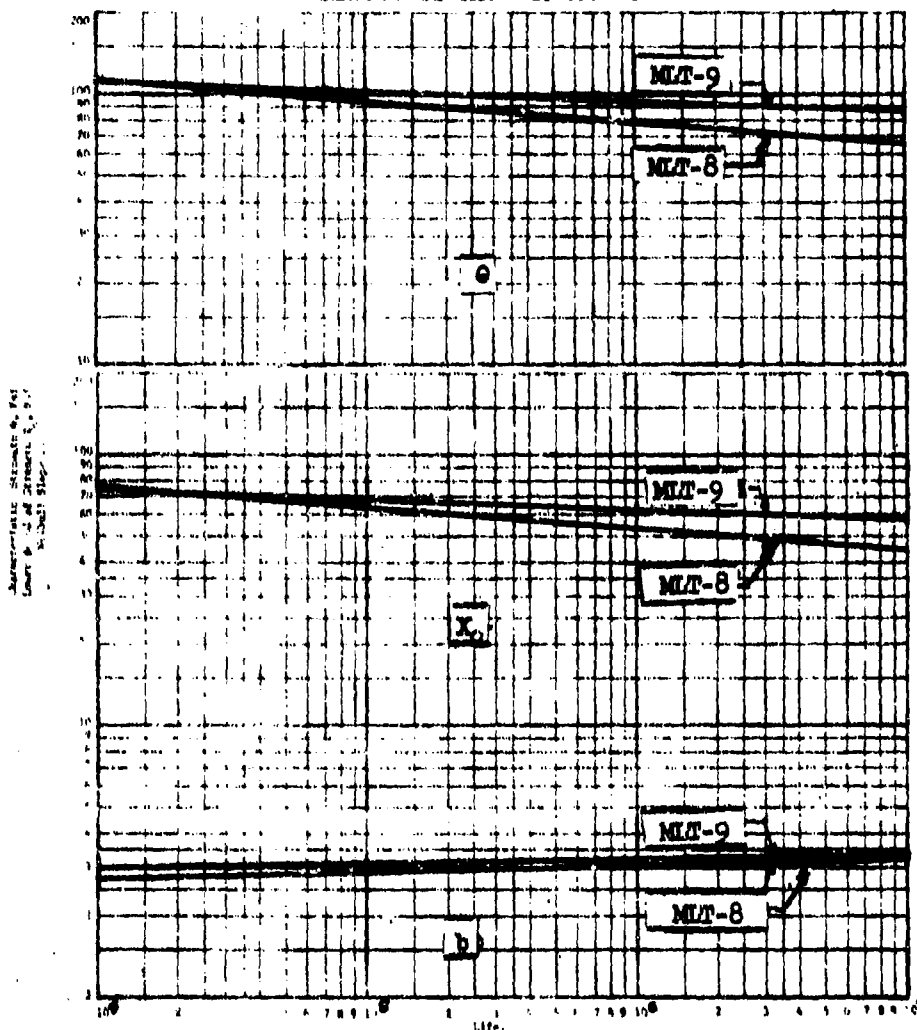
Figure 4.45 (For Tabulated Data See Page 218)

4340 Steel

# FATIGUE STRENGTH

$S_{u8} = 222 \text{ ksi}$ ,  $S_{u9} = 207 \text{ ksi}$

## Effect of Melt Practice



Rotatory Beam Bending

Lathe Turned

Mechanically Polished

Melt Practice: See Page 199, Item 8A

Unnotched

Composition:

Heat Treatment: A See Page 199, Item 8A

(.37-.44)% C, (.55-.90)%  $M_n$   
 (.20-.35)% Si, (1.55-2.0)%  $N_i$   
 (.65-.95)% Cr, (.20-.30)%  $M_o$

Figure: 6.46 (For Tabulated Data See Page 218)

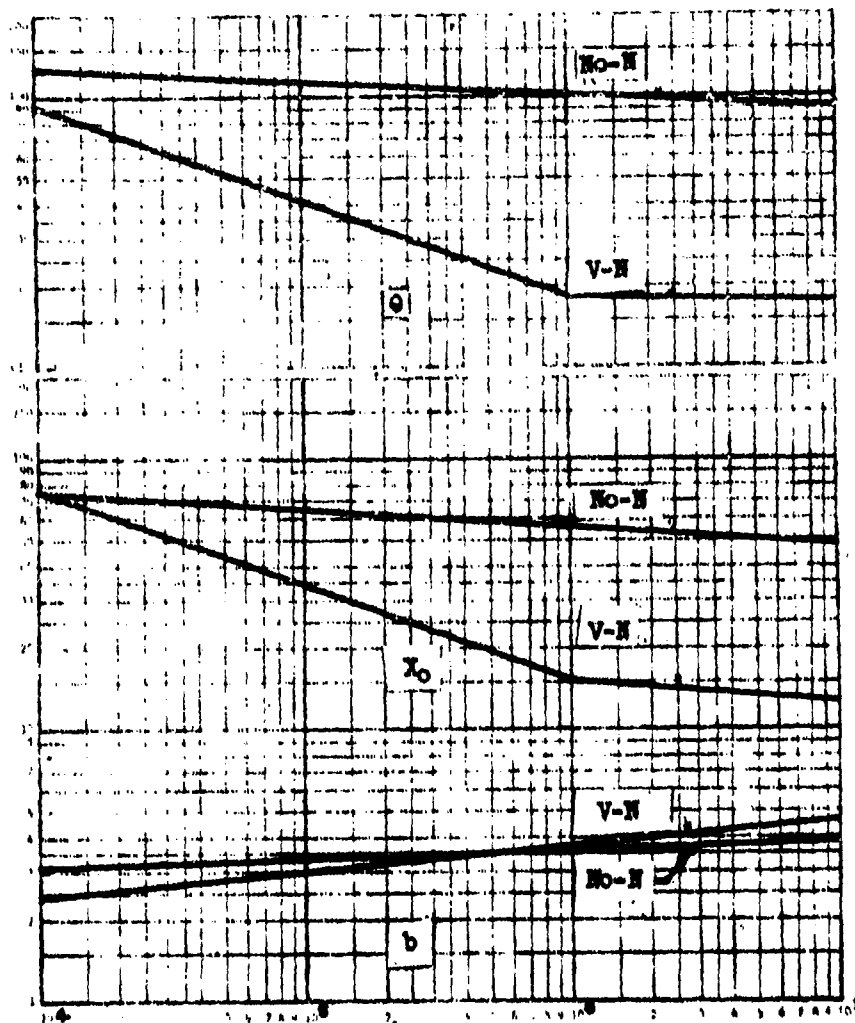


4340 Steel

# FATIGUE STRENGTH

$S_u = 246 \text{ ksi}$

## Effect of Stress Concentration



Rotary Beam Bending  
Melt Practice  
Air Melt-Vacuum Arc Remelt  
Composition:  
(.37-.44)% C, (.55-.90)% Mn  
(.20-.35)% Si, (1.55-2.0)% P  
(.65-.95)% Cr, (.20-.30)% Ni

Lathe Turned  
Mechanically Polished  
Mean Stress = 0  
Heat Treatment:  
A: Normalize 1550°F, OQ,  
Temper. 400°F, Air Cool

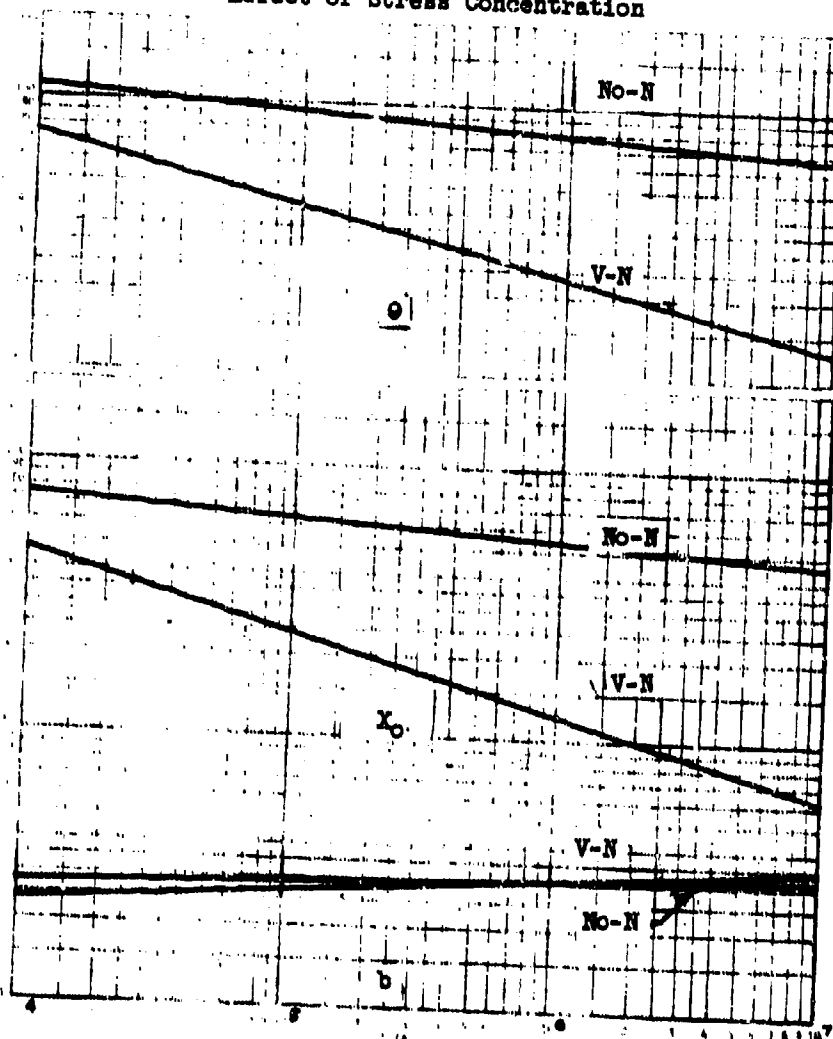
Figure: 6.47 (For Tabulated Data See Page 218)

4340 Steel

# FATIGUE STRENGTH

$S_u = 222 \text{ ksi}$

## Effect of Stress Concentration



Rotary Beam Bending

Melt Practice: Air Melt, Vacuum Arc  
Remelt

Composition:

(.37-.44)% C, (.55-.90)% Mn  
(.20-.35)% Si, (1.55-2.0)% Ni  
(.65-.95)% Cr, (.20-.30)% Mo

Lathe Turned  
Mechanically Polished

Heat Treatment:

Et: Normalize 1550°F,  
Quenched, Temper. 775°F, AC

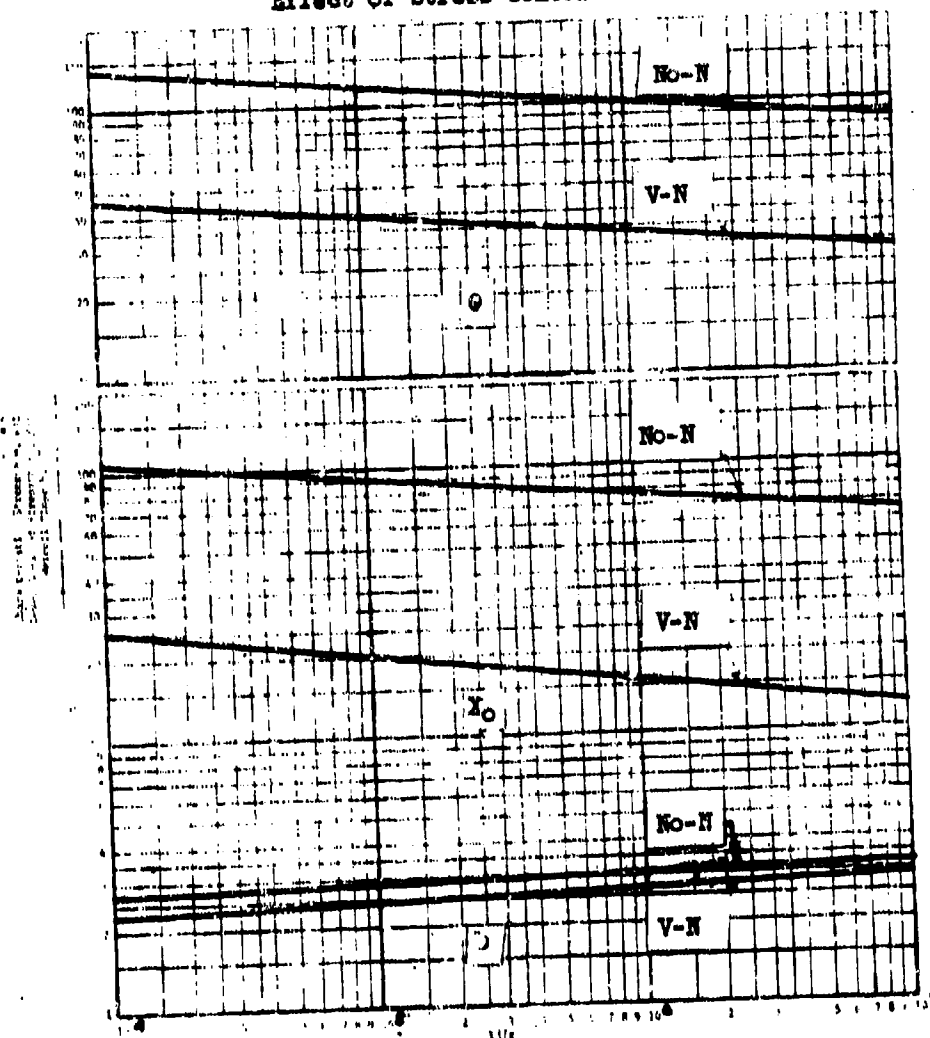
Figure: 6.48 (For Tabulated Data See Page 218)

4340 Steel

# FATIGUE STRENGTH

$S_u = 238 \text{ ksi}$

## Effect of Stress Concentration



Rotary Beam Bending

Melt Practice: Vacuum Induction Melt

Composition:

(.37-.44)% C, (.55-.90)% Mn  
(.20-.35)% Si, (1.55-2.0)% Ni  
(.65-.95)% Cr, (.20-.30)% Mo

Lathe Turned, Mech. Polished  
Mean Stress = 0  
Heat Treatment:  
A: Normalize 1550°F, OQ,  
Temper: 400°F, Air Cool

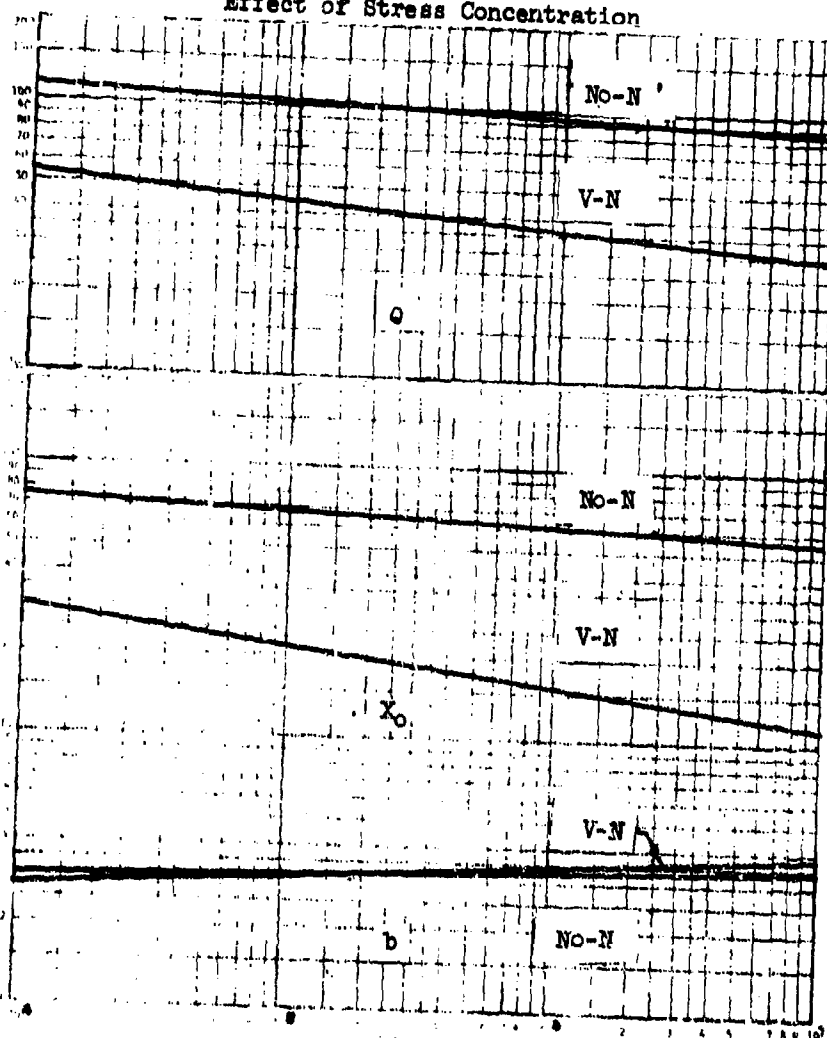
Figure: 6.49 (For Tabulated Data See Page 719)

4340 Steel

# FATIGUE STRENGTH

$S_u = 207 \text{ ksi}$

## Effect of Stress Concentration



### Rotary Beam Bending

Melt Practice: Vacuum Induction Melt  
Composition:

(.37-.44)% C, (.55-.90)% Mn  
(.20-.35)% Si, (1.55-2.0)% Ni  
(.650-.95)% Cr, (.20-.30)% Mo

Lathe Turned  
Mechanically Polished

Mean Stress = 0

Heat Treatment:

8: Normalize 1550°F,  
Quenched, Temper. at 775°F, AC

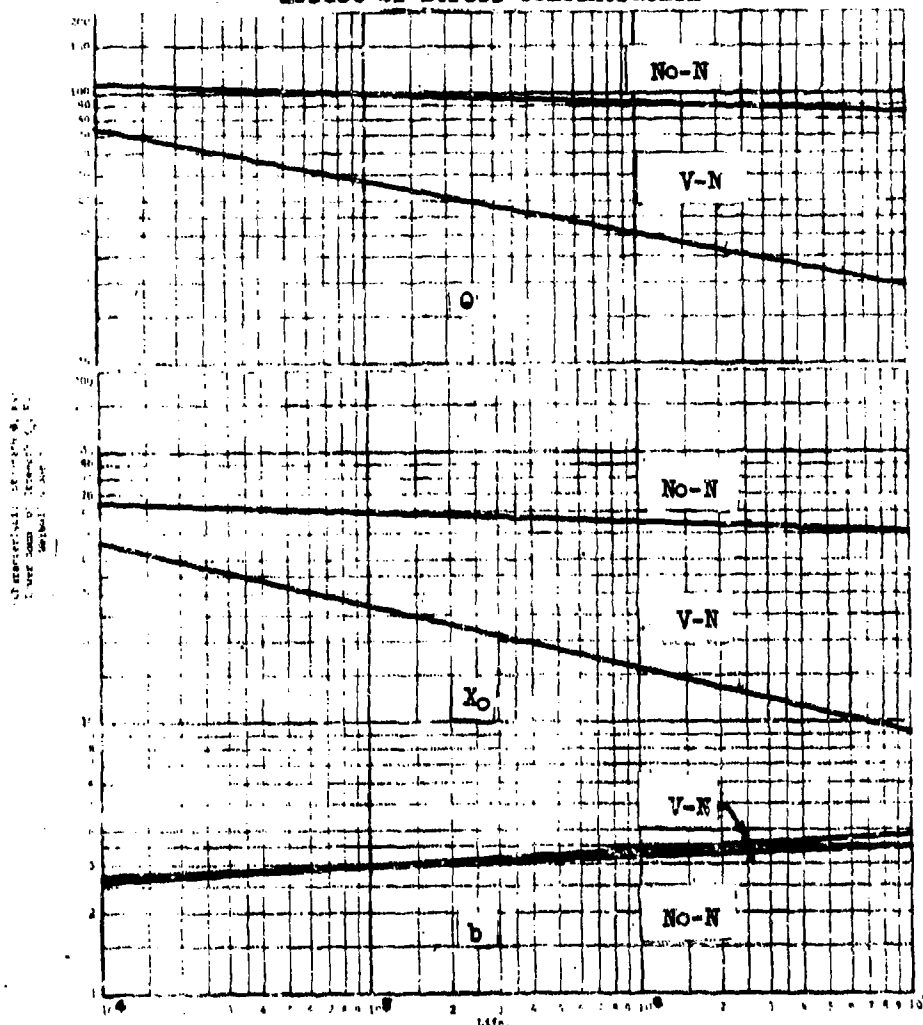
Figure: 6.50 (For Tabulated Data See Page 219)

4340 Steel

# FATIGUE STRENGTH

$S_u = 264 \text{ ksi}$

## Effect of Stress Concentration



Rotary Beam Bending

Lathe Turned  
Mechanically Polished

Melt Practice: Vacuum Induction Melt  
Composition:

Mean STRESS = 0  
Heat Treatment:  
At Normalise 1550°F, OQ,  
Temper. 400°F, AC

(.37-.44)% C, (.55-.90)%  $M_n$   
(.20-.35)%  $S_i$ , (1.55-2.0)%  $M_n$   
(.65-.95)%  $C_r$ , (.20-.30)%  $S_o$

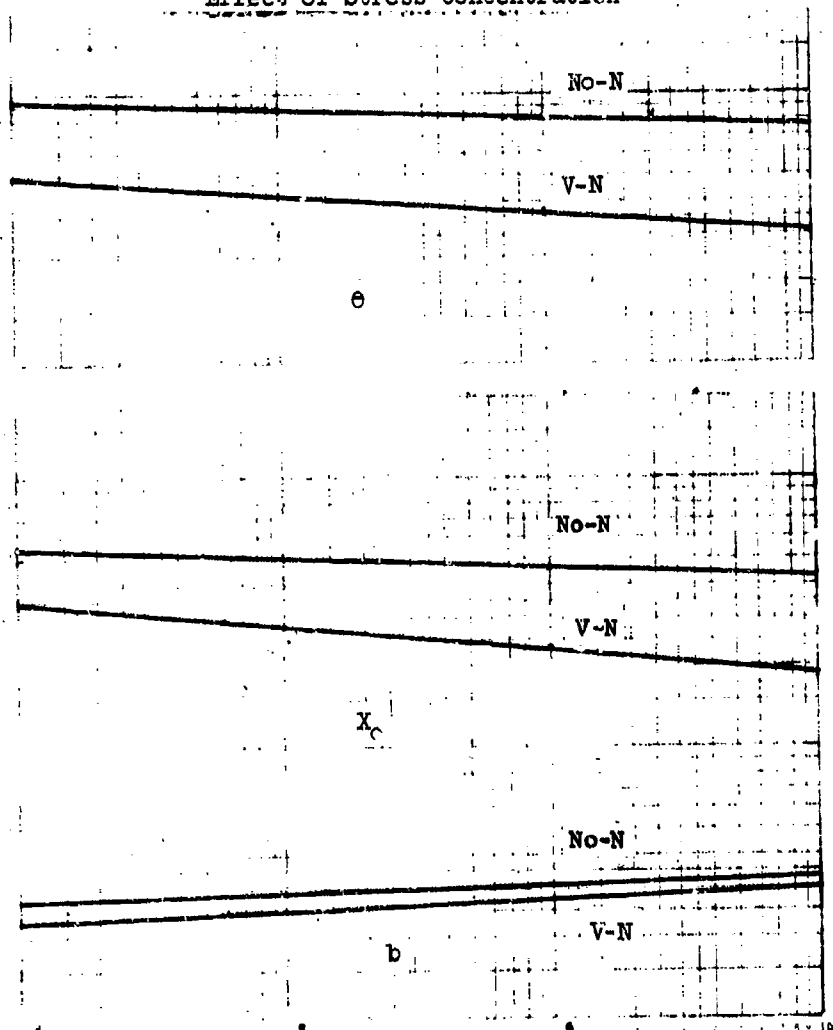
Figure 6.51 (For Tabulated Data See Page 219)

4340 Steel

FATIGUE STRENGTH

$S_u = 206 \text{ ksi}$

Effect of Stress Concentration



Rotary Beam Bending

Lathe Turned

Melt Practice:

Mean Stress = 0

Vacuum Induction Melt

Composition:

Heat Treatment:

(.37-.44)% C, (.55-.90)%  $M_n$   
 (.20-.35)% S<sub>1</sub>, (1.55-2.0)% N<sub>1</sub>  
 (.65-.95)% C<sub>r</sub>, (.20-.30)% M<sub>o</sub>

B: Normalize 1550°F, Quenched,  
 Temper. 775°F, AC

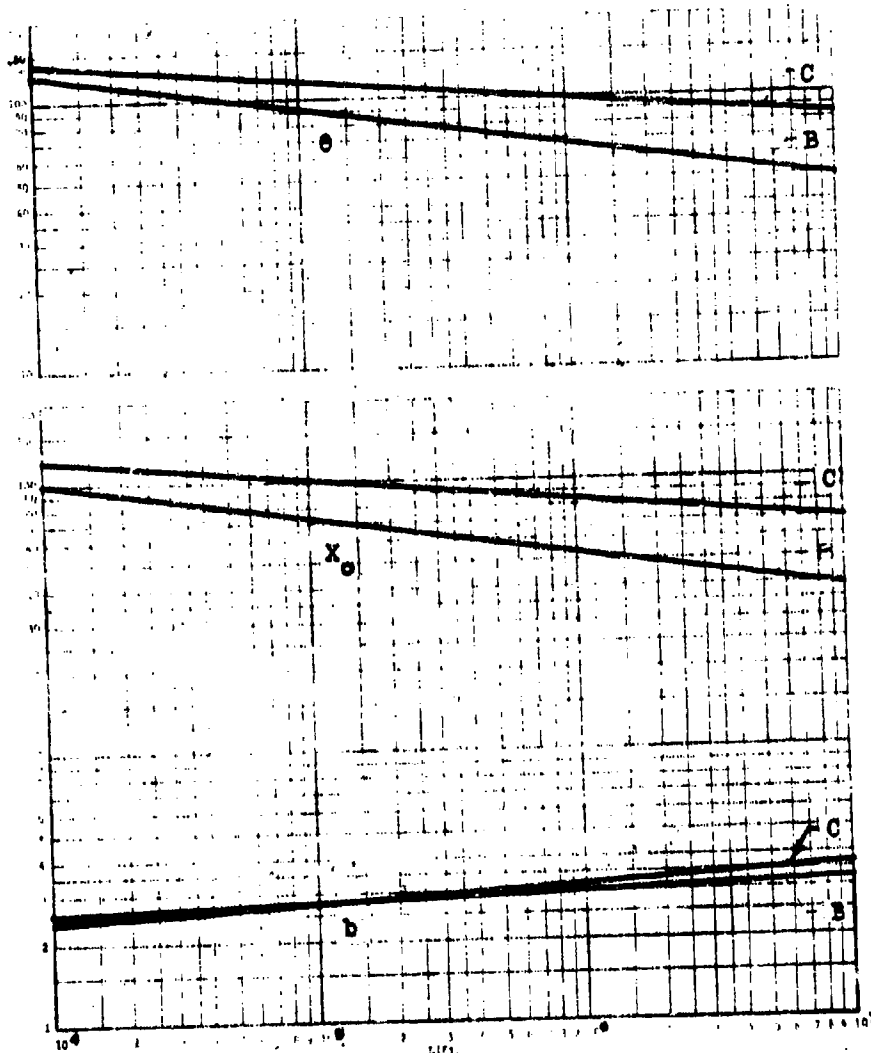
Figure: 6.52 (For Tabulated Data See Page 219)

# FATIGUE STRENGTH

AISI 4340 Steel

$S_u$ : B - 158 ksi, C - 171 ksi

Effect of Heat Treatment



Rotary Beam Bending  
 Stress Conc. Factor  $K_t = 1.0$   
 Composition:

.37-.44% C, .55-.90% Mn  
 .20-.35% Si, 1.55-2.0% Ni,  
 .63-.97% Cr, .20-.30% Mo

Hot Rolled and Lathed  
 Mean Stress = 0  
 Heat Treatment:  
 See Page 199, Item 8B

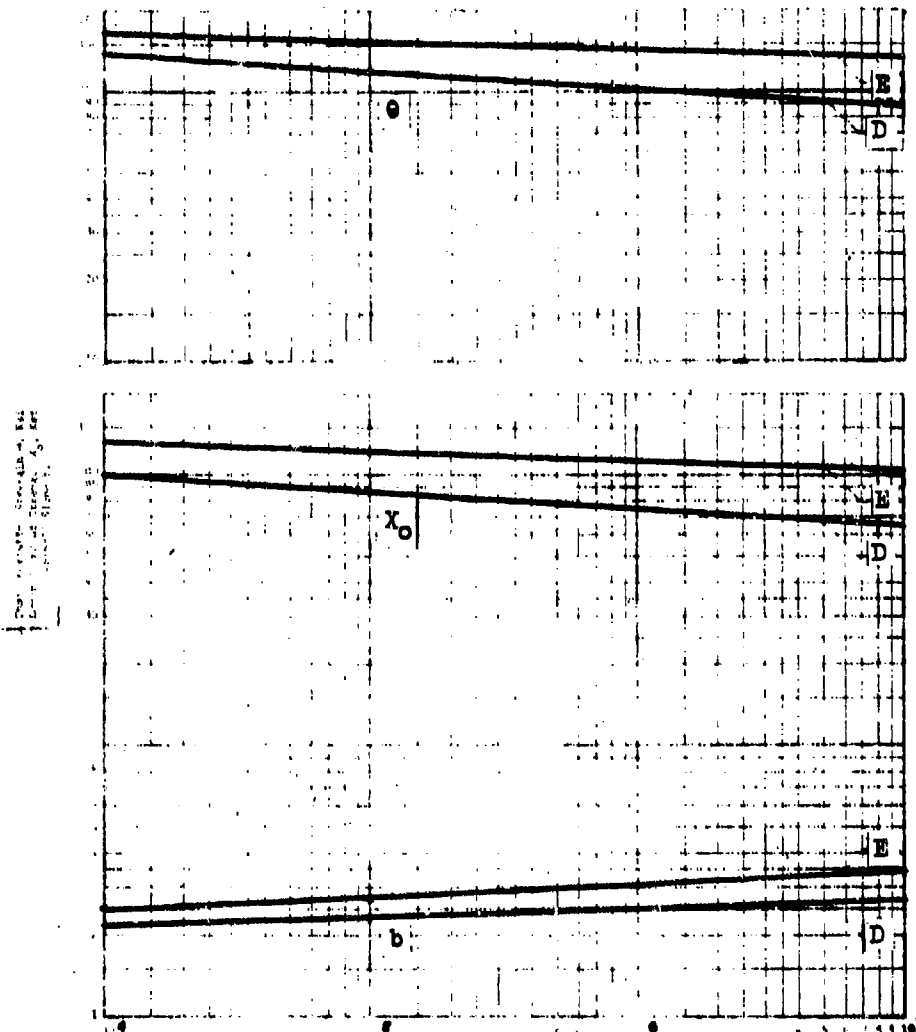
Figure 4.33 (For Tabulated Data See Page 220)

# FATIGUE STRENGTH

AISI 4340 Steel

$S_u$ : D - 275 ksi, E - 290 ksi

Effect of Heat Treatment



Rotary Beam Bending  
Stress Conc. Factor  $K_t = 1.0$   
Composition:

.37-.44% C, .55-.90% Mn,  
.20-.35% Si, 1.55-2.0% Ni,  
.65-.95% Cr, .20-.30% Mo

Forged and Ground

Mean Stress = 0

Heat Treatment:

See Page 199, Item 3B

Figure:6.54 (For Tabulated Data See Page 1220 )

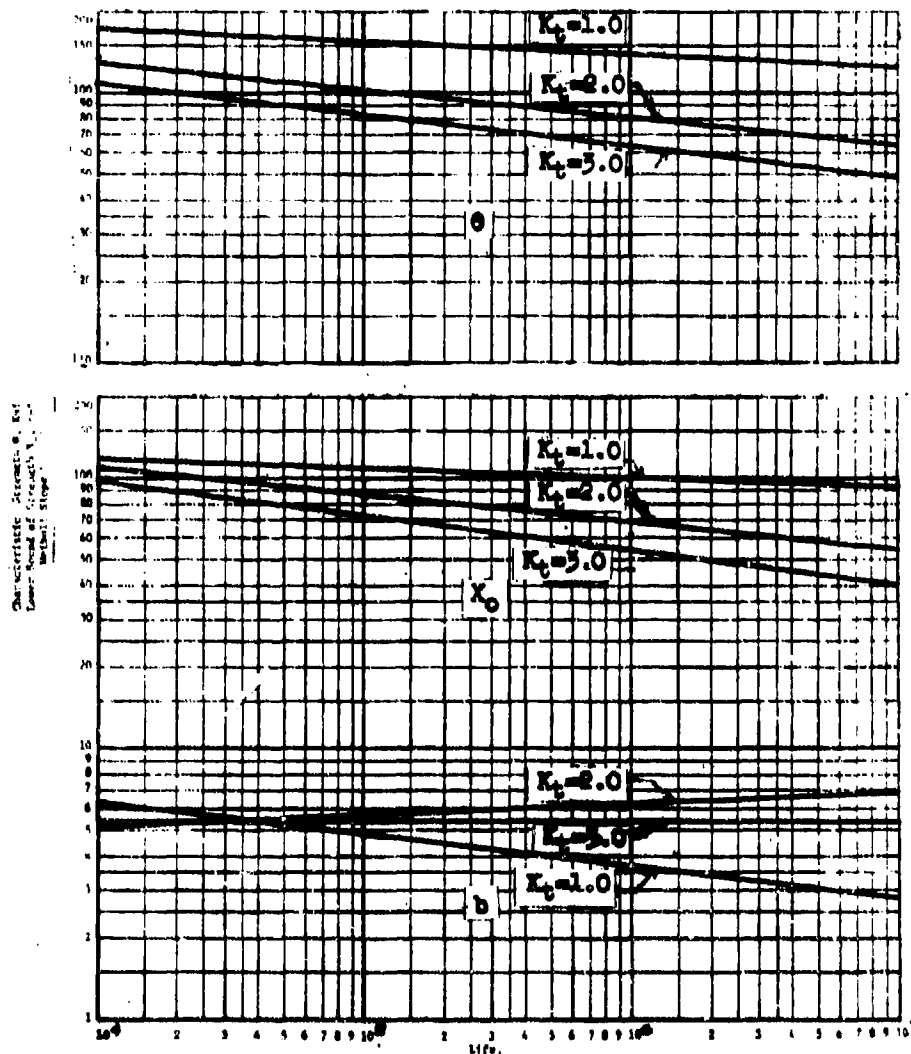


# FATIGUE STRENGTH

Thermold J

$S_u = 294 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed.

Temperature = 80°F

Composition:

.37-.44% C, .55-.90% Mn, .20-.35% Si

1.55-2.0% Ni, .65-.95% Cr, .20-.30% Mo

Figure: 6.55 (For Tabulated Data See Page 221)

Mean Stress = 0

Heat Treatment:

See Page 199, Item 9

# FATIGUE STRENGTH

Fe, 5.5% Mn, 2.5% Cr, .5% C

$S_u$  314 ksi,  $S_y$  = 267 ksi

## Effect of Stress-Concentration

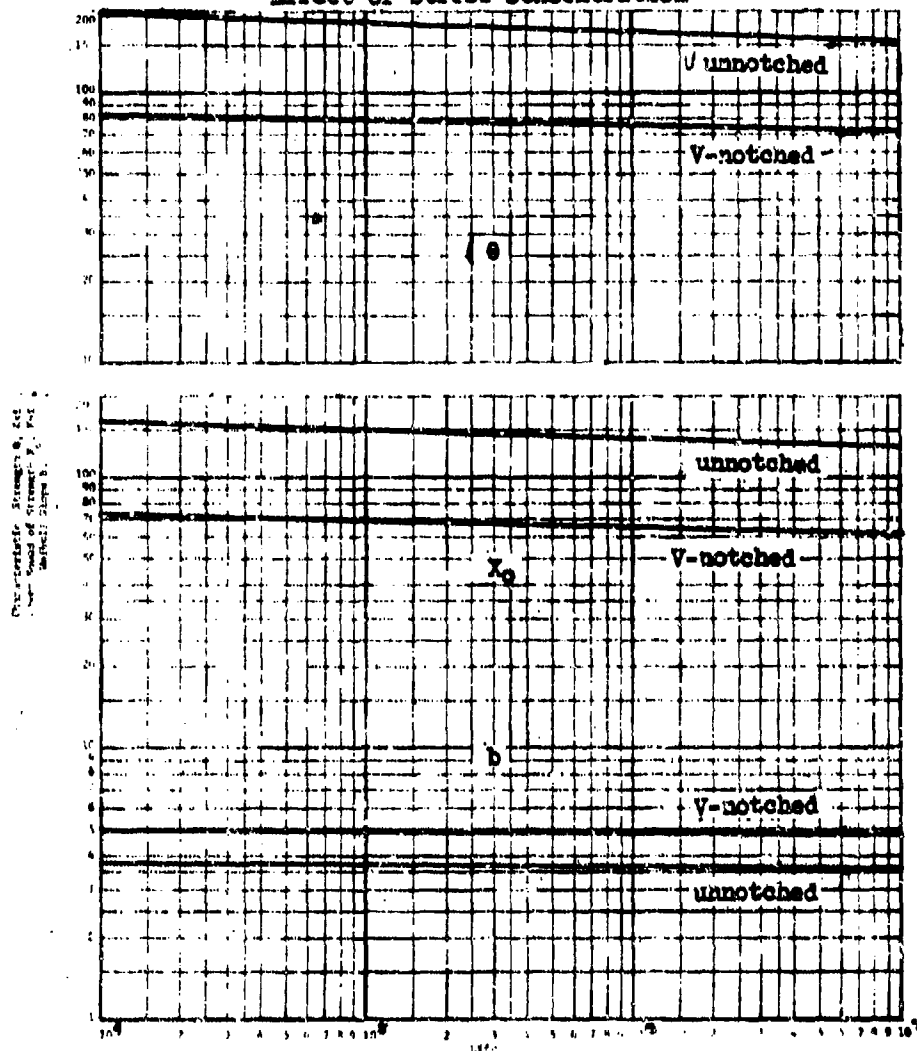


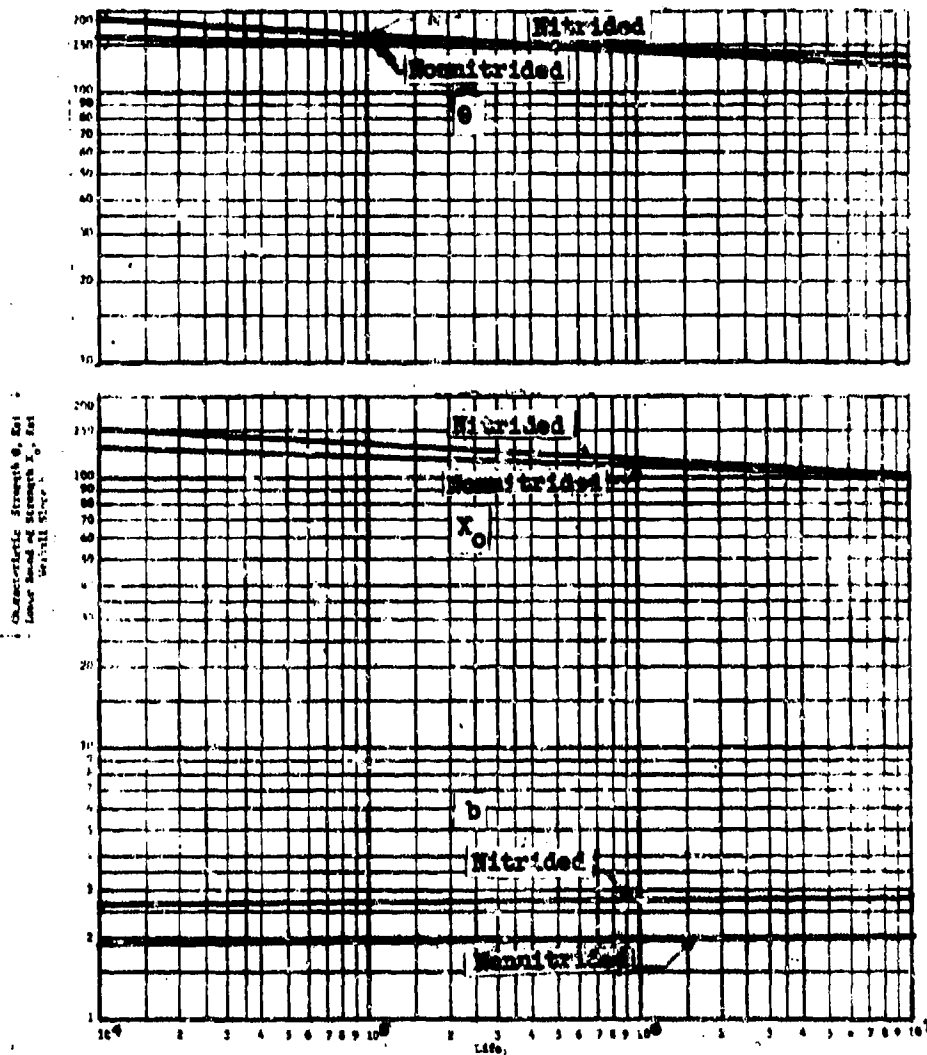
Figure:6.56 (For Tabulated Data See Page 222 )

# FATIGUE STRENGTH

M-10 Tool Steel

$S_{ut} = 330 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending  
Stress Conc. Factor  $K_t = 1.0$   
Composition:  
.8% C, .4% Cr, 2% V, .8% Mo

Forged and Staged, Lathe Turned  
Mean Stress = 0  
Heat Treatment:  
See Page 200, Item 11

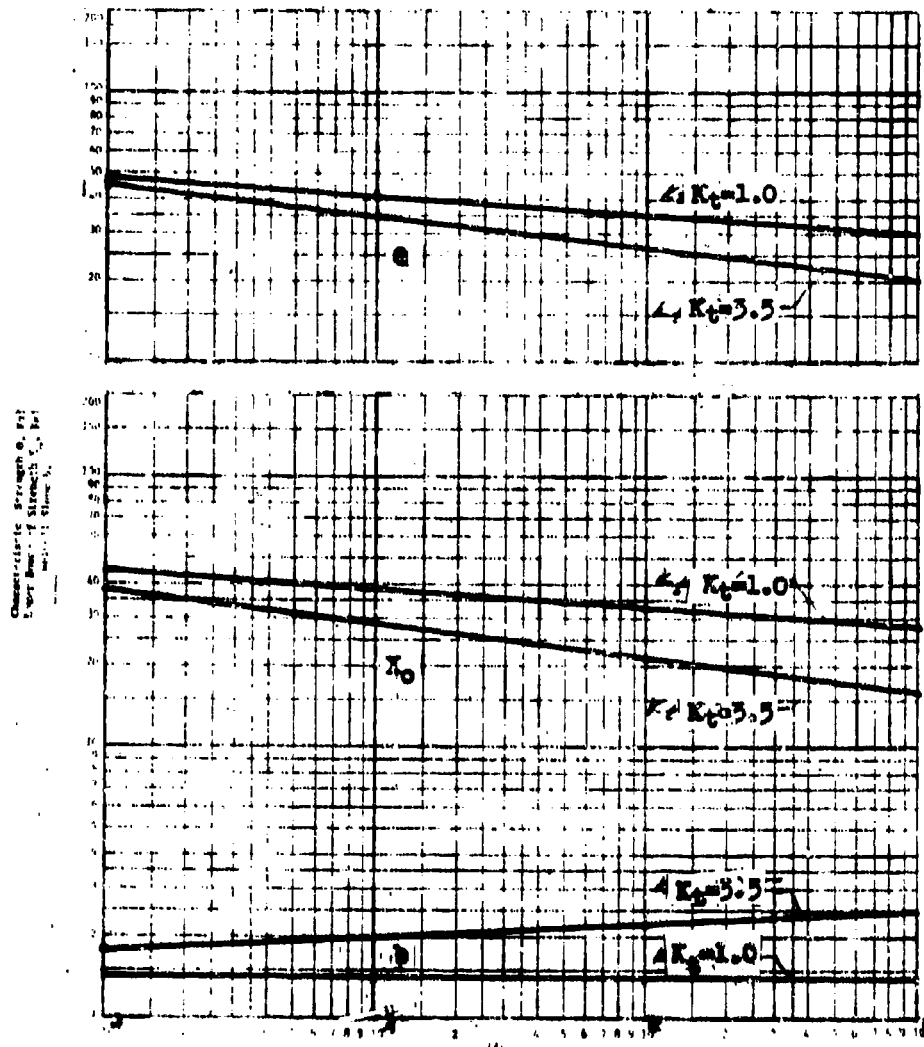
Figure 6.57 (For Tabulated Data See Page 220)

**STAINLESS STEEL**  
**STRESS-STRAIN**

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

**Effect of Stress Concentration**



Axial Load, Completely Reversed

Temperature = 80°F

Composition:

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Mechanically Polished

Mean Stress = 0

Heat Treatment: See Page 200,  
Hot Rolled, Annealed Item 11.

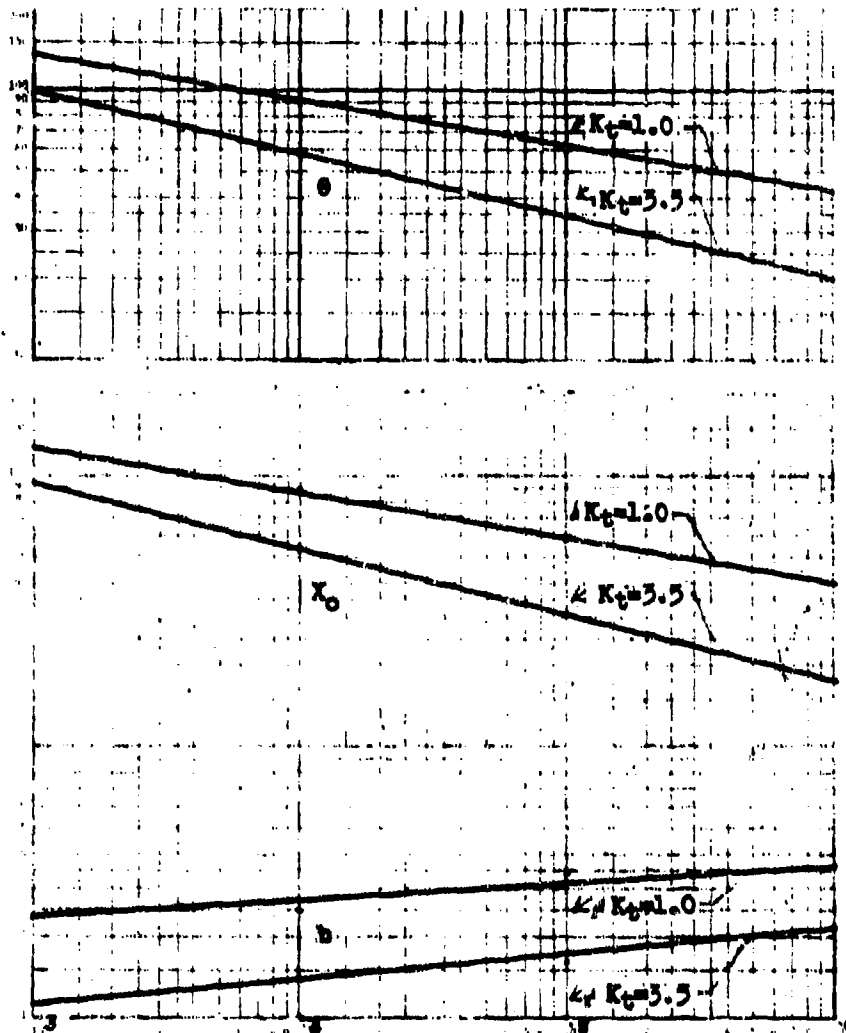
Figure: 6.58 (For Tabulated Data See Page 224)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 53 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Mechanically Polished

Temperature = -320°F

Mean Stress = 0

Composition:

Heat Treatment: See Page 200,

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Hot Rolled, Annealed

Item 12

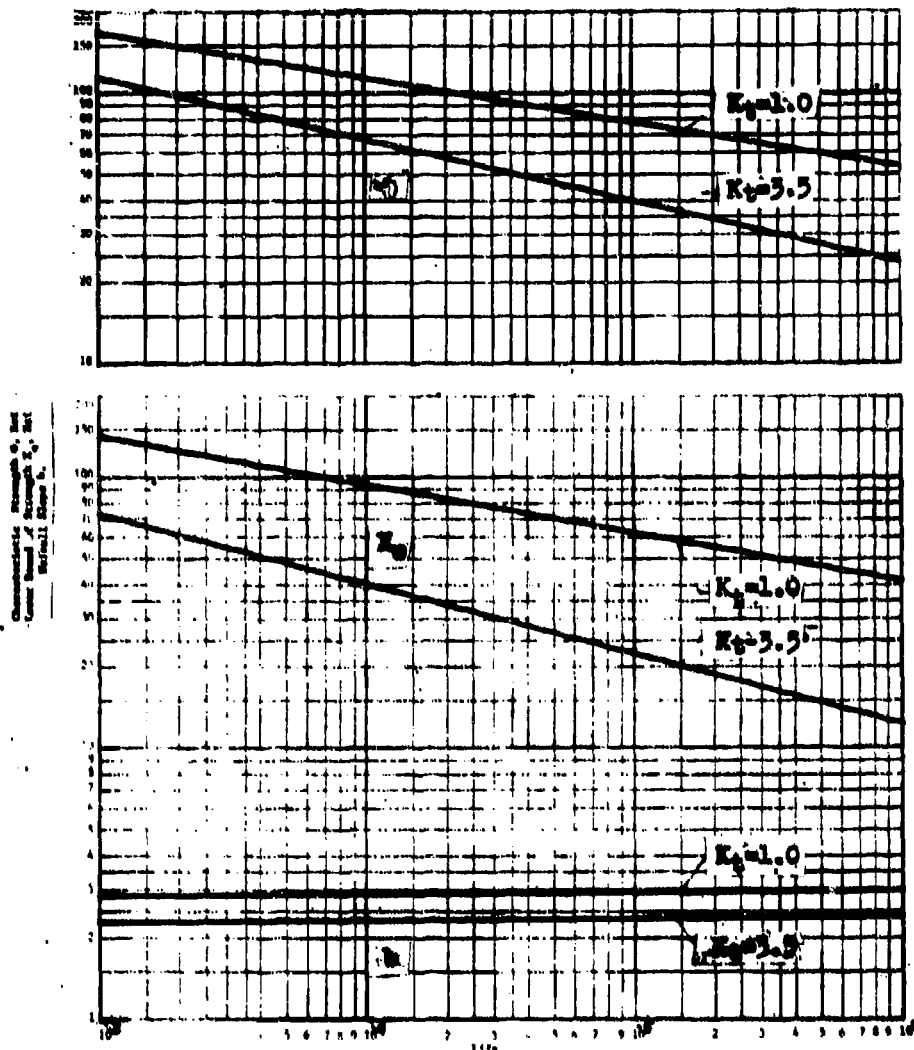
Figure: 6.59 (For Tabulated Data See Page 200)

# FATIGUE STRENGTH

301 Stainless Steel

$$\sigma_u = 86 \text{ ksi}, \quad \sigma_y = 38 \text{ ksi}$$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Temperature = -423°F

Composition:

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Mechanically Polished

Mean Stress = 0

Heat Treatment: See Page 200,  
Item 12  
Hot Rolled, Annealed

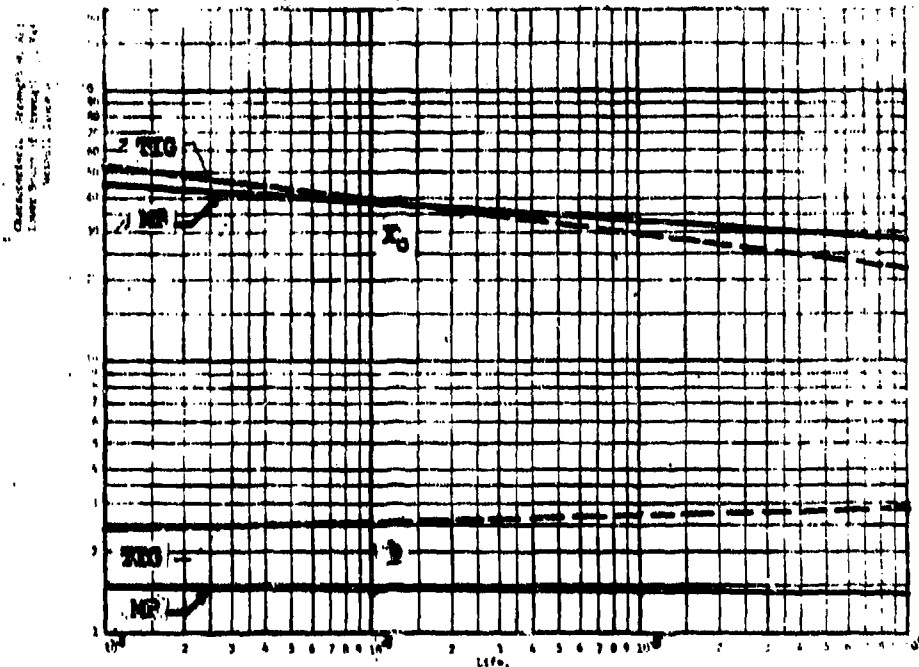
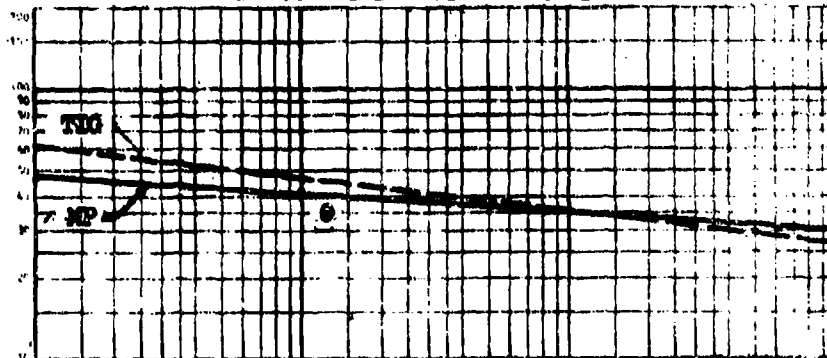
Figure: 6.60 (For Tabulated Data See Page 224)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

Affect of Process at 80° F



Axial Load, Completely Reversed

TIG = Tungsten Inert Gas Welded  
MP = Mechanically Polished

Stress Conc. Factor  $K_t = 1.10$

Mean Stress = 0

Composition:

Heat Treatment: See Page 200, Item 12

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Hot Rolled, Annealed

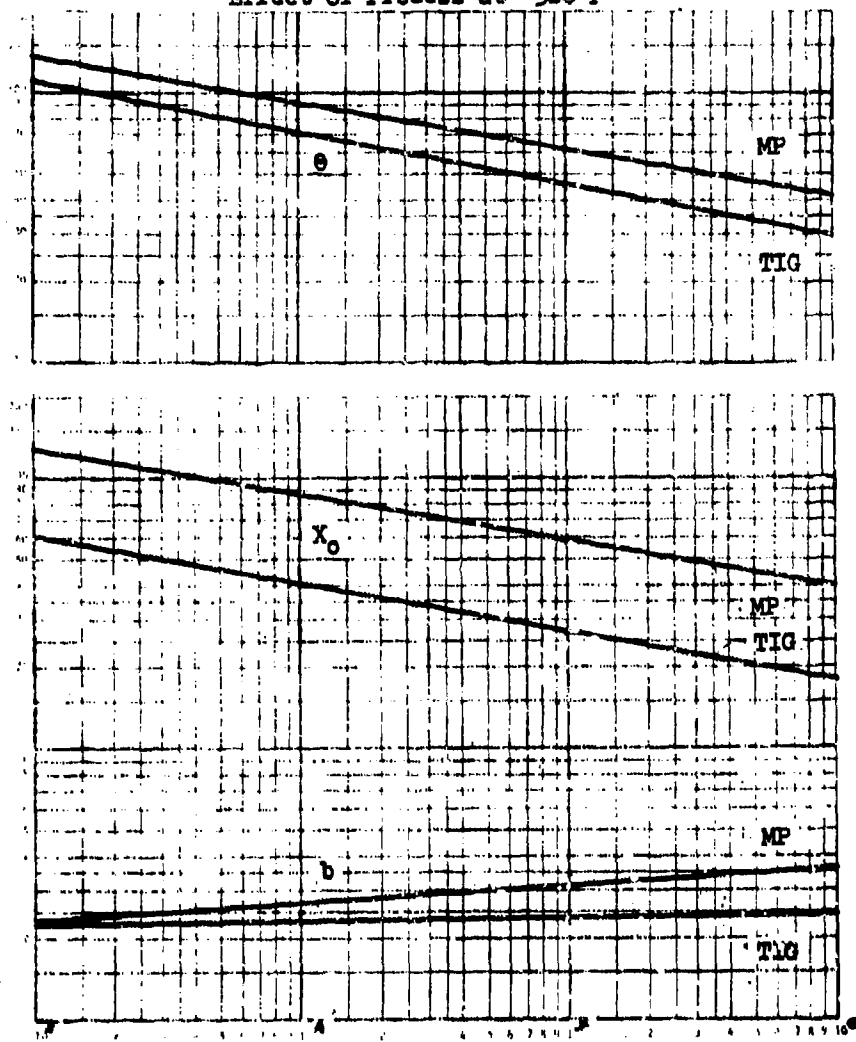
Figure: 6.61 (For Tabulated Data See Page 224)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

Effect of Process at  $-320^\circ\text{F}$



Axial Load, Completely Reversed

TIG = Tungsten Inert Gas Welded

Stress Conc. Factor  $K_t = 1.0$

MP = Mechanically Polished

Composition:

Mean Stress = 0

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.05% C

Heat Treatment: See Page 200, Item 12

Hot Rolled, Annealed

Figure: 6.62 (For Tabulated Data See Page 224)

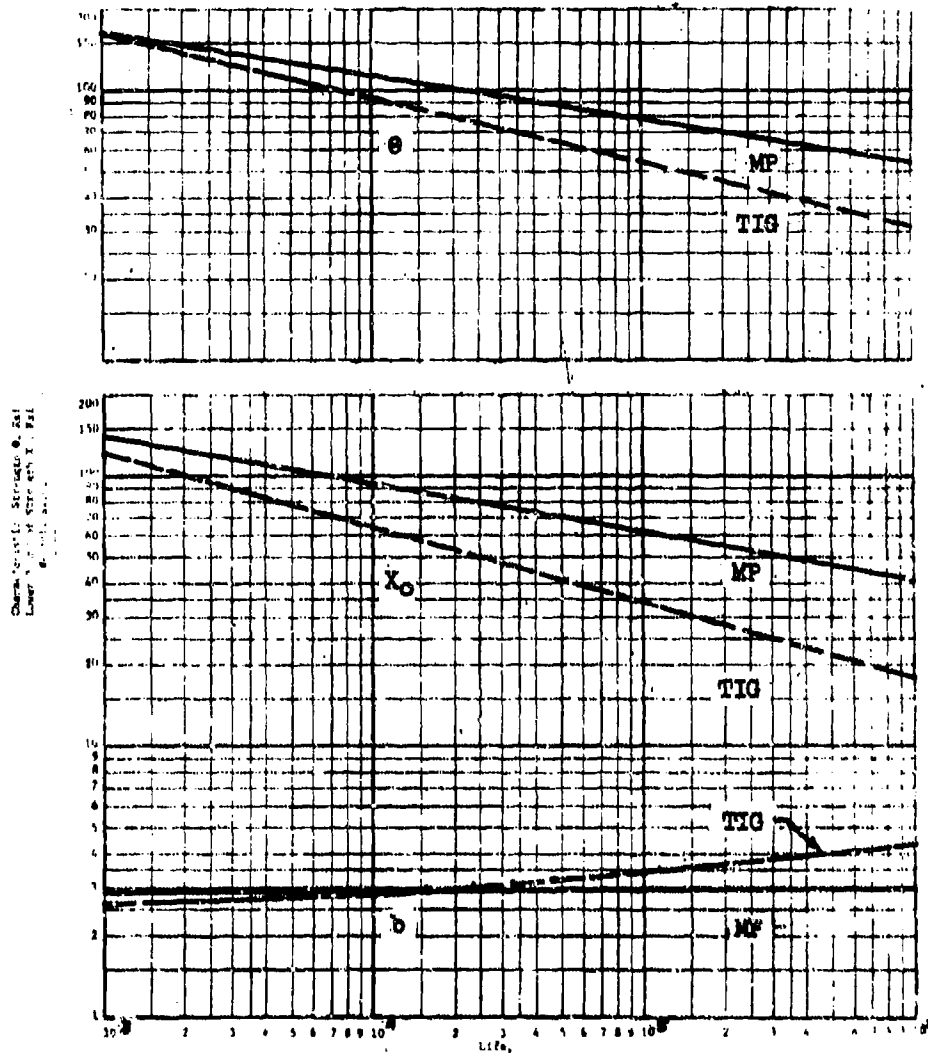


# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

Effect of Process at  $-423^\circ\text{F}$



Axial Load, Completely Reversed

TIG = Tungsten Inert Gas Welded

MP = Mechanically Polished

Stress Conc. Factor  $K_t = 1.0$

Mean Stress = 0

Composition:

Heat Treatment: See Page 200, Item 12.

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Hot Rolled, Annealed

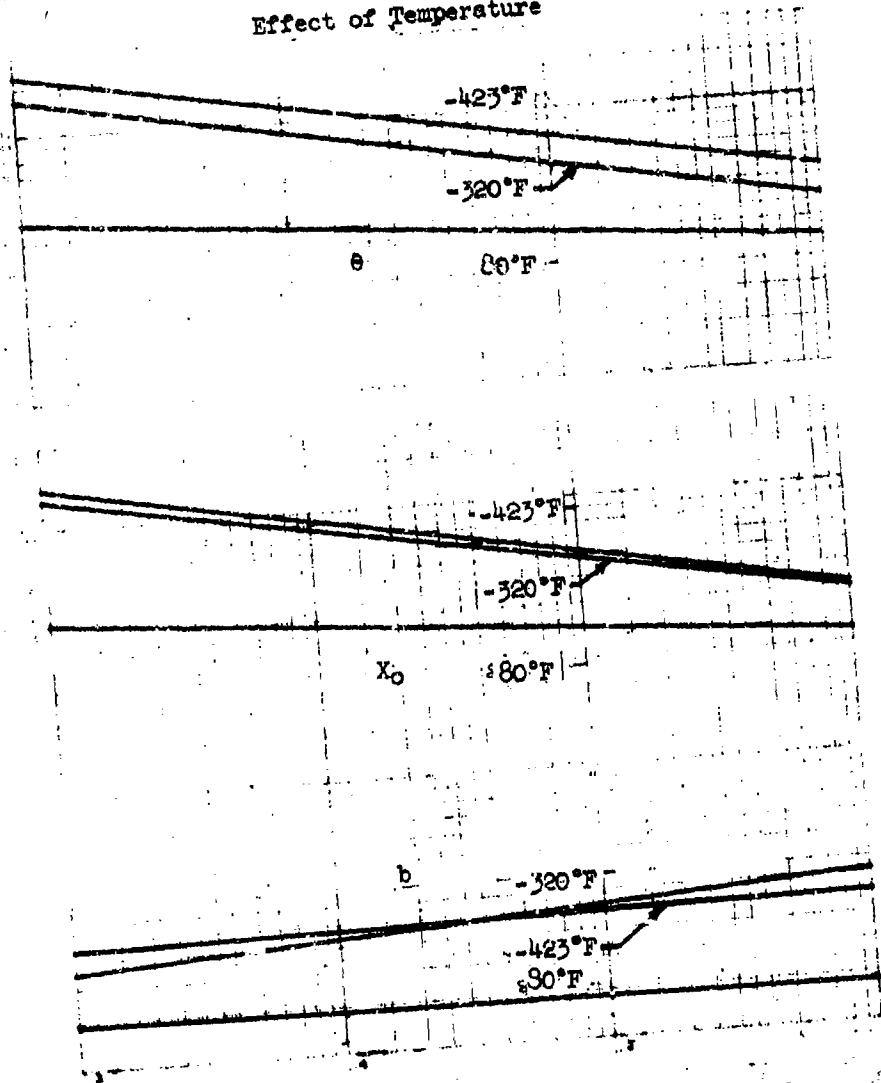
Figure: 6.63 (For Tabulated Data See Page 224)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

Effect of Temperature



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 1.0$

Composition:

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Mechanical Polish

Mean Stress = 0

Heat Treatment: See Page 200,  
Hot Rolled, Annealed Item 12,

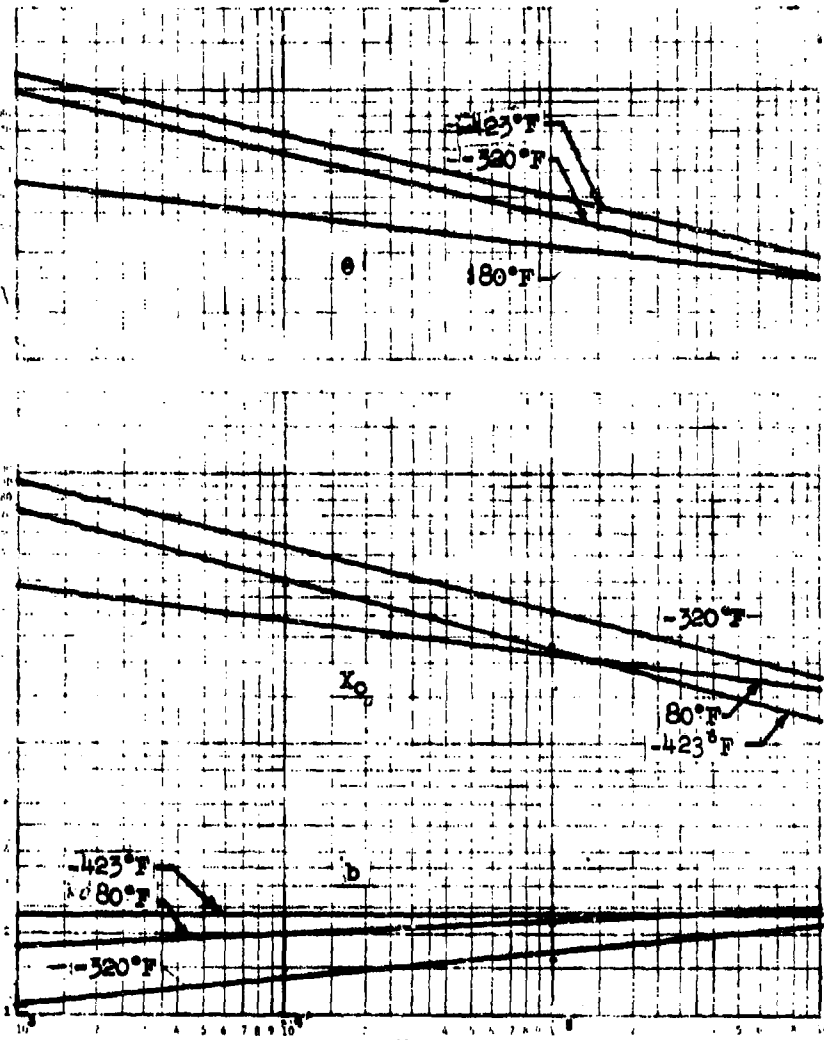
Figure: 6.64 (For Tabulated Data See Page 224)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 58 \text{ ksi}$

## Effect of Temperature



Axial Load, Completely Reversed

Mechanically Polished

Stress Conc. Factor  $K_t = 3.5$

Mean Stress = 0

Composition:

Heat Treatment: (See Page 200,

18% Cr, 10% Ni, 2% Mn, 1% Si,

Hot Rolled, Annealed

Item 12.

.08% C

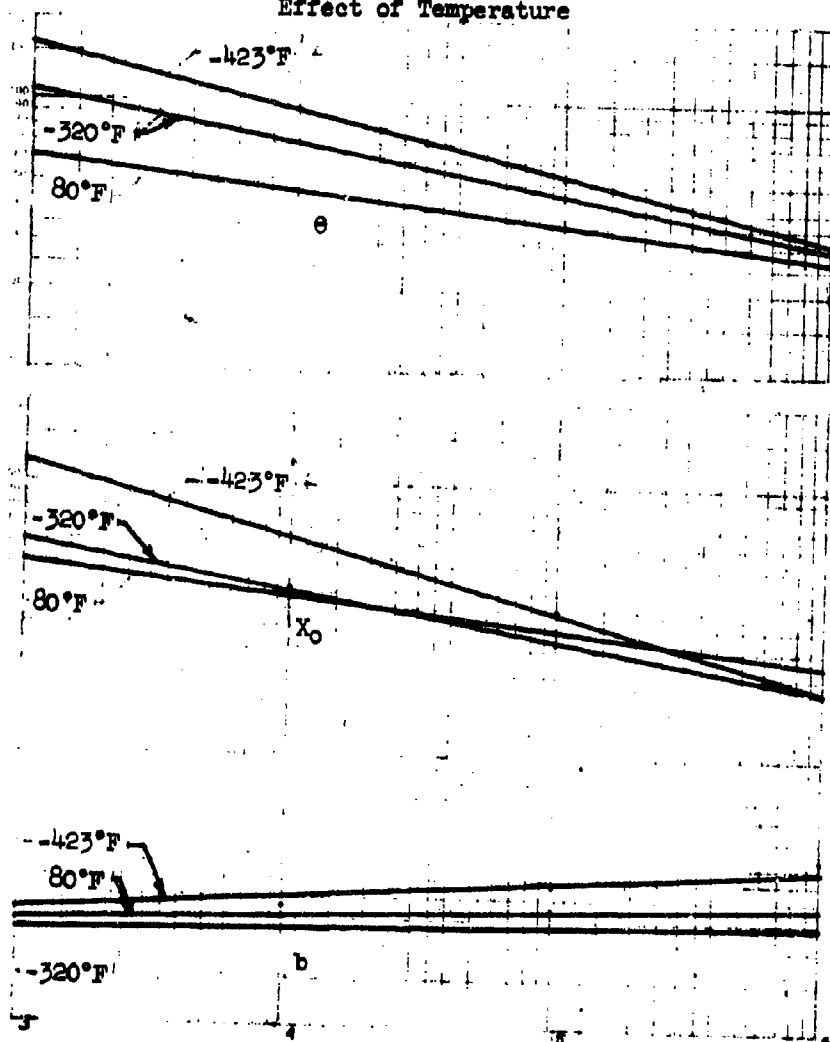
Figure 6.65 (For Tabulated Data See Page 324)

# FATIGUE STRENGTH

321 Stainless Steel

$S_u = 86 \text{ ksi}$ ,  $S_y = 38 \text{ ksi}$

Effect of Temperature



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 1.0$

Composition:

18% Cr, 10% Ni, 2% Mn, 1% Si,  
.08% C

Tungsten Inert Gas Welded

Mean Stress = 0

Heat Treatment: See Page 200,

Hot Rolled, Annealed Item 12.

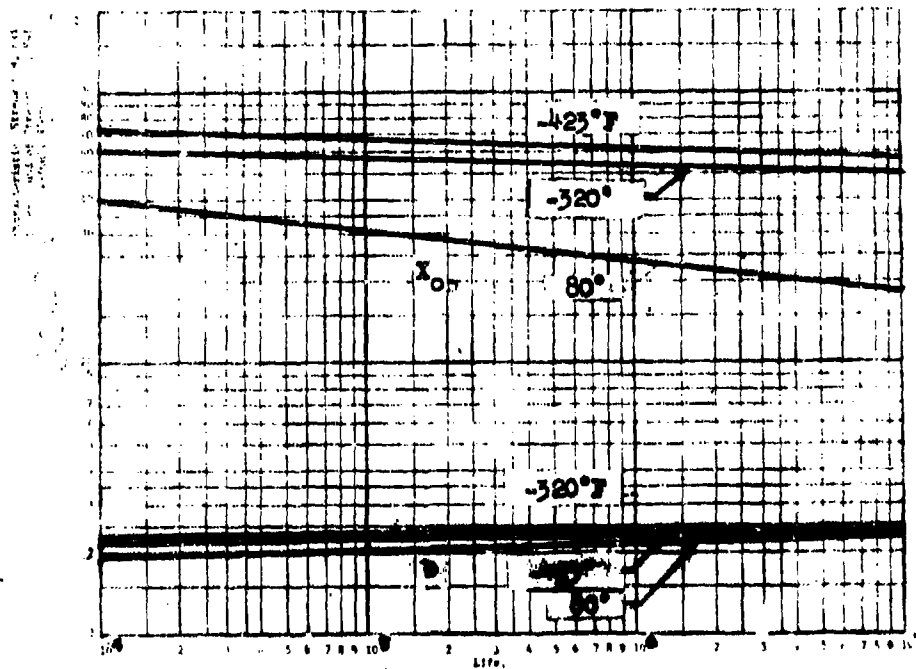
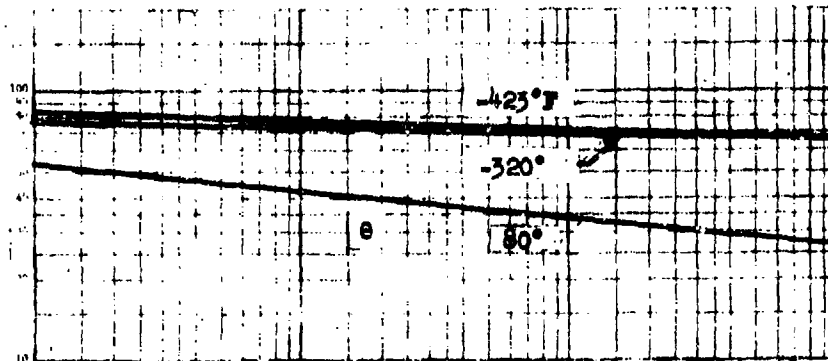
Figure: 6.66 (For Tabulated Data See Page 225 )

A-286 Stainless Steel

FATIGUE STRENGTH

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Temperature



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 1.0$

Composition:

15% Cr, 26% Ni, 1.2% Mo, 2% Ti,  
.2% Al

Mechanically Polished

Mean Stress = 0

Heat Treatment: Page 200, Item 13

Solution Treated, Hot Rolled

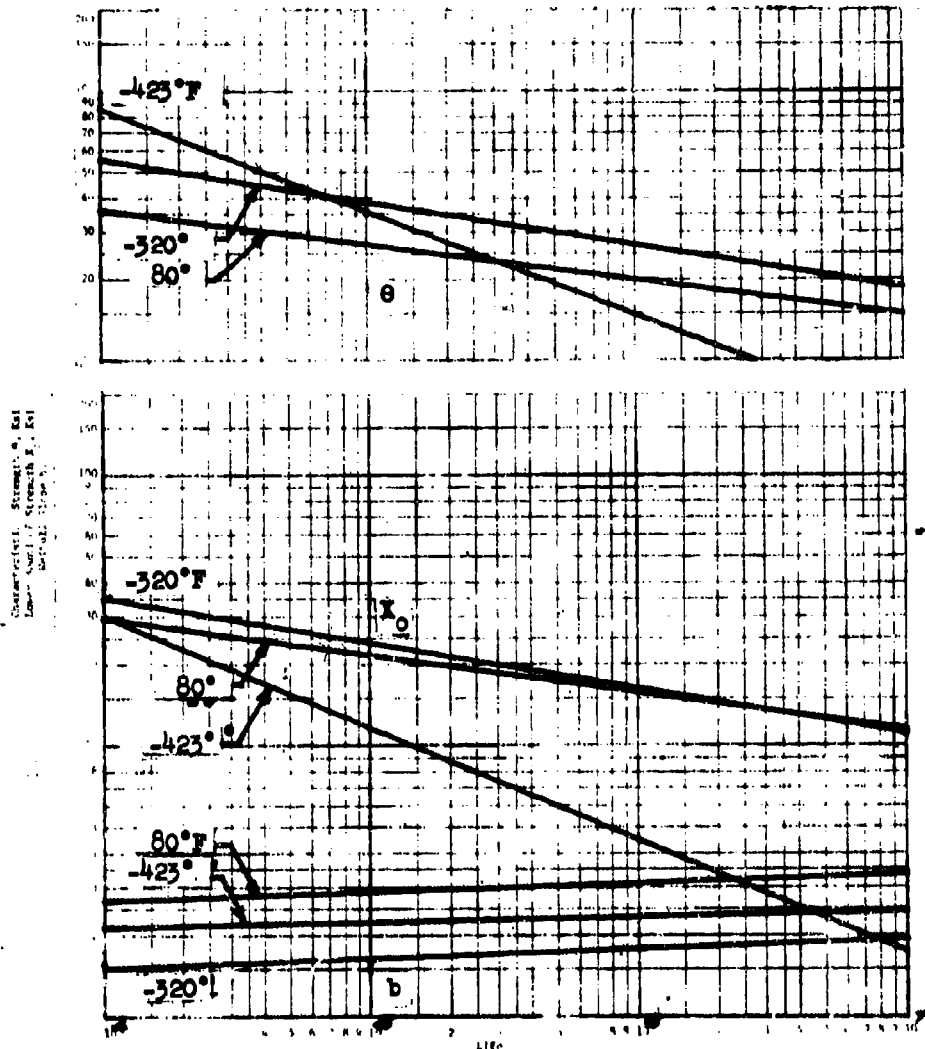
Figure: 6.67 (For Tabulated Data See Page 286)

# FATIGUE STRENGTH

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Temperature



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 3.5$

Composition:

17% Cr, 26% Ni, 1.25% Mo, 2% Ti,  
.25% Al

Mechanically Polished

Mean Stress = 0

Heat Treatment: Page 200, Item 13

Solution Treated, Hot Rolled

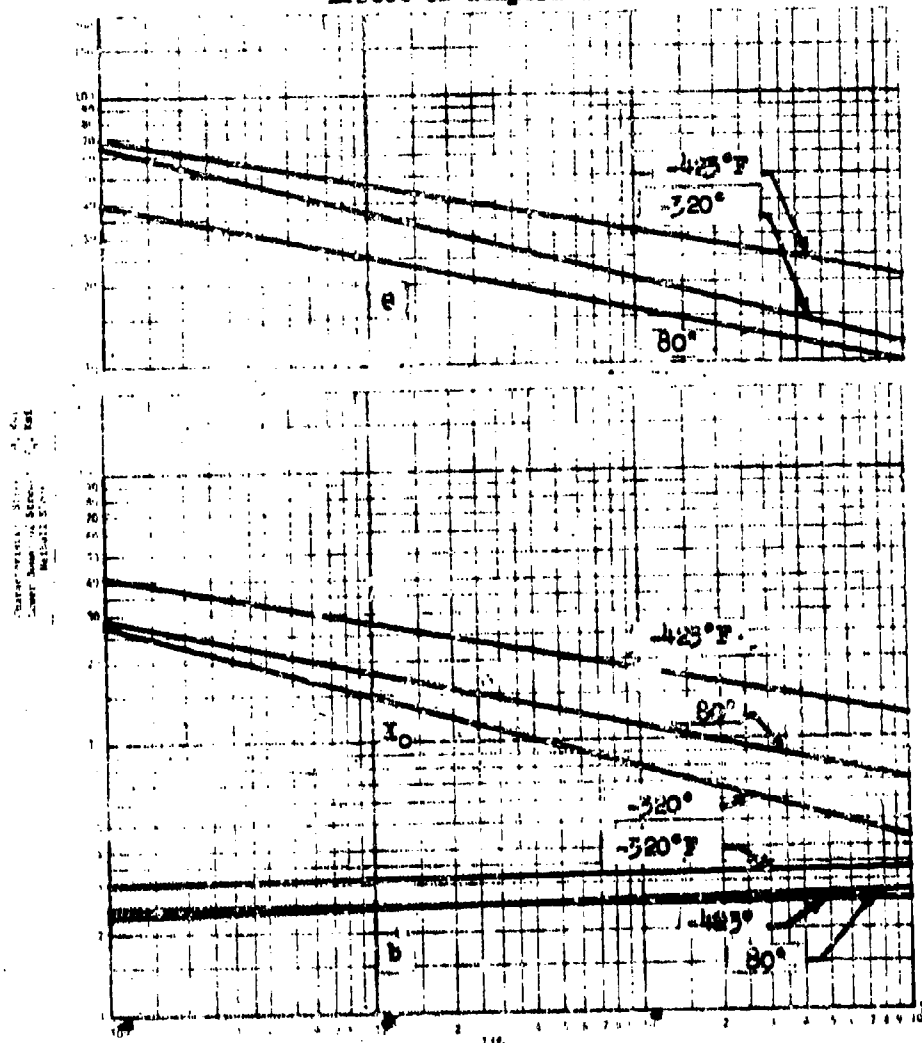
Figure: 6.68 (For Tabulated Data See Page 226 )

# FATIGUE STRENGTH

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

## Effect of Temperature



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 1.0$

Composition:

1% Cr, 26% Ni, 1.25% Mo, 2% Ti,  
.25% Al

Tungsten Inert Gas Welded

Mean Stress = 0

Heat Treatment: Page 200, Item 13

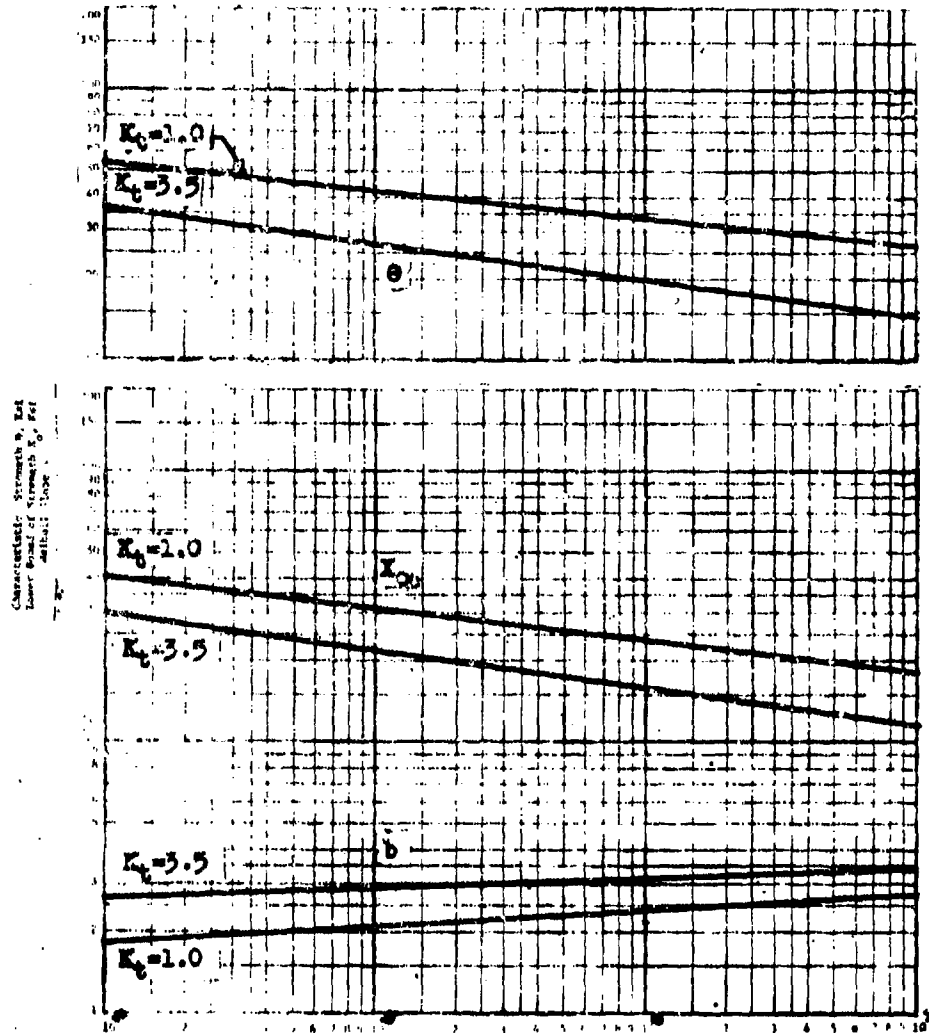
Solution treated, Not Rolled

Figure: 6.69 (For Tabulated Data See Page 243)

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Stress Concentration



Axial Load, Completely Reversed

Mechanically Polished

Temperature = 80°F

Mean Stress = 0

Composition:

15% Cr, 26% Ni, 1.25% Mo, 2% Ti,  
.25% Al

Heat Treatment: Page 200, Item 13  
Solution Treated, Hot Rolled

Figure: 6.70 (For Tabulated Data See Page 226)

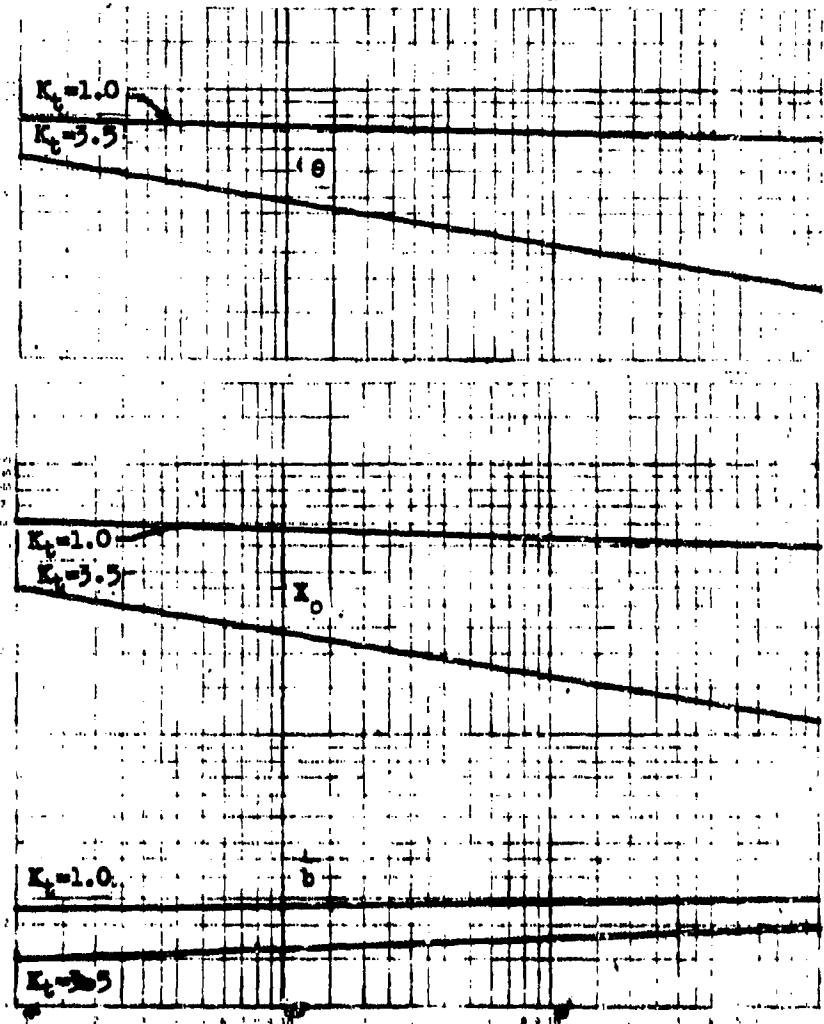


# FATIGUE STRENGTH

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Temperature = -320°F

Composition:

15% Cr, 26% Ni, 1.25% Mo, 2% Ti,  
.25% Al

Mechanically Polished

Mean Stress = 0

Heat Treatment: Page 200, Item 13

Solution Treated, Hot Rolled

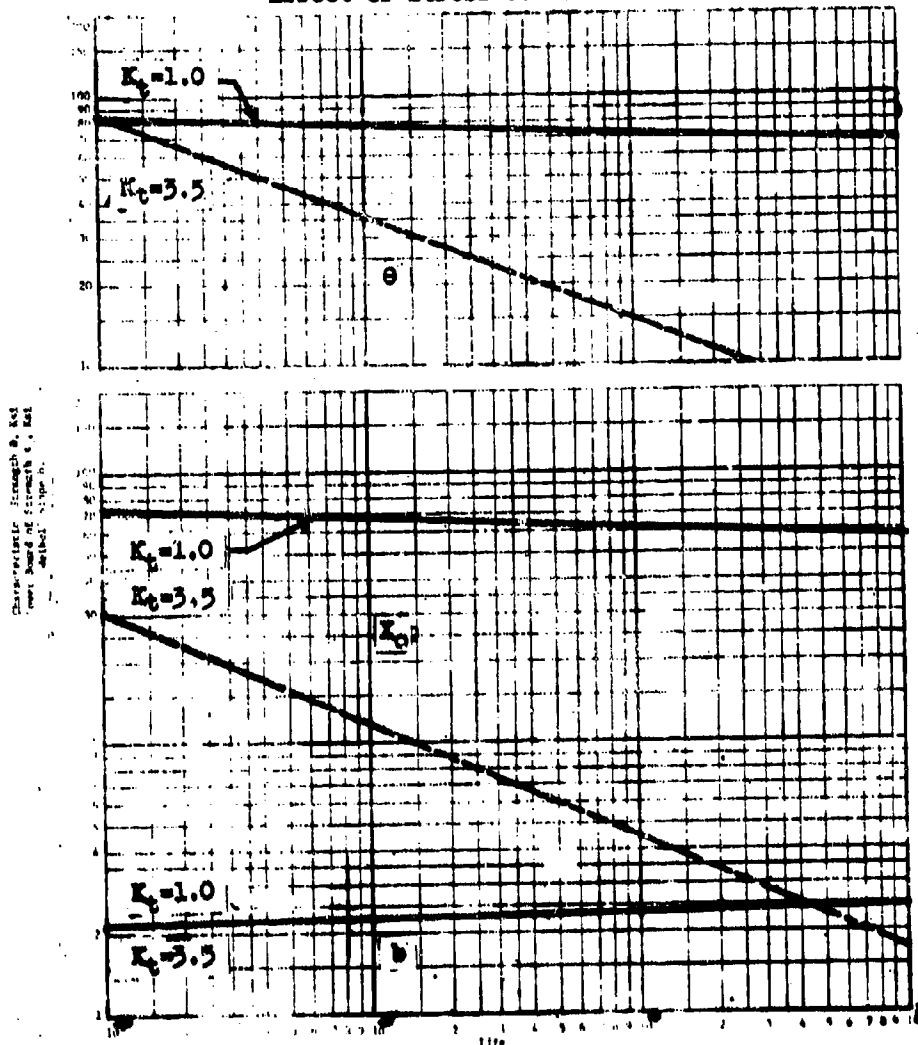
Figure: 6.71 (For Tabulated Data See Page 226)

# FATIGUE STRENGTH

A-286 Stainless Steel

$R_u = 90 \text{ ksi}$   $R_y = 46 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Mechanically Polished

Temperature = -423°F

Mean Stress = 0

Composition:

Heat Treatment: Page 200 Item 1.3

15% Cr, 26% Ni, 1.25% Mo, 2% Ti,

Solution Treated, Not Rolled

.25% Al

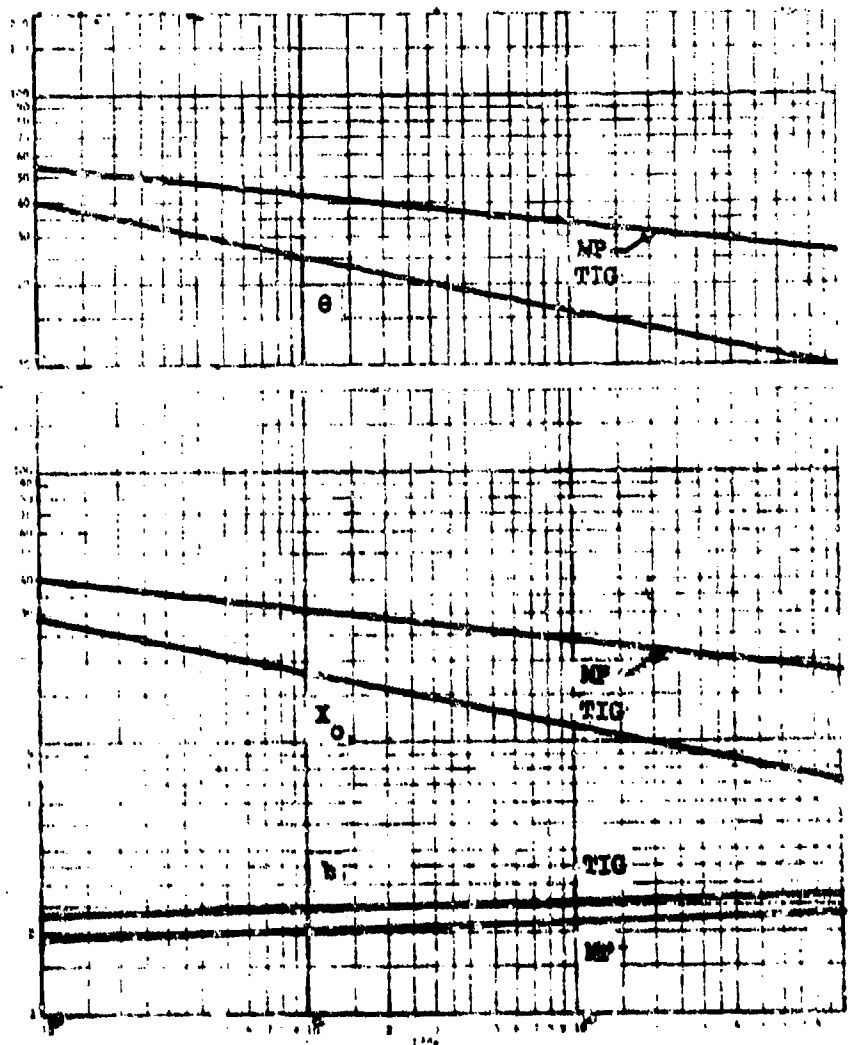
Figure: 6.72 (For Tabulated Data See Page 226)

# FATIGUE STRENGTH

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Process at 80°F



Axial Load, Completely Reversed

TIG = Tungsten Inert Gas  
Welded

NP = Mechanically Polished

Mean Stress = 0

Stress Conc. Factor  $K_f = 1.0$

Composition:

15% Cr, 20% Ni, 1.25% Mo, 2% Ti,  
.85% Al

Heat Treatment: Page 200, Item 13  
Solution Treated, Not Rolled

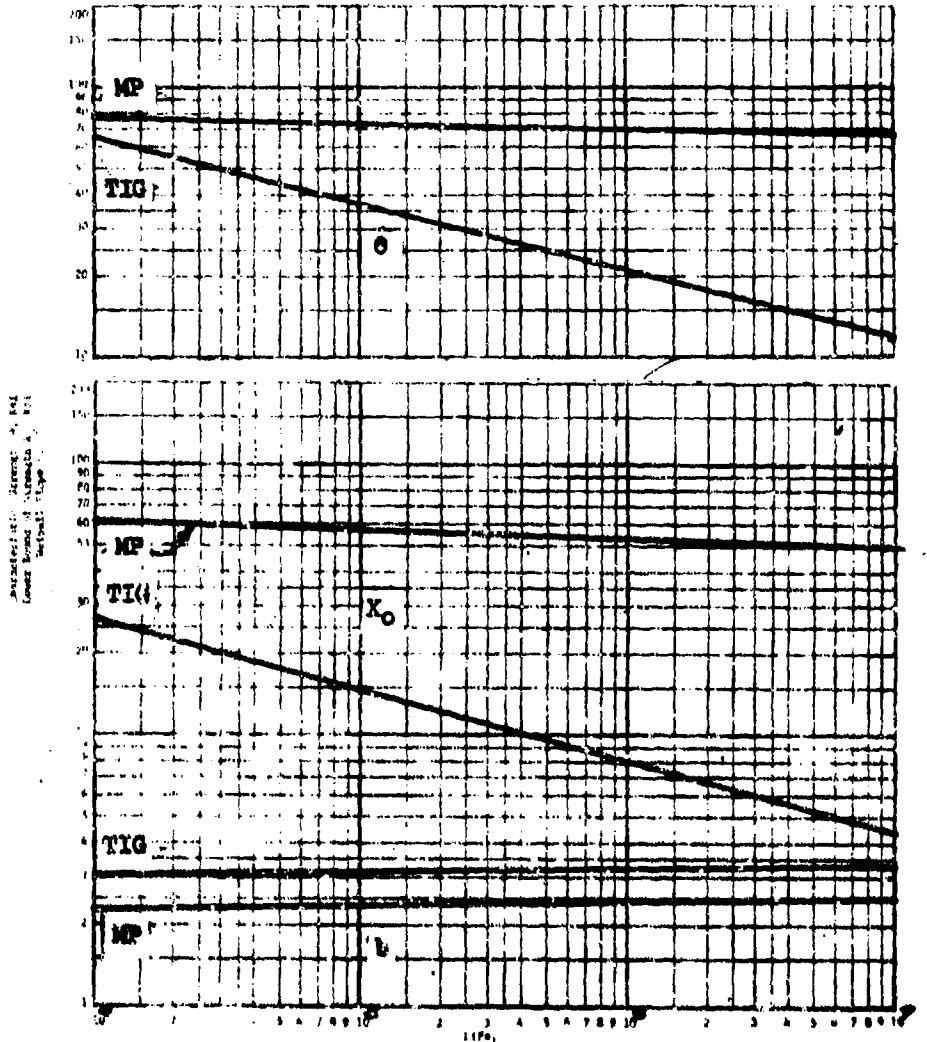
Figure: 6.13 (For Tabulated Data See Page 216)

# FATIGUE STRENGTH

A-286 Stainless Steel

$S_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Process at  $-320^\circ\text{F}$



Axial Load, Completely Reversed

Stress Conc. Factor  $K_t = 1.0$

Composition:

15% Cr, 26% Ni, 1.25% Mo, 2% Ti,  
.25% Al

TIG = Tungsten Inert Gas  
Welded

MP = Mechanically Polished

Mean Stress = 0

Heat Treatment: Page 200, Item 13

Solution Treated, Hot Rolled

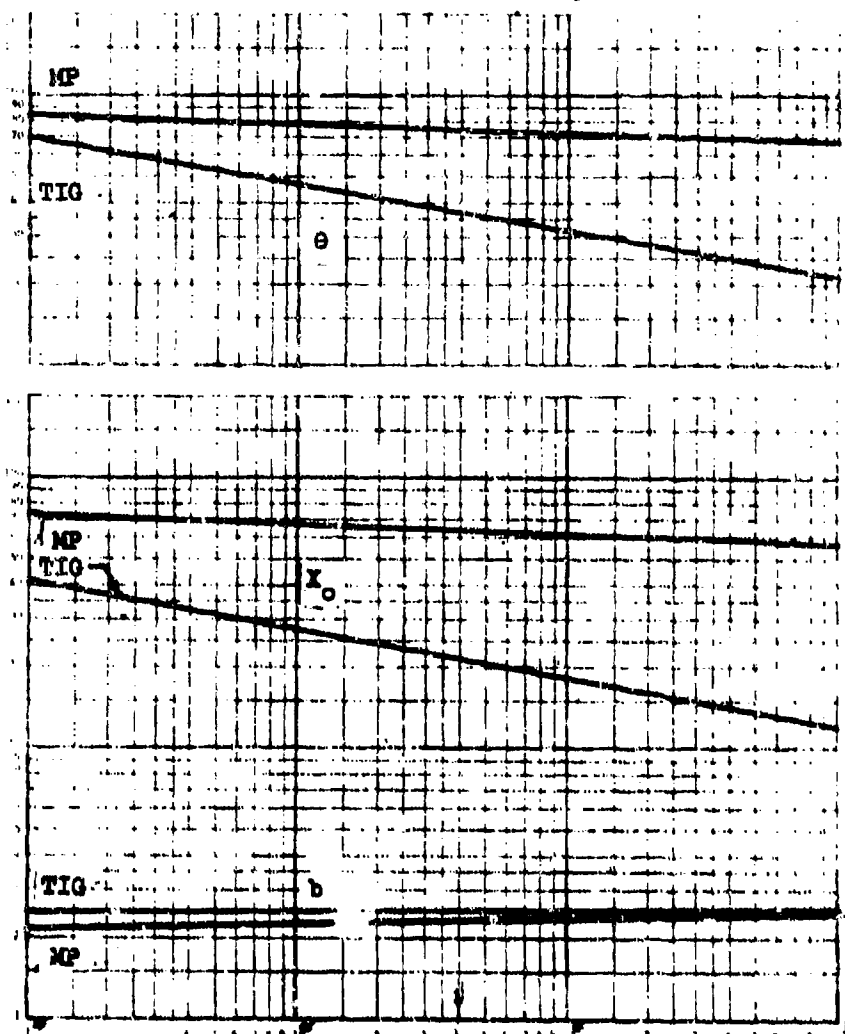
Figure 6.74 (For Tabulated Data See Page 226.)

# FATIGUE STRENGTH

A-286 Stainless Steel

$R_u = 90 \text{ ksi}$   $S_y = 46 \text{ ksi}$

Effect of Process at  $-423^\circ\text{F}$



Axial Load, Completely Reversed

TIG = Tungsten Inert Gas  
Welded

MP = Mechanically Polished

Mean Stress = 0

Stress Conc. Factor  $K_t = 1.0$

Composition:

17% Cr, 28% Ni, 1.25% Mo, 2% Ti,  
.25% Al

Heat Treatment: Page 200, Item 13  
Solution Treated, Hot Rolled

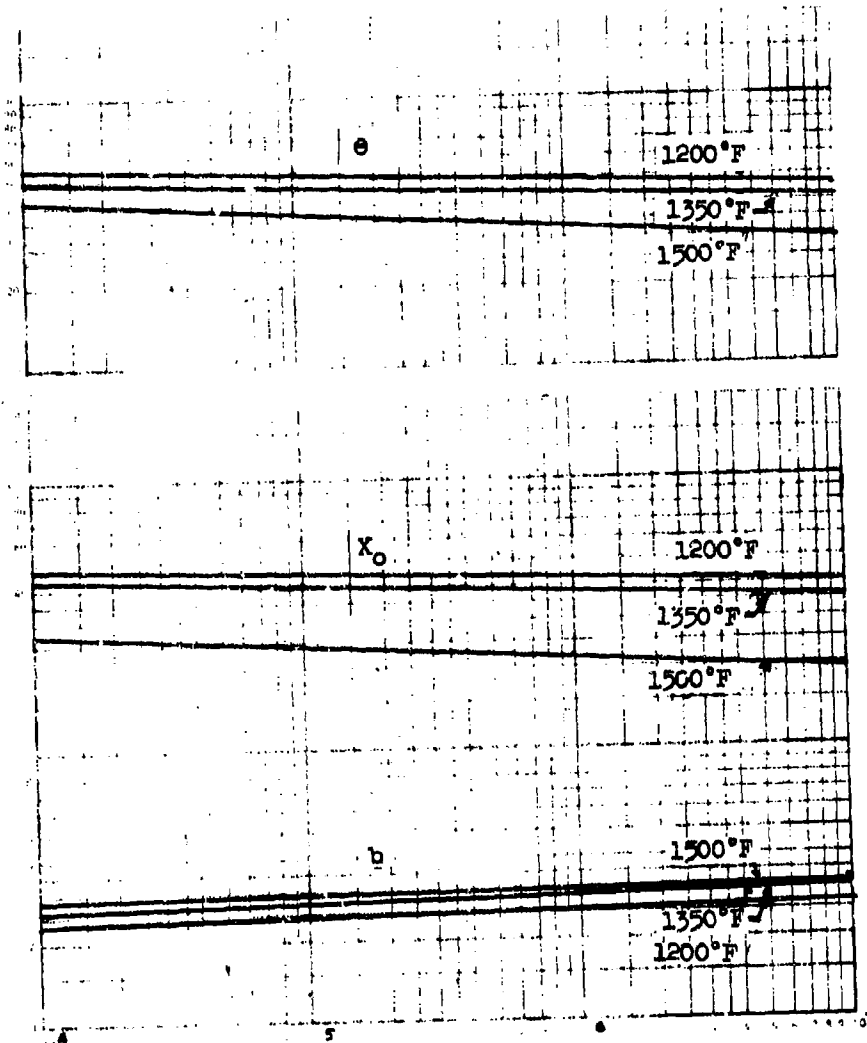
Figure 6.73 (For Tabulated Data See Page 226)

FATIGUE STRENGTH

Multiment N-155

$S_u = 119 \text{ ksi}$      $S_y = 60 \text{ ksi}$

Effect of Temperature



Axial Load, Completely Reversed  
Composition:

21% Cr, 20% Ni, 20% Co,  
5% Si, 3% Mo, 3% W,  
1.5% Mn, 1% Cb, .15% C

Lathe Turned or Bored,  
Mechanically Polished

Mean Stress = 0

Heat Treatment:

See Page 200, Item 15A

Figure: 6.76 (For Tabulated Data See Page 228 )

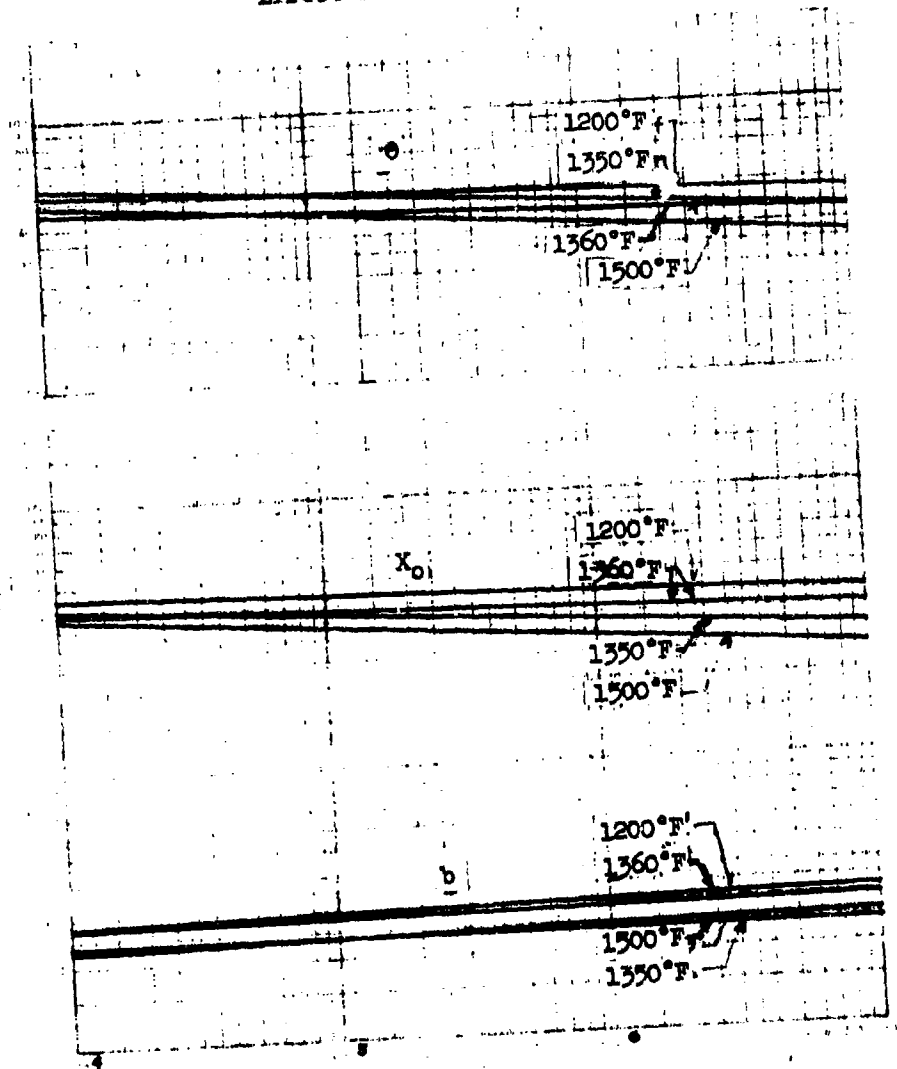
# FATIGUE STRENGTH

Multiment N-155

$S_u = 119 \text{ ksi}$

$S_y = 60 \text{ ksi}$

## Effect of Temperature



Rotary Beam Bending

Composition:

21% Cr, 20% Ni, 20% Co  
5% Si, 3% Mo, 3% W,  
1.5% Mn, 1% Cu, .15% C

Figure 6.77

Lathe Turned or Bored,  
Mechanically Polished

Mean Stress = 0

Heat Treatment:

See Page 200 Item 15A

(For Tabulated Data See Page 228)

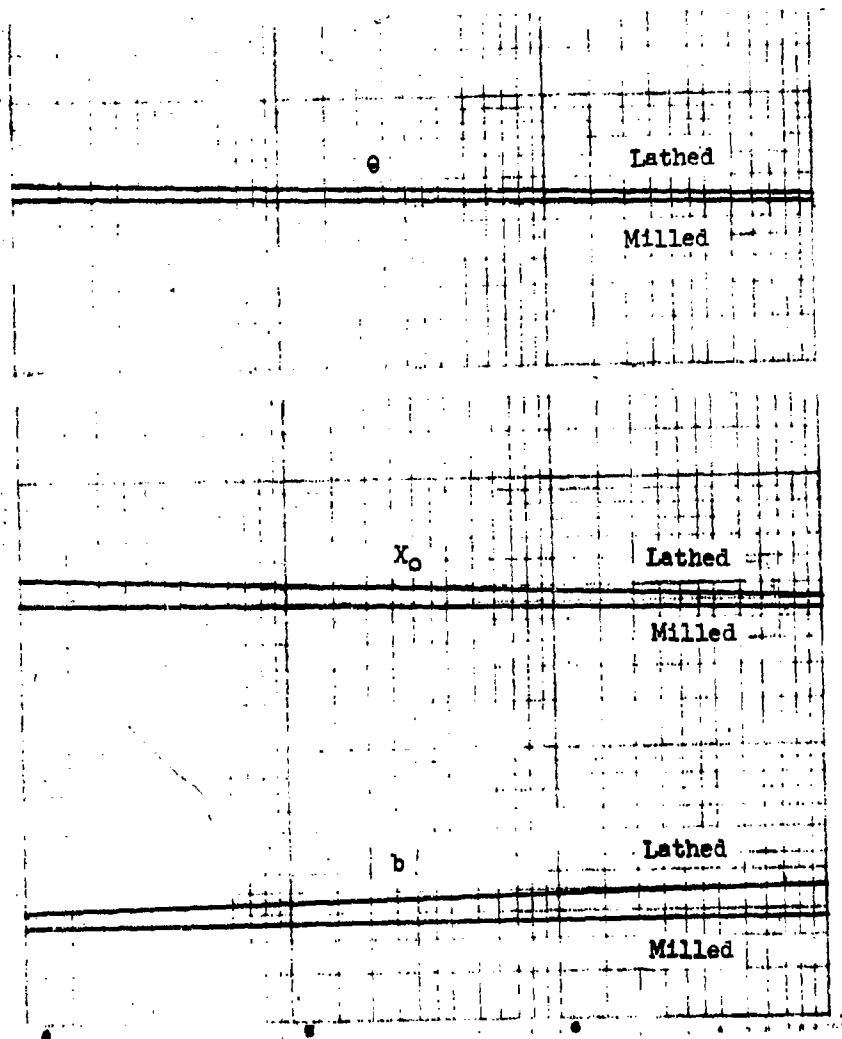
# FATIGUE STRENGTH

Multiment N-155

$S_u = 119 \text{ ksi}$

$S_y = 60 \text{ ksi}$

Effect of Surface Treatment



Axial Load Completely Reversed

Temperature = 1350°F

Composition:

21% Cr, 20% Ni, 20% Co,

5% Si, 3% Mo, 3% W,

1.5% Mn, 1% Cb, .15% C

Figure: 6.78

(For Tabulated Data See Page 228 )

Lathe Turned or Bored,

Mean Stress = 0

Heat Treatment:

See Page 200, Item 15A



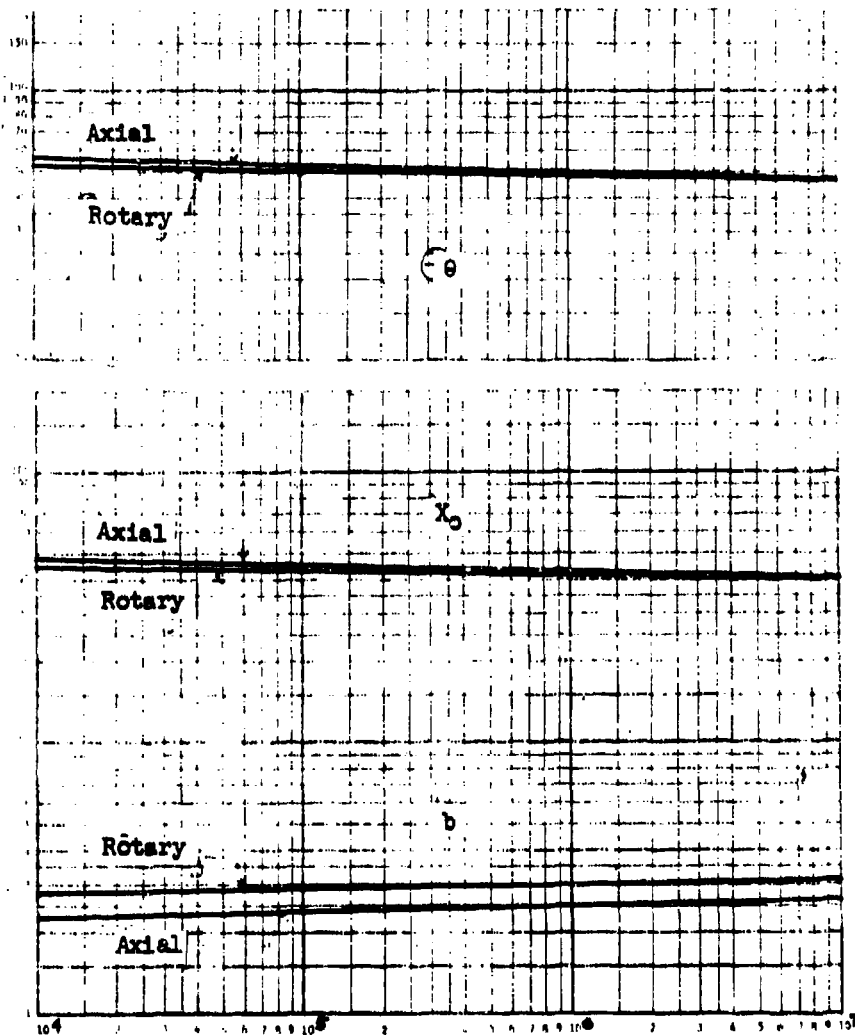
# FATIGUE STRENGTH

Multiment N-155

$S_u = 119 \text{ ksi}$

$S_y = 60 \text{ ksi}$

## Effect of Type of Loading



Temperature = 1200°F

Composition:

21% Cr, 20% Ni, 20% Co

5% Si, 3% Mo, 3% W,

1.5% Mn, 1% Cu, .15% C

Lathe Turned or Bored,  
Mechanically Polished  
Mean Stress = 0

Heat Treatment:

See Page 200, Item 15A

Figure: 6.79

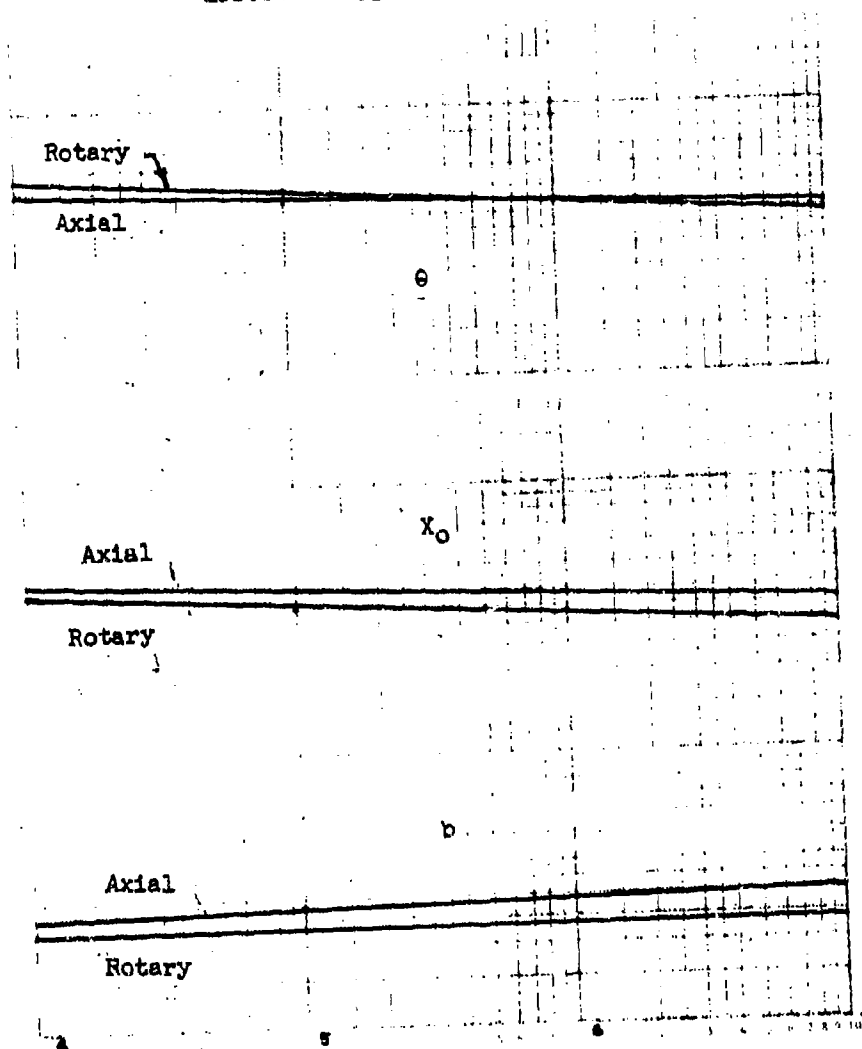
(For Tabulated Data See Page 228.)

# FATIGUE STRENGTH

Multiment N-155

$S_u = 119 \text{ ksi}$        $S_y = 60 \text{ ksi}$

## Effect of Type of Loading



Temperature = 1350°F  
 Composition:  
 21% Cr, 20% Ni, 20% Co,  
 5% Si, 3% Mo, 3% W,  
 1.5% Mn, 1% Cb, .15% C  
 Figure: 6.80

Lathe Turned or Bored,  
 Mechanically Polished  
 Mean Stress = 0  
 Heat Treatment:  
 See Page 200, Item 15A

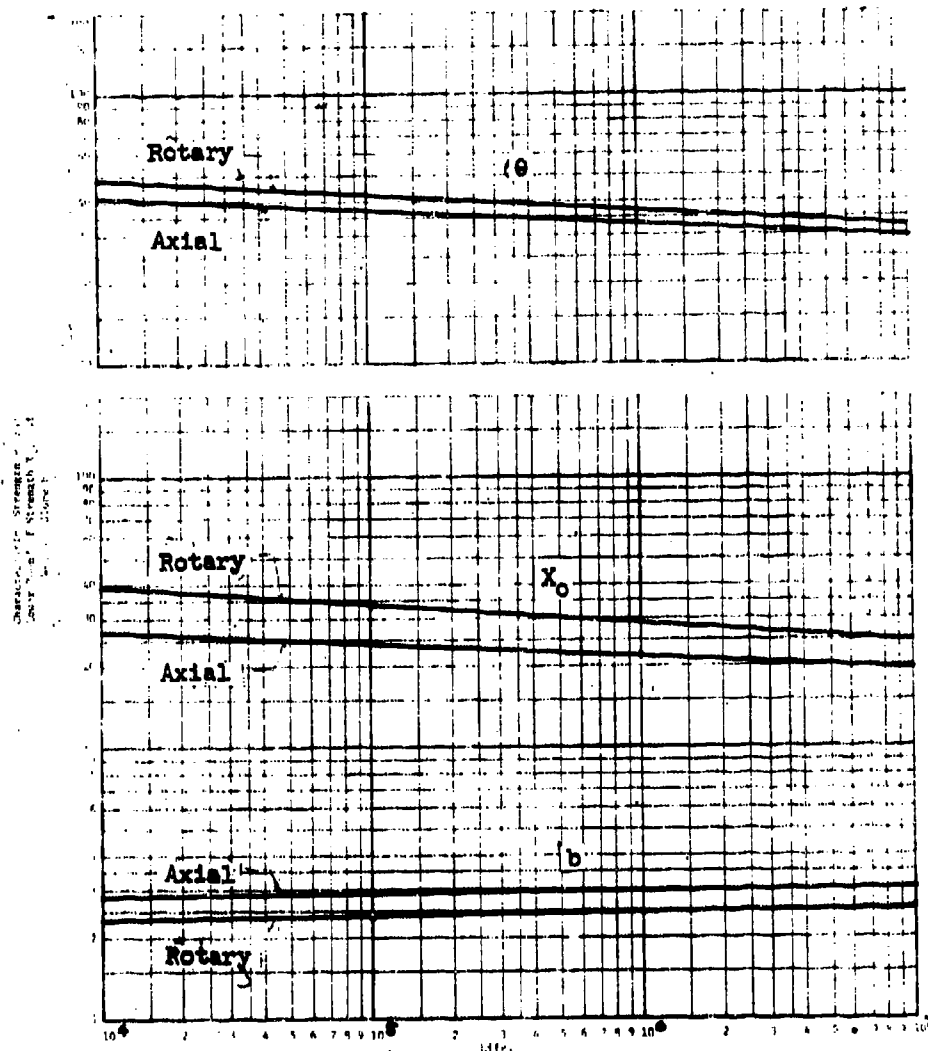
(For Tabulated Data See Page 228 )

# FATIGUE STRENGTH

Multiment N-135

$S_u = 119 \text{ ksi}$      $S_y = 60 \text{ ksi}$

Effect of Type of Loading



Temperature = 1500°F

Composition:

21% Cr, 20% Ni, 20% Co

5% Si, 3% Mo, 3% W,

1.5% Mn, 1% Cb, .15% C

Lathe Turned or Bored,

Mechanically Polished

Mean Stress = 0

Heat Treatment:

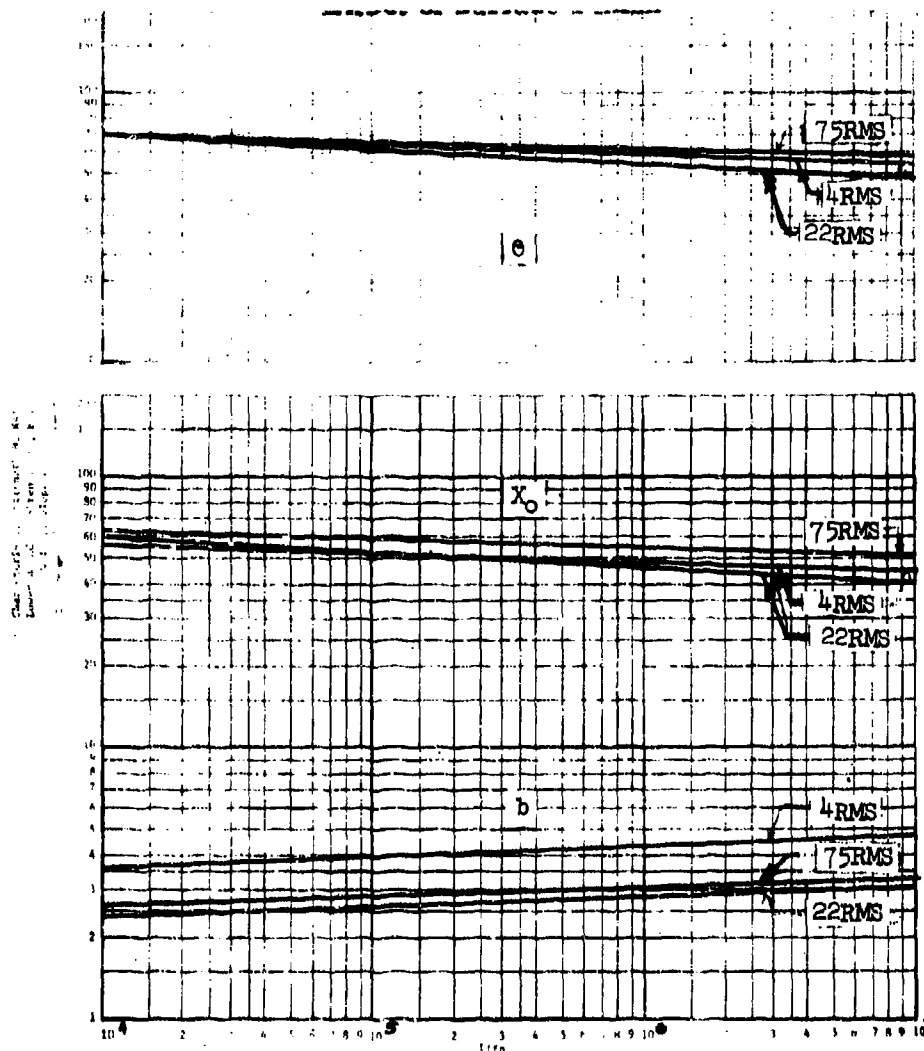
See Page 200, Item 15A

Figure: 6.81

(For Tabulated Data See Page 228)

Multiment N-155 Stainless **FATIGUE STRENGTH**  
 $S_u = 119 \text{ ksi}$   $S_y = 60 \text{ ksi}$

Effect of Surface Finish



Axial Loaded, Completely Reversed

Mean Stress = 0

Stress Conc. Factor  $K_t = 1.0$

Heat Treatment: Sol. Treated  
 2200°F, 1 hr, WQ, Aged 1400°F  
 16 hrs, A.C.

Composition: 21% Cr, 20% Ni, 20% Co  
 3% Mo, 3% W, 1.5% Mn, 1% Cu, .15% C

Figure:6.82 (For Tabulated Data See Page.229 )

Multiment N-155 Stainless FATIGUE STRENGTH  $S_u = 126 \text{ ksi}$   $S_y = 73 \text{ ksi}$

Effect of Surface Finish

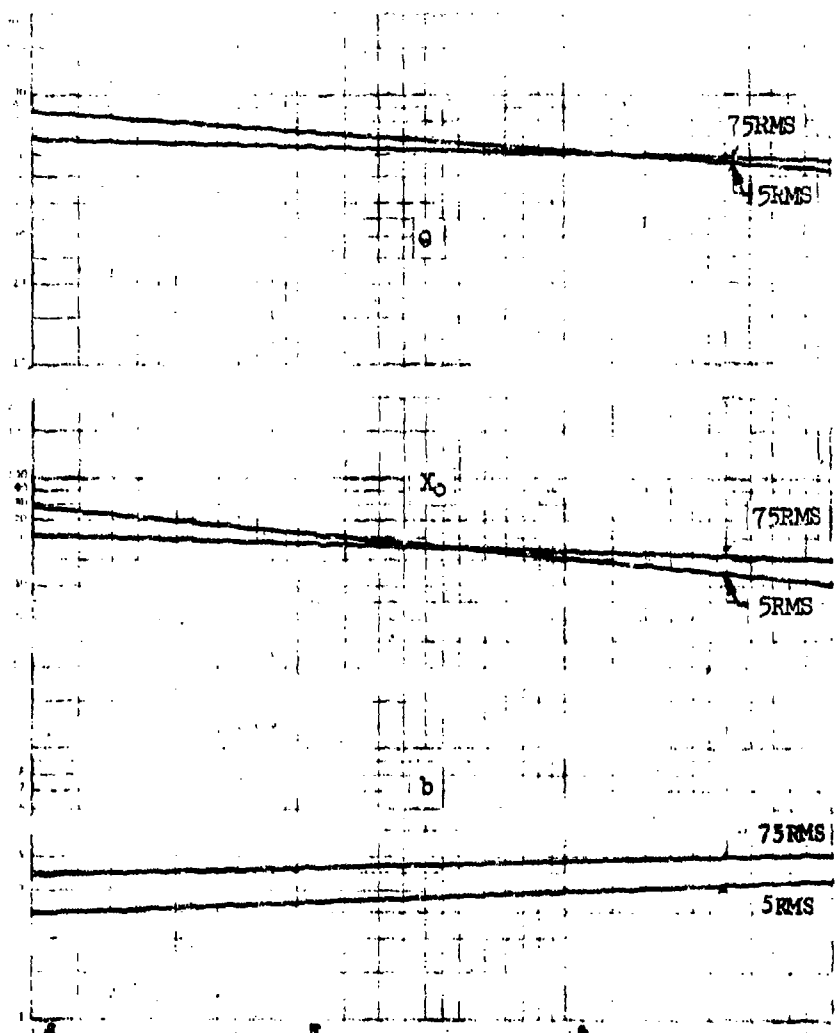


Plate Bending, Completely Reversed

Mean Stress = 0

Stress Conc. Factor,  $K_t = 1.0$

Heat Treatment: sol.  
Treated, 2200°F, 1 hr, WQ,  
Aged 1400°F, 16 hrs, A.C.

Composition: 21% Cr, 20% Ni, 20% Co,  
3% Mo, 3% W, 1.5% Mn, 1% C, .15% C.

Figure: 6.83 (For Tabulated Data See Page 229)

Multiment N-155 Stainless

# FATIGUE STRENGTH

$S_u = 119 \text{ ksi}$

$S_y = 60 \text{ ksi}$

## Effect of Surface Finish

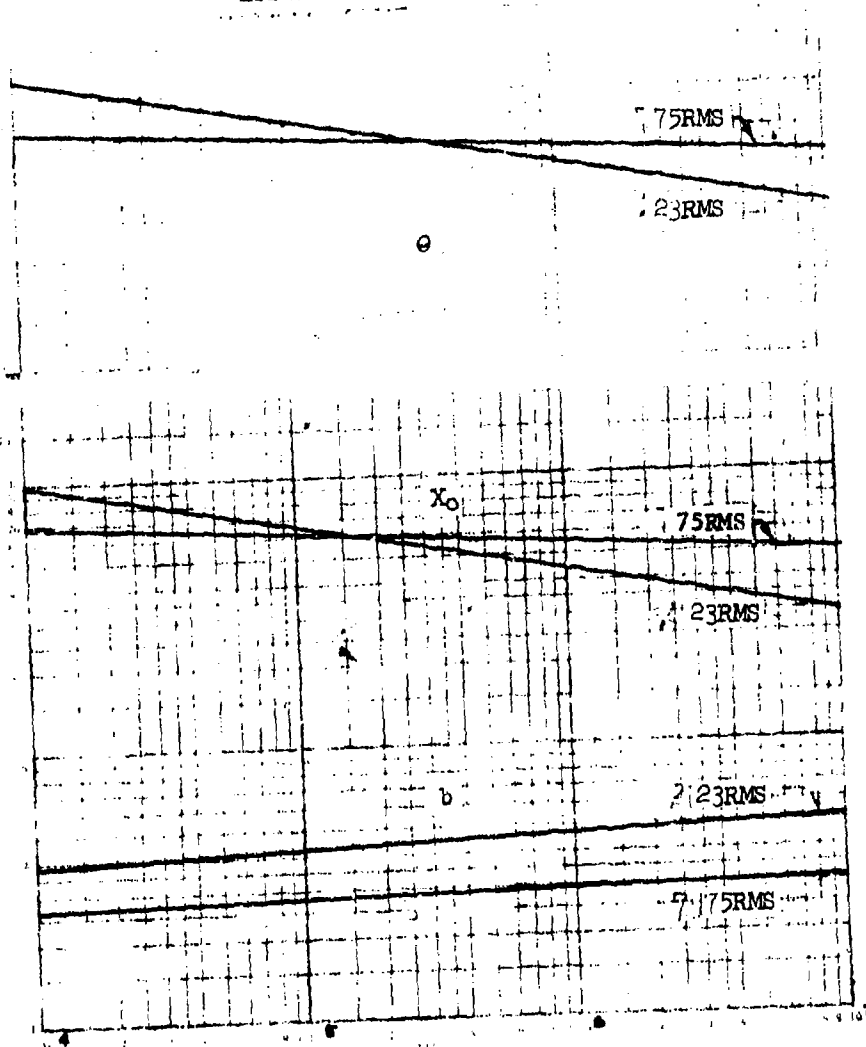


Plate Bending, Completely Reversed

Mean Stress = 0

Stress Conc. Factor  $K_t = 1.0$   
Composition: 21% Cr, 20% Ni, 20% Co  
3% Mo, 3% W, 1.57% Mn, 1% Cu, .15% C

Heat Treatment: Sol. Treated  
2200°F, 1 hr, WQ, Aged 1400°F  
16 hrs, A.C.

Figure: 6.84 (For Tabulated Data See Page 229)

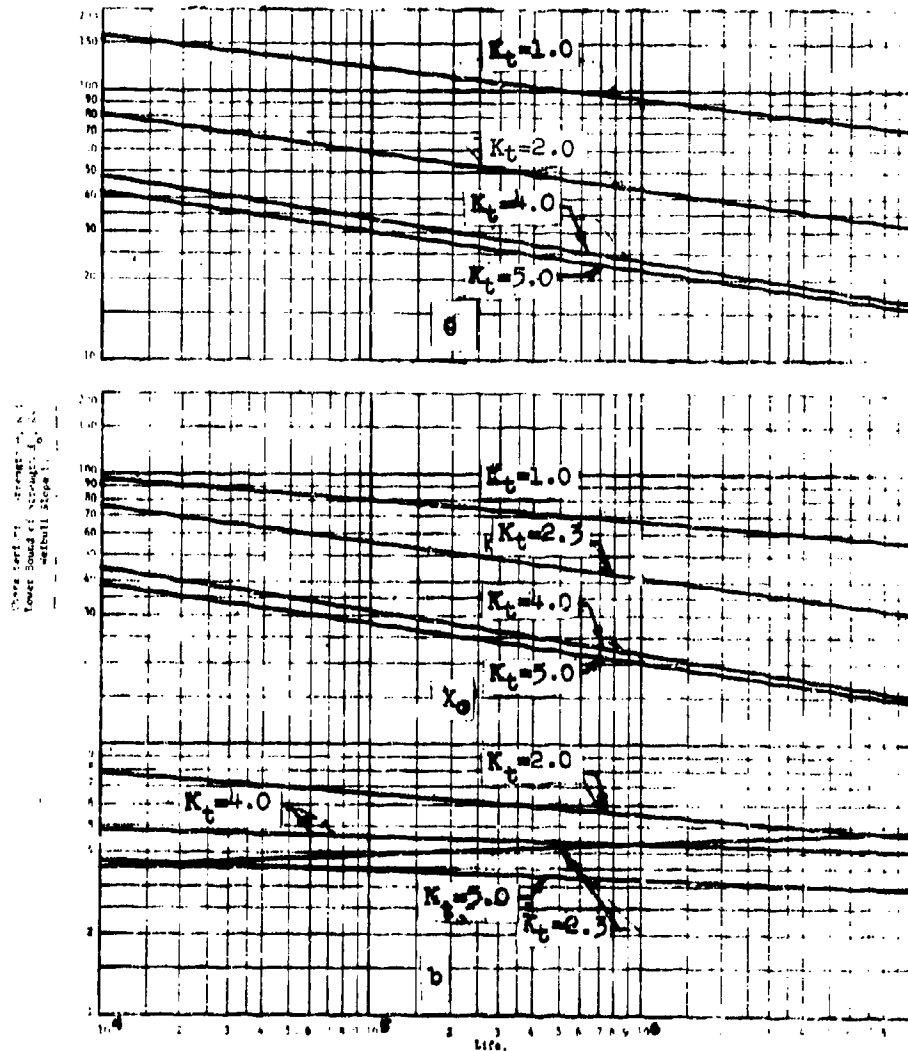
# FATIGUE STRENGTH

17-7 PH

$S_u = 205 \text{ ksi}$

$S_y = 195 \text{ ksi}$

## Effect of Stress Concentration



Axial Load, Completely Reversed

Temperature =  $800^\circ\text{F}$

Composition:

17% Cr, 7% Ni, 1.15% Al,

.4% Si, .7% Mn, .07% C

Hand Polished—Longitudinal

Mean Stress = 0

Heat Treatment:

See Page 201, Item 17

Figure: 6.85

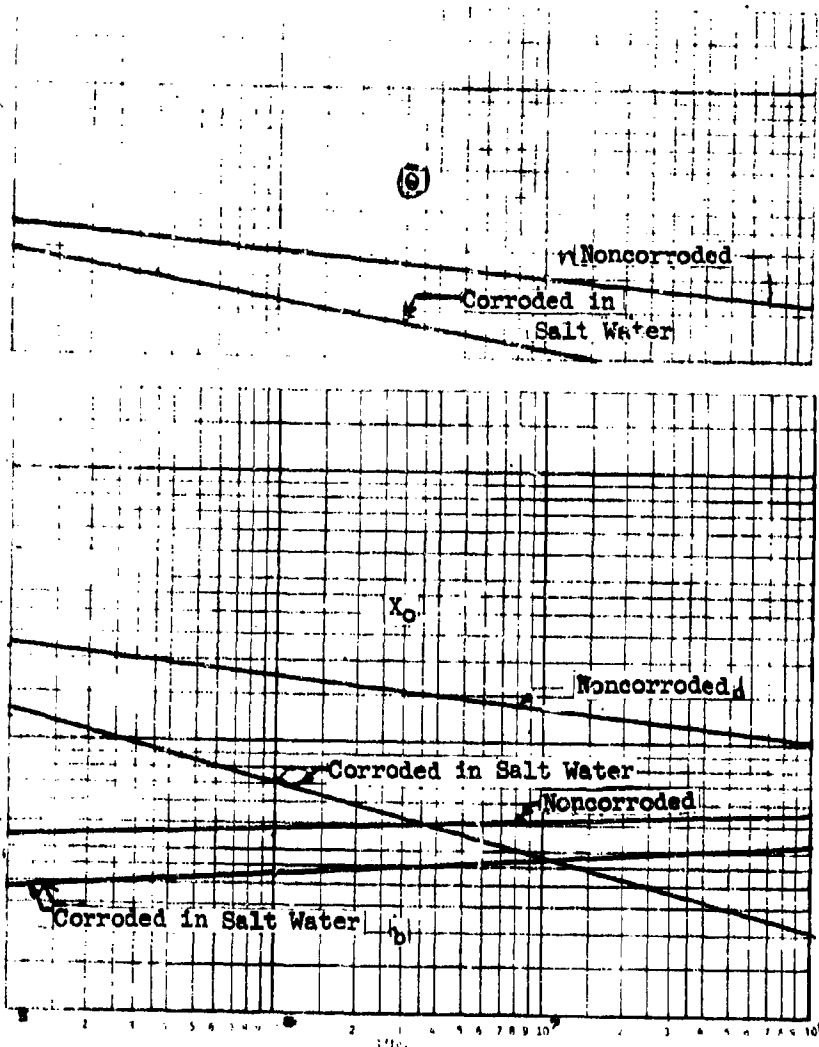
(For Tabulated Data See Page 232)

# FATIGUE STRENGTH

Duralumin

$S_{ut}$ ,  $S_y$  - Unknown

Effect of Salt Water Corrosion



Rotary Beam Bending  
Stress Conc. Factor  $K_t$  - Unknown  
Composition:  
Al, Cu, Mn, Mg

Specimen Condition:  
Unknown  
Mean Stress - Unknown  
Heat Treatment:  
Unknown

Figure:6.86 (For Tabulated Data See Page 236.)

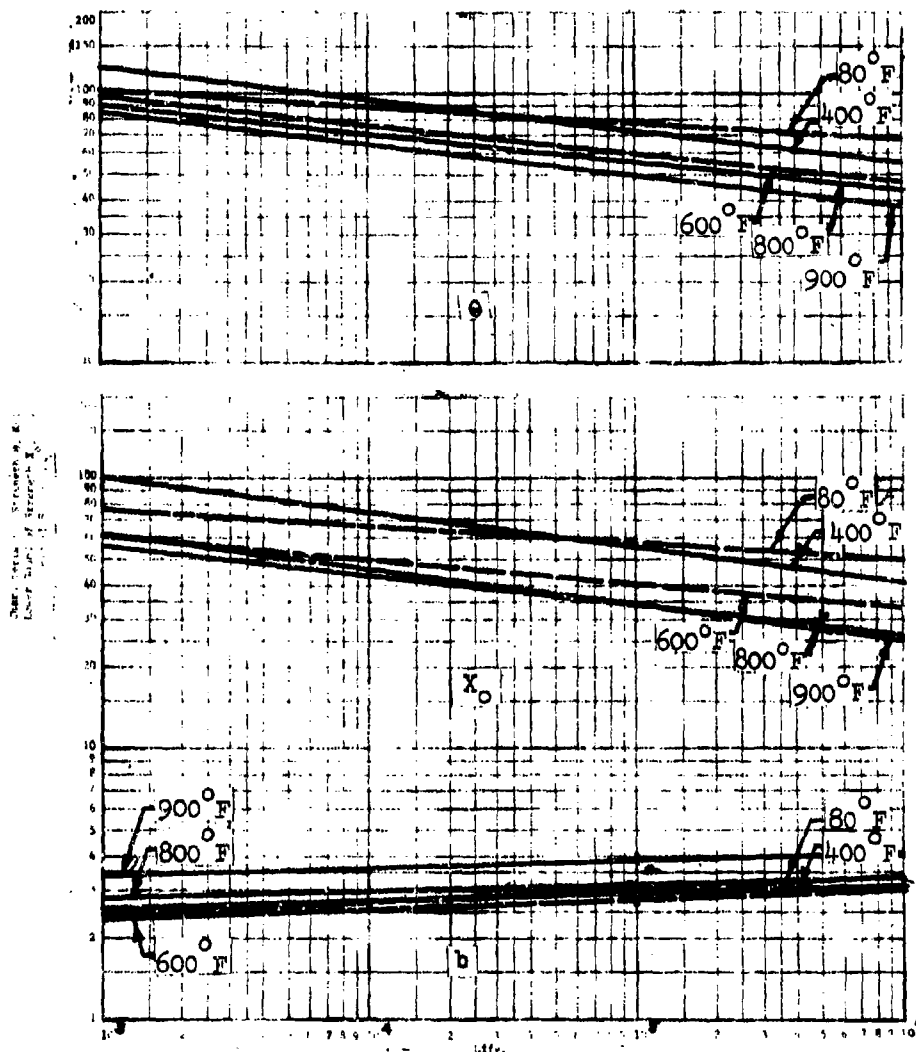


Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177$  ksi,  $S_y = 166$  ksi

## Effect of Temperature



Rotary Beam Bending

Hot Rolled

Composition:  
6% Al, 4% V, Max .07% N<sub>1</sub>, max .10% C,  
max .015% H, max .40% Fe, max .30% O

Mean Stress = 0  
Heat Treatment:  
(A: sol. treated 1690°F, 12 min.  
WQ, aged 900°F, 4 hrs. air cooled)

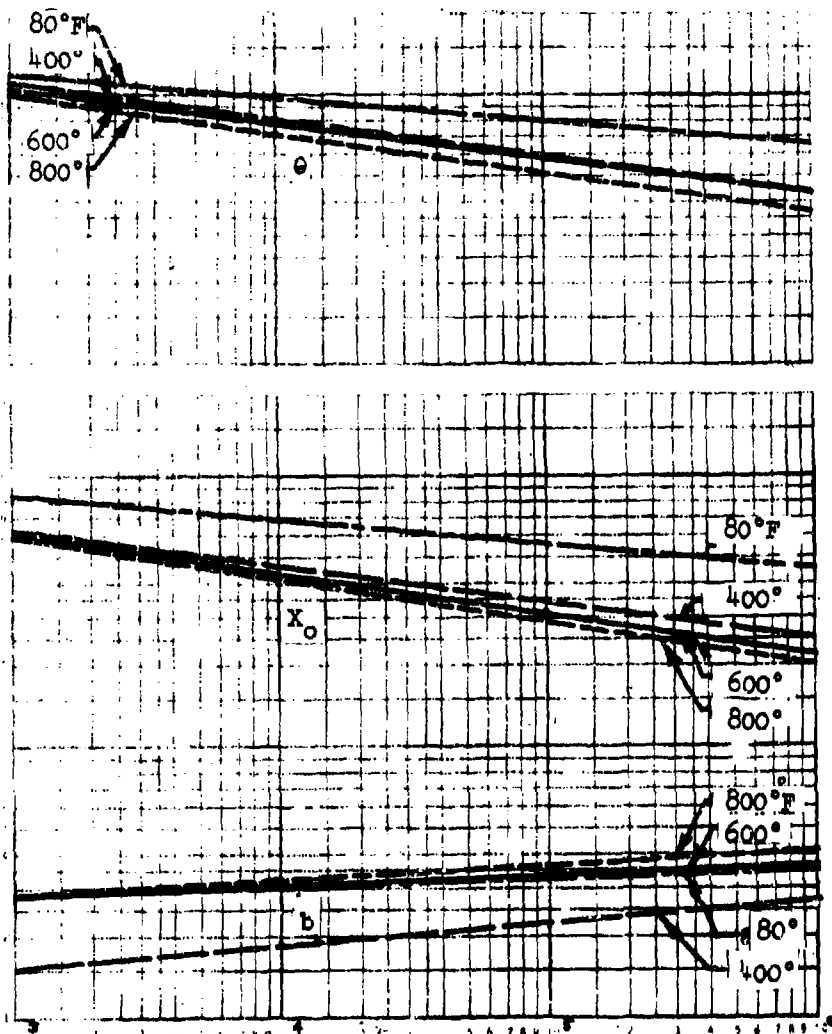
Figure 6.87 (For Tabulated Data See Page 237.)

TI-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Temperature



Rotary Beam Bending

Composition:

6%Al, 4%V, max. 0.07% N<sub>2</sub>, max. 0.10%

C, max. 0.015% H, max. 0.04% Fe, max. 0.30% O

Hot Rolled

Mean Stress = 0

Heat Treatment:

B: sol. treated 1675°F, 20 min.

WQ, aged 900°F, 4 hrs  
air cooled

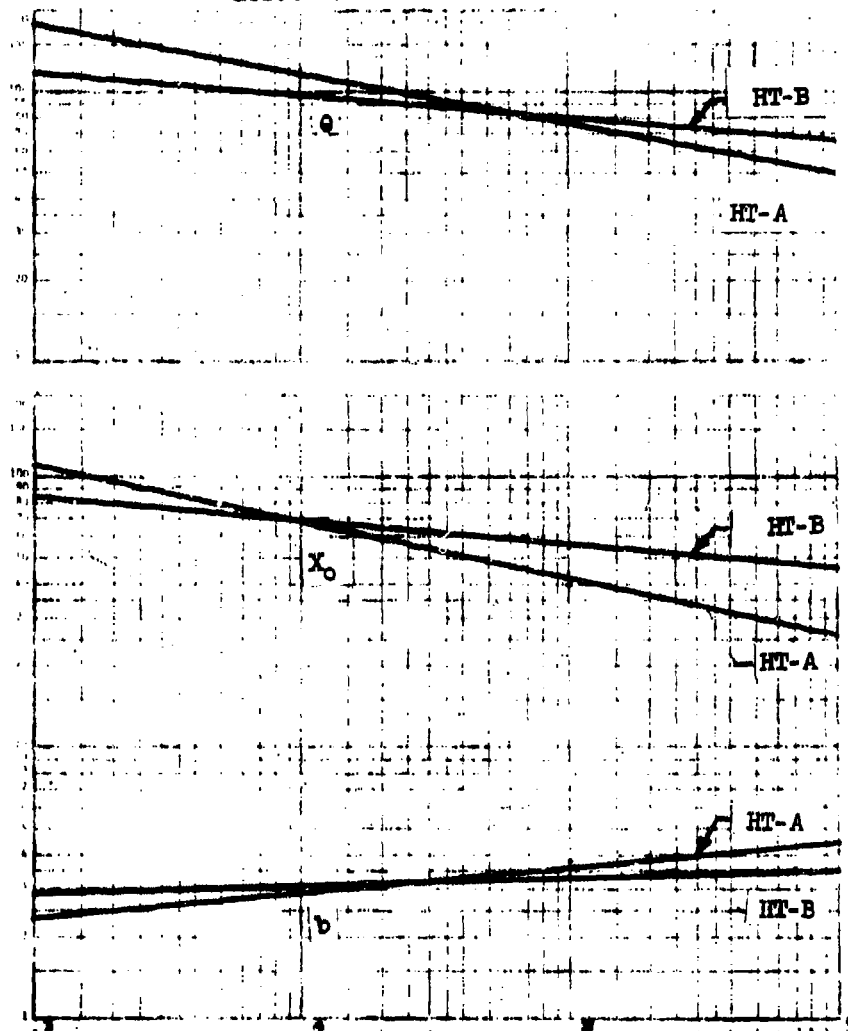
Figure: 6.88 (For Tabulated Data See Page 237)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Temperature = 80°F

Composition:

6%Al, 4%V, max .07%N, max .10% C,  
max .015%N, max .40%Fe, max .30% O

Not Rolled

Mean Stress = 0

Heat Treatment:

HT-A and HT-B

See Page 202, Item 28

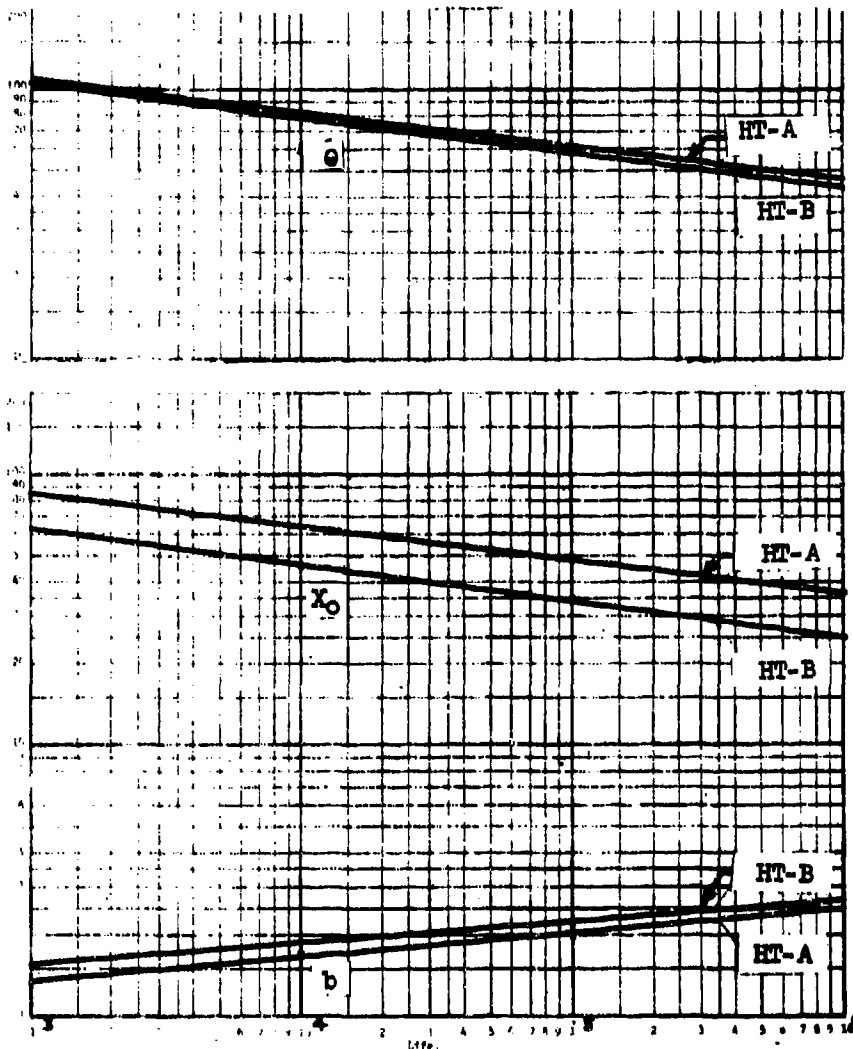
Figure: 6.89 (For Tabulated Data See Page 237)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Temperature = 400°F

Composition:

6%Al, 4%V, max .07%N, max .10% C,

max .015%H, max .40%Fe, max .30% O

Not Notched

Mean Stress = 0

Heat Treatment:

HT-A and HT-B

See Page 202, Item 28

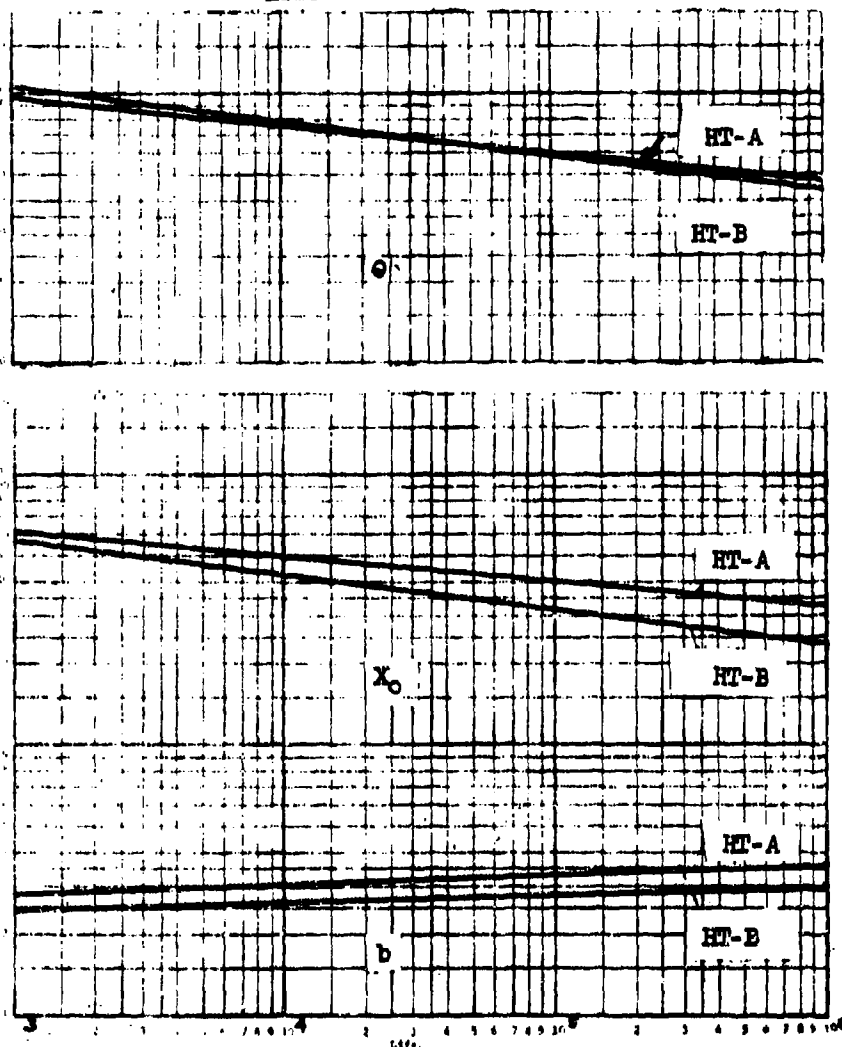
Figure: 6.90 (For Tabulated Data See Page 238)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending  
 Temperature = 600°F  
 Composition:  
 6%Al, 4%V, max .07%N, max .10% C,  
 max .015%H, max .40%Fe, max .30% O

Not Rolled  
 Mean Stress = 0  
 Heat Treatment:  
 HT-A and HT-B  
 See Page 202, Item 28

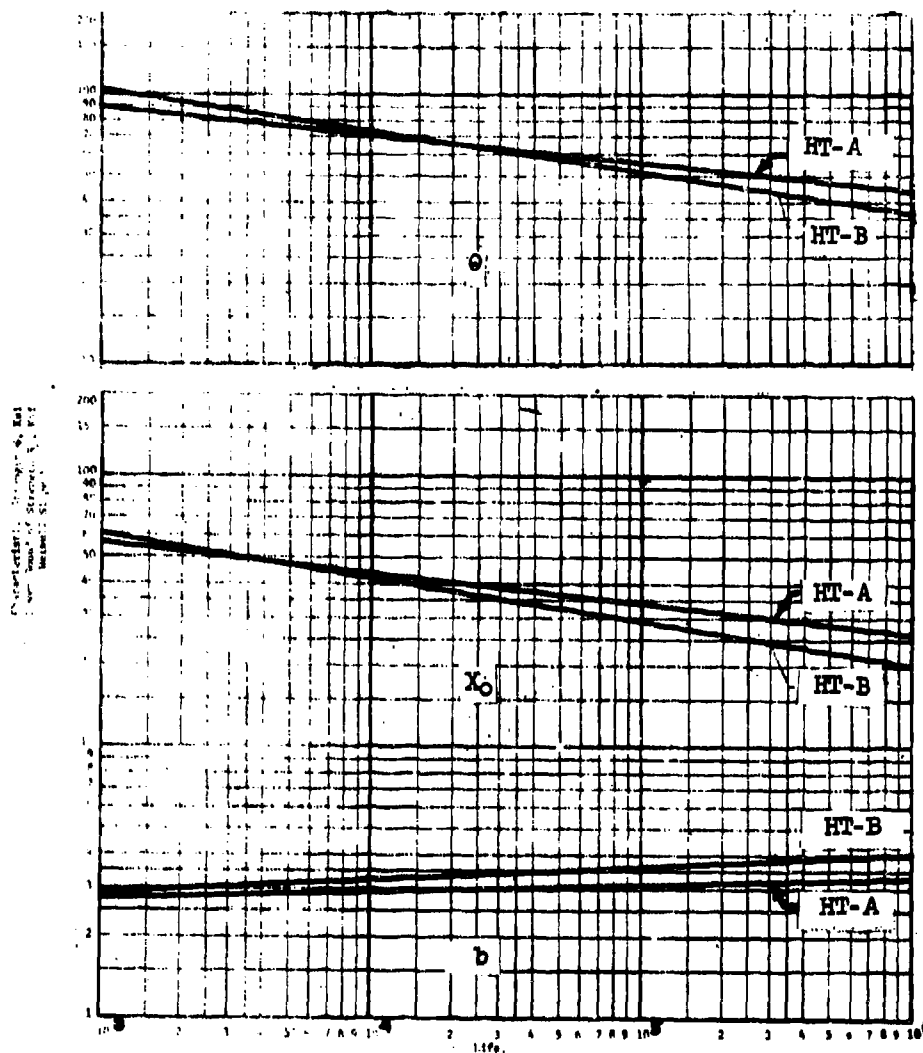
Figure: 6.91 (For Tabulated Data See Page 238)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Temperature = 800°F

Composition:

6%Al, 4%V, max .07%N<sub>1</sub>, max .10%C,  
max .015%H, max .40%Fe, max .30%O

Hot Rolled

Mean Stress = 0

Heat Treatment:

HT-A and HT-B

See Page 202, Item 28

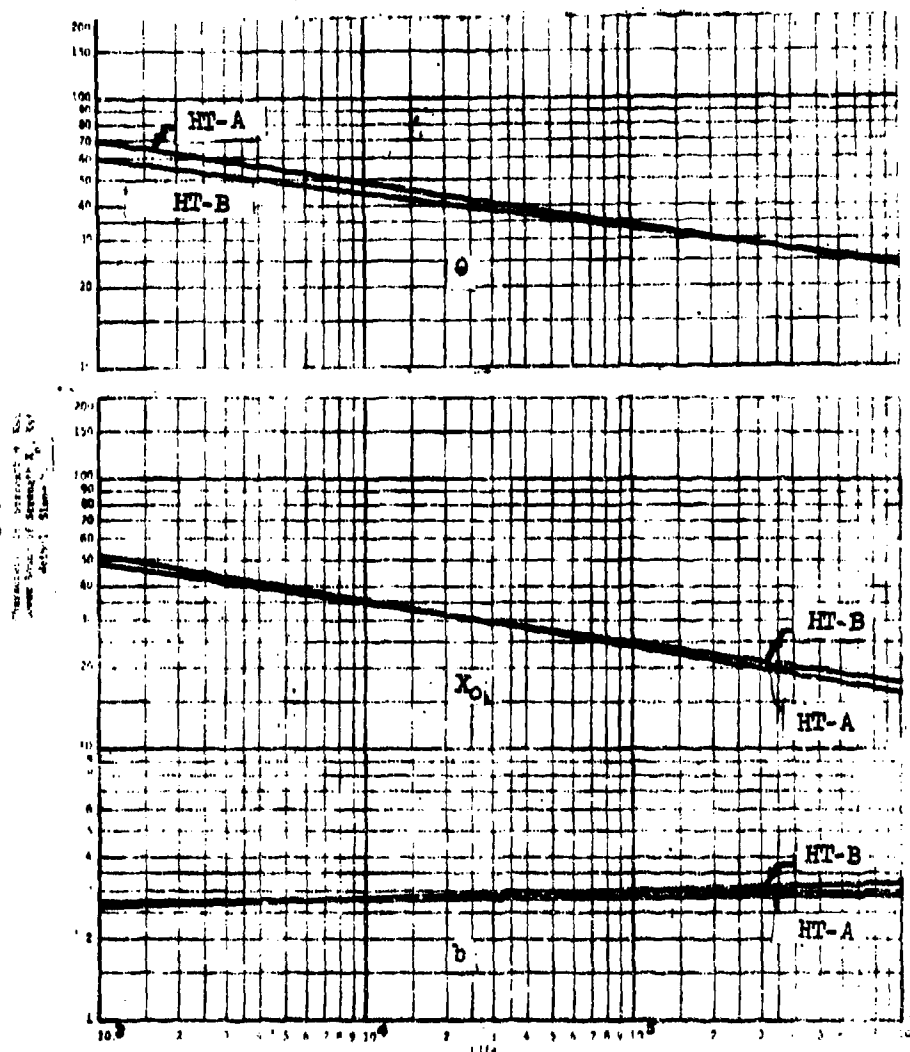
Figure:6.92 (For Tabulated Data See Page 238 )

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177$  ksi,  $S_y = 166$  ksi

## Effect of Heat Treatment



Rotary Beam Bending

Temperature = 80°F

Composition:

6%Al, 4%V, max .07%N, max .10% C,  
max .015%H, max .40%Fe, max .30% O

Hot Rolled

Mean Stress = 82-107 ksi

Heat Treatment:

HT-A and HT-B

See Page 202, Item 28

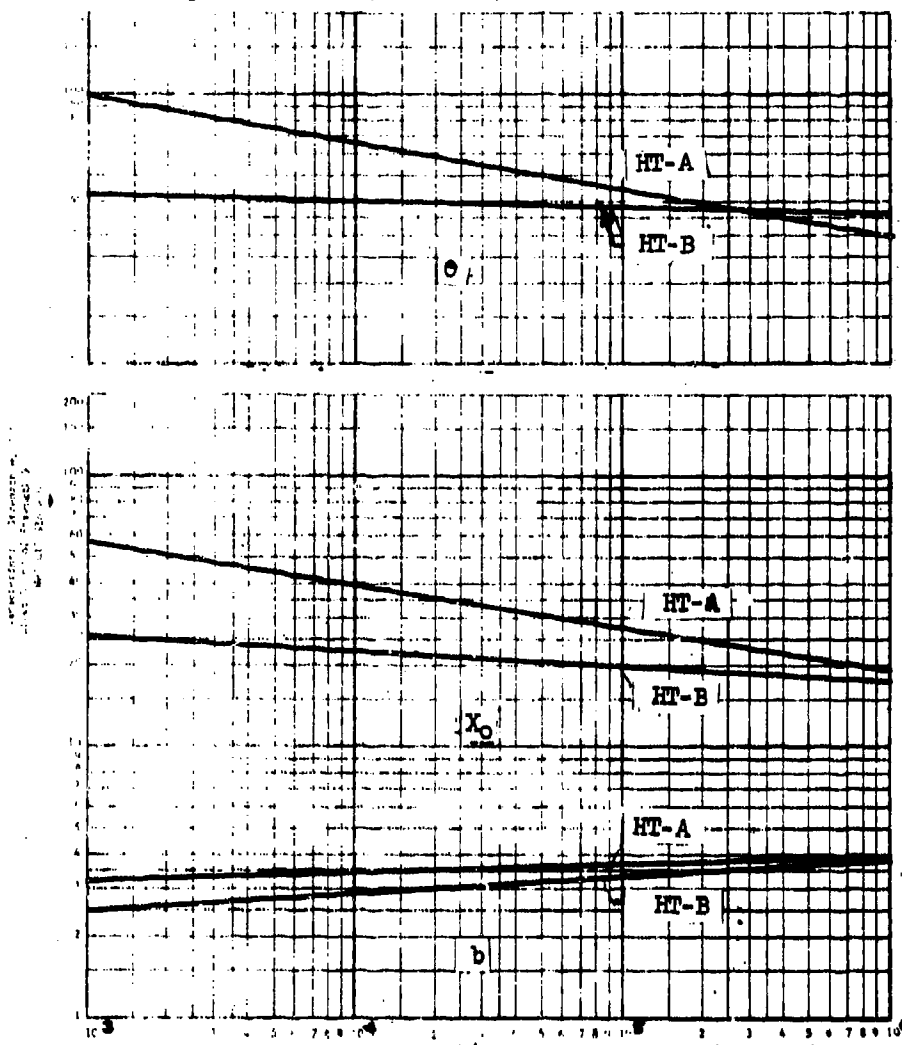
Figure: 6.93 (For Tabulated Data See Page 238)

Ti-6Al-4V

# FATIGUE STRENGTH

$\bar{S}_u = 177$  ksi,  $S_y = 166$  ksi

## Effect of Heat Treatment



Rotary Beam Bending  
 Temperature = 400°F  
 Composition:  
 6%Al, 4%V, max .07%Ni, max .10% C,  
 max .015%N, max .40%Fe, max .30% O

Hot Rolled  
 Mean Stress = 30-42 ksi  
 Heat Treatment:  
 HT-A and HT-B  
 See Page 202, Item 28.

Figure:6.94 (For Tabulated Data See Page 238.)

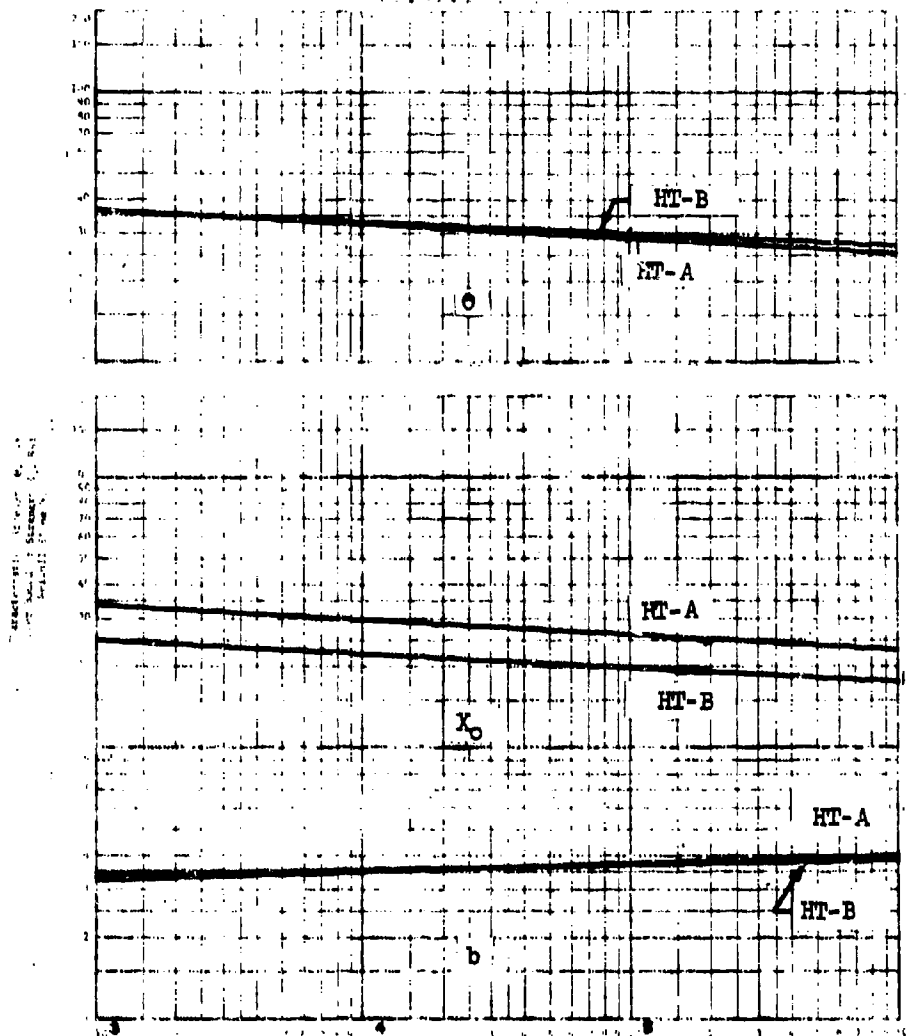


Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending

Temperature = 400°F

Composition:

6%Al, 4%V, max .07%N, max .10% C,  
max .015%H, max .40%Fe, max .30% O

Hot Rolled

Mean Stress = 77-100 ksi

Heat Treatment:

HT-A and HT-B

See Page 202, Item 28

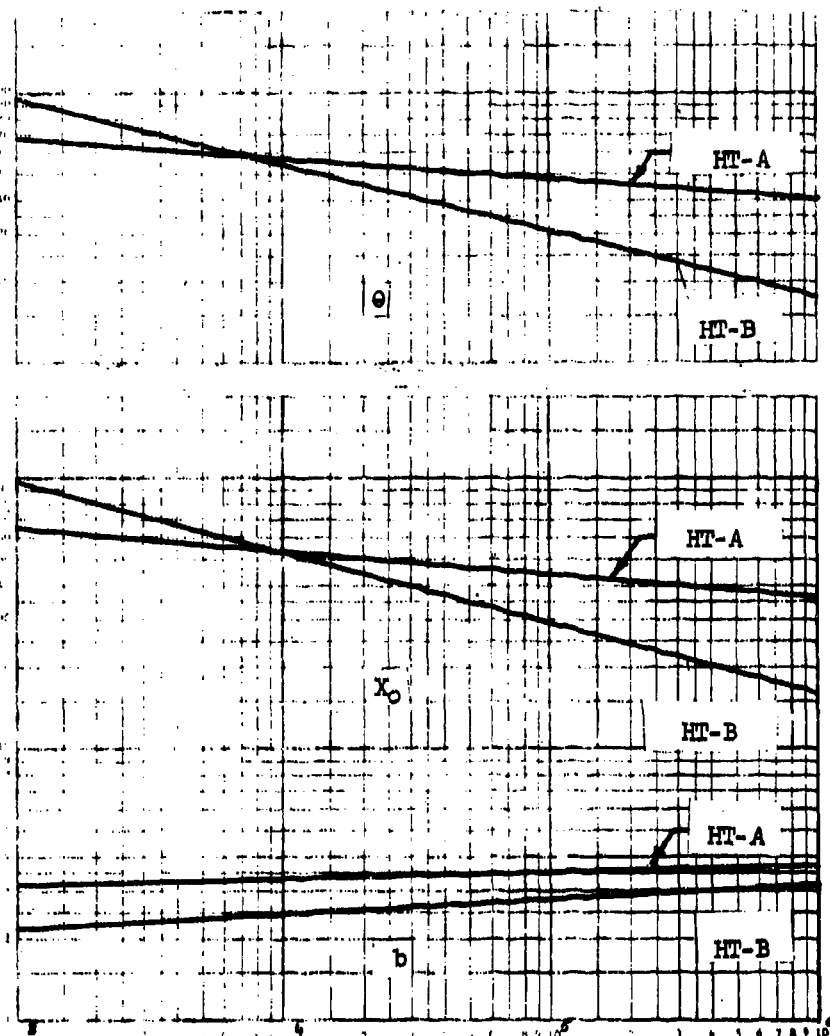
Figure:6.95 (For Tabulated Data See Page 238 )

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Effect of Heat Treatment



Rotary Beam Bending  
 Temperature = 800°F  
 Composition:  
 6%Al, 4%V, max .07%N, max .10% C,  
 max .015%H, max .40%Fe, max .30% O

Hot Rolled  
 Mean Stress = 40-64 ksi  
 Heat Treatment:  
 HT-A and HT-B See Page

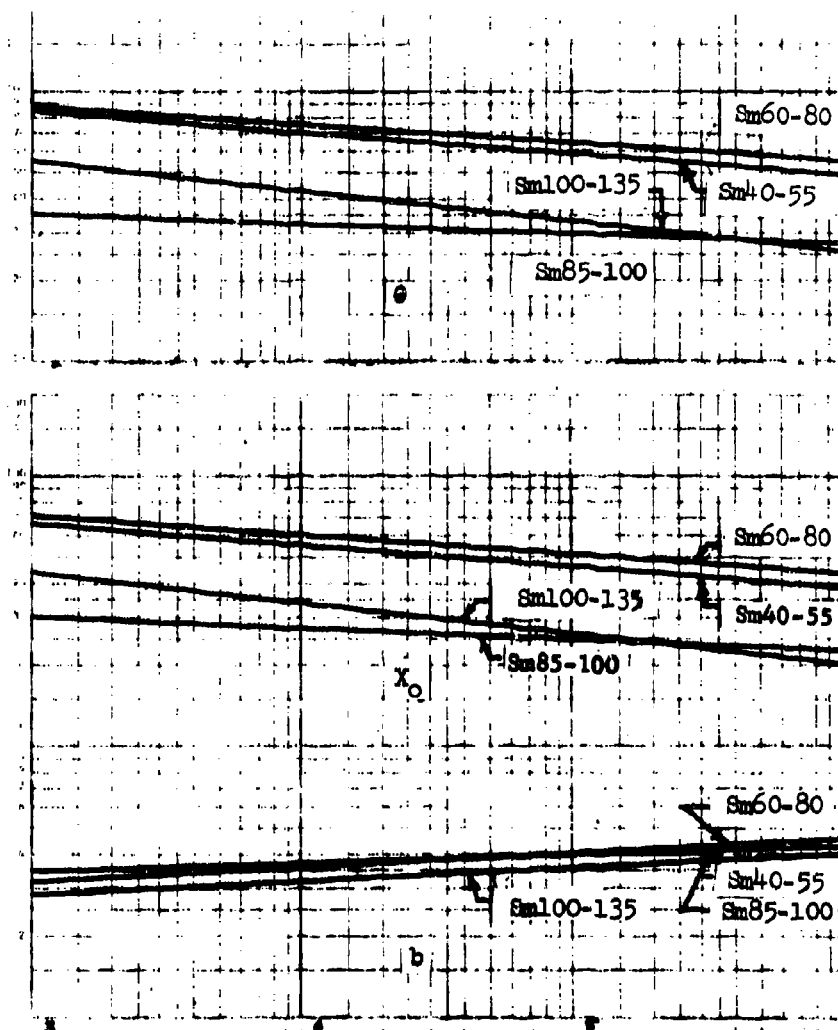
Figure: 6.96 (For Tabulated Data See Page 238)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Miscellaneous Results (Effect of Mean Stress)



Rotary Beam Bending

Temperature = 80°F

Composition:

6%Al, 4%V, max .07%Ni, max .10% C,  
max .015%H, max .40%Fe, max .30% O

Not Rolled

Heat Treatment:

A: Sol. treated 1690°F, 12 min.

WQ, aged 900°F, 4 hrs. air cooled

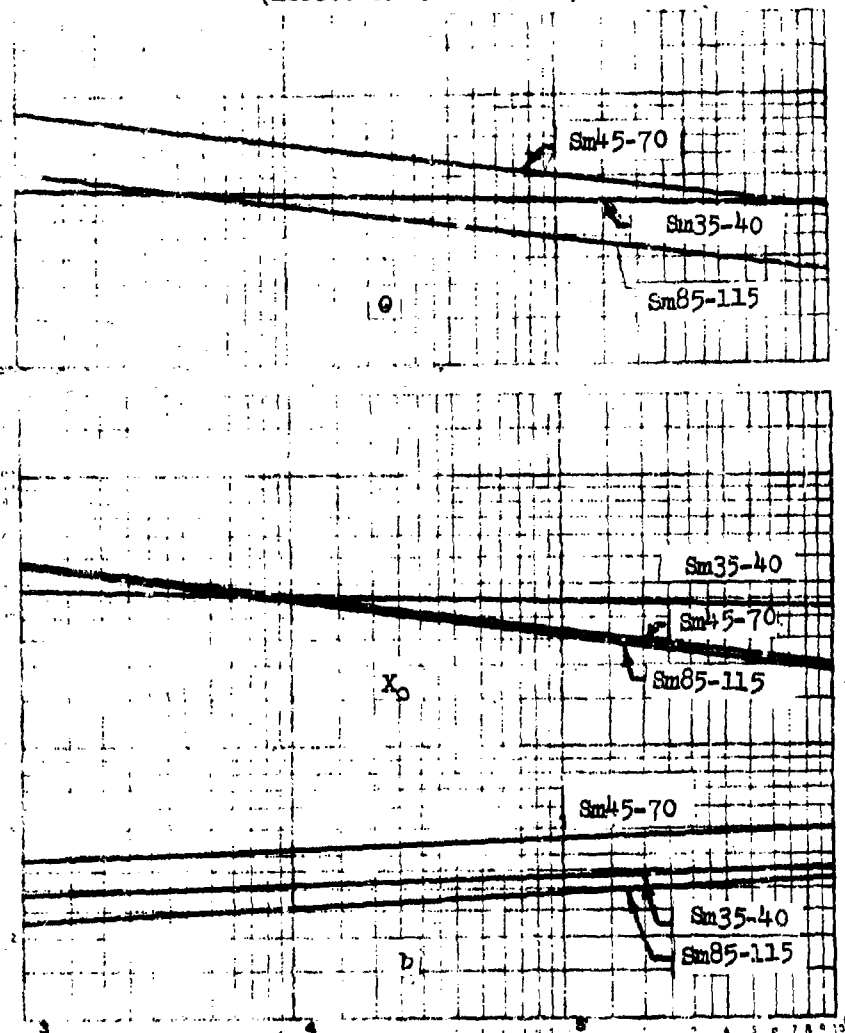
Figure:6.97 (For Tabulated Data See Page 238 )

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177$  ksi,  $S_y = 166$  ksi

## Miscellaneous Results (Effect of Mean Stress)



Rotary Beam Bending

Temperature = 400°F

Composition:

6%Al, 4%V, max .07%N<sub>1</sub>, max .10% C, max  
.015%H, max .40%Fe, max .30% O

Hot Rolled

Heat Treatment:

A: sol. treated 1690°F, 12 min.

WQ, aged 900°F, 4 hrs, air cooled

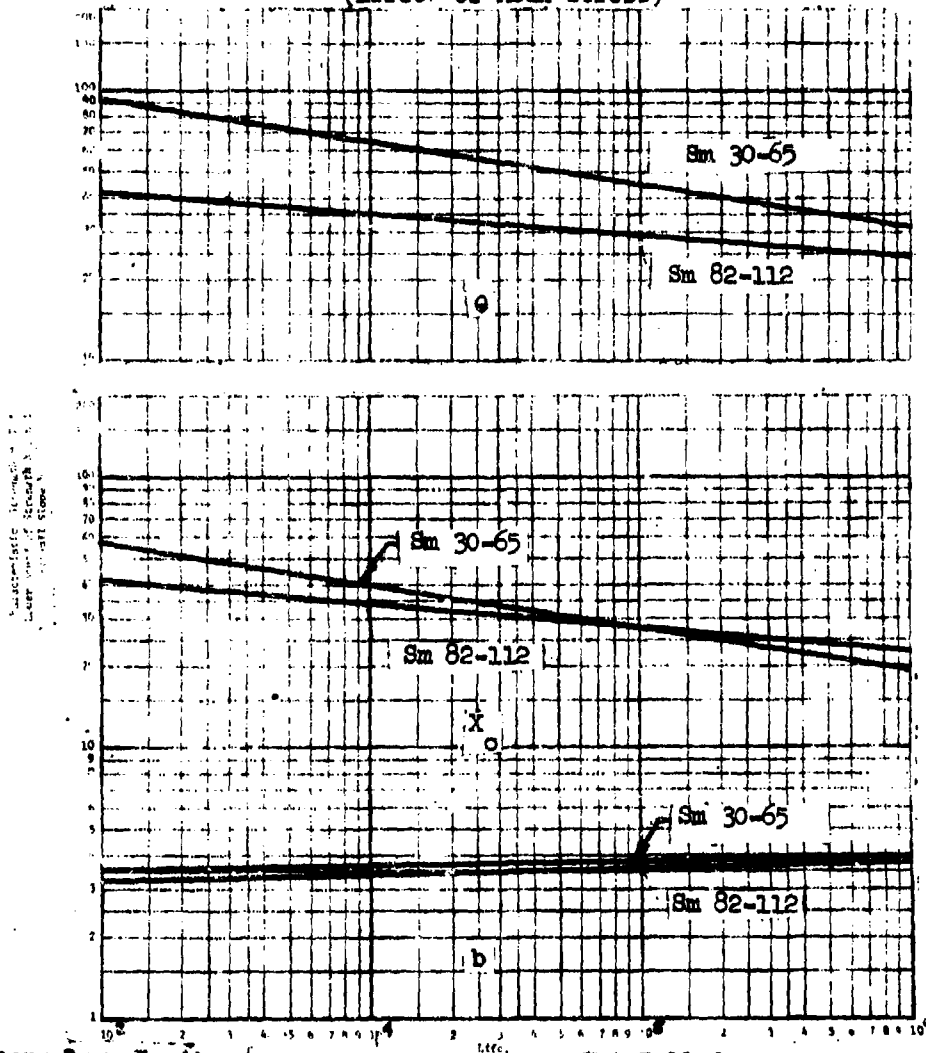
Figure: 6.98 (For Tabulated Data See Page 239)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Miscellaneous Results (Effect of Mean Stress)



Rotary Beam Bending

Temperature - 600°F

Composition:

6%Al, 4%V, max .07%N, max .10%C,  
.015%H, max .40%Fe, max .30%O

Hot Rolled

Heat Treatment:

B: sol. treated 1675°F, 20 min.  
WQ, aged 900°F, 4hrs. air cooled

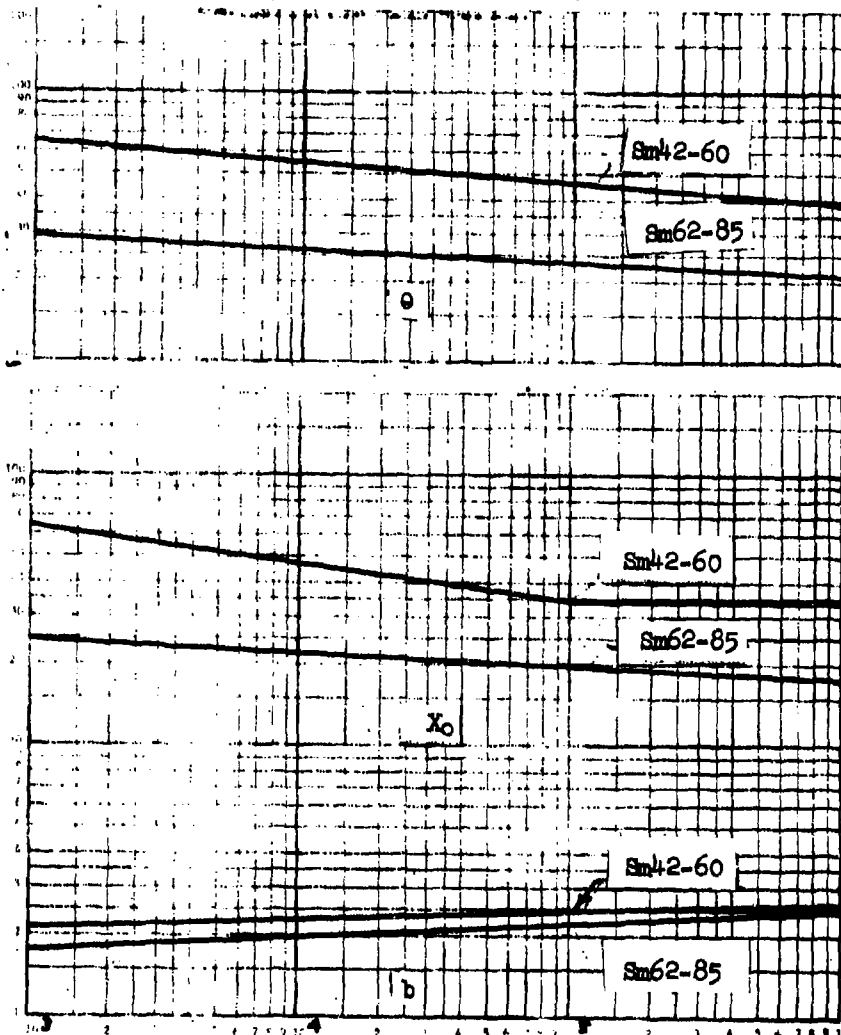
Figure:6.99 (For Tabulated Data See Page 239)

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Miscellaneous Results (Effect of Mean Stress)



Rotary Beam Bending

Temperature = 800°F

Composition:

6%Al, 4%V, max .07%N<sub>1</sub>, max .10%C, max

.015%H, max .40%Fe, max .30%O

Hot Rolled

Heat Treatment:

A: sol. treated 1690°F, 12 min.

WQ, aged 900°F, 4 hrs. air cooled

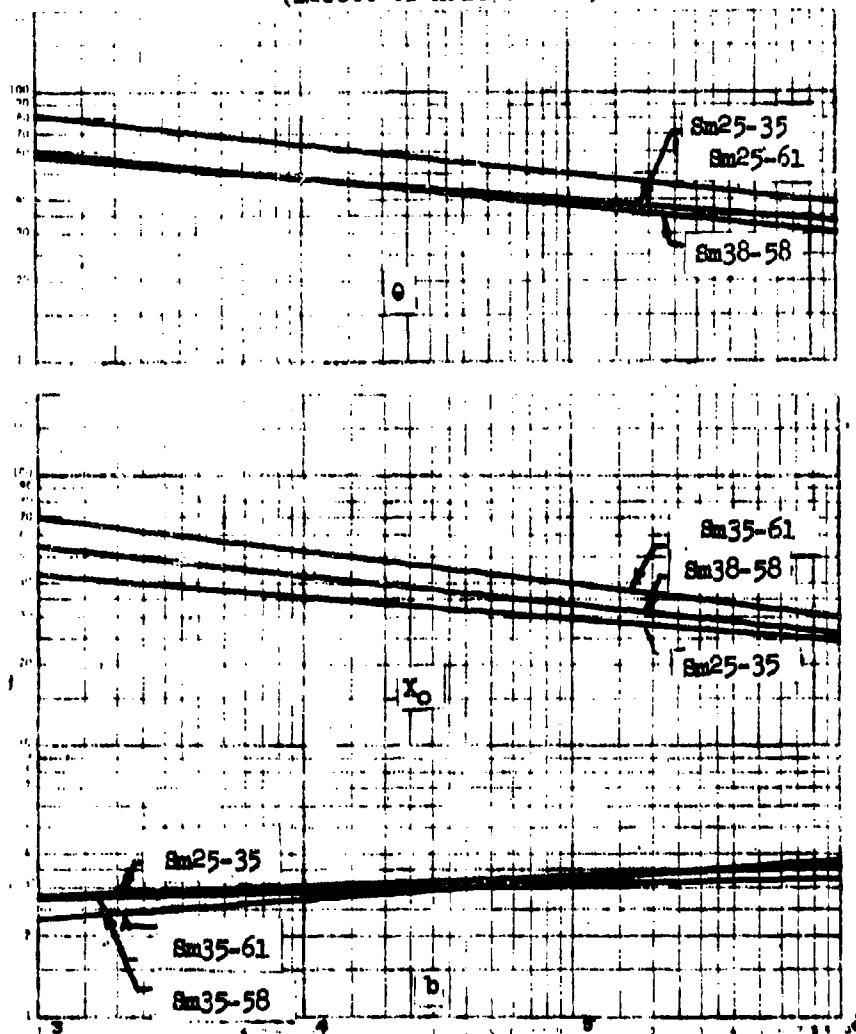
Figure:6.100 (For Tabulated Data See Page 239 )

Ti-6Al-4V

# FATIGUE STRENGTH

$S_u = 177 \text{ ksi}$ ,  $S_y = 166 \text{ ksi}$

## Miscellaneous Results (Effect of Mean Stress)



Rotary Beam Bending

Temperature = 900°F

Composition:

6%Al, 4%V, max .07%N, max .10% C,  
max .015% H, max .40% Fe, max .30% O

Hot Rolled

Heat Treatment:

A: sol. treated 1690°F, 12 min.

WQ, aged 900°F, 4 hrs. air cooled

Figure:6.101 (For Tabulated Data See Page 239.)

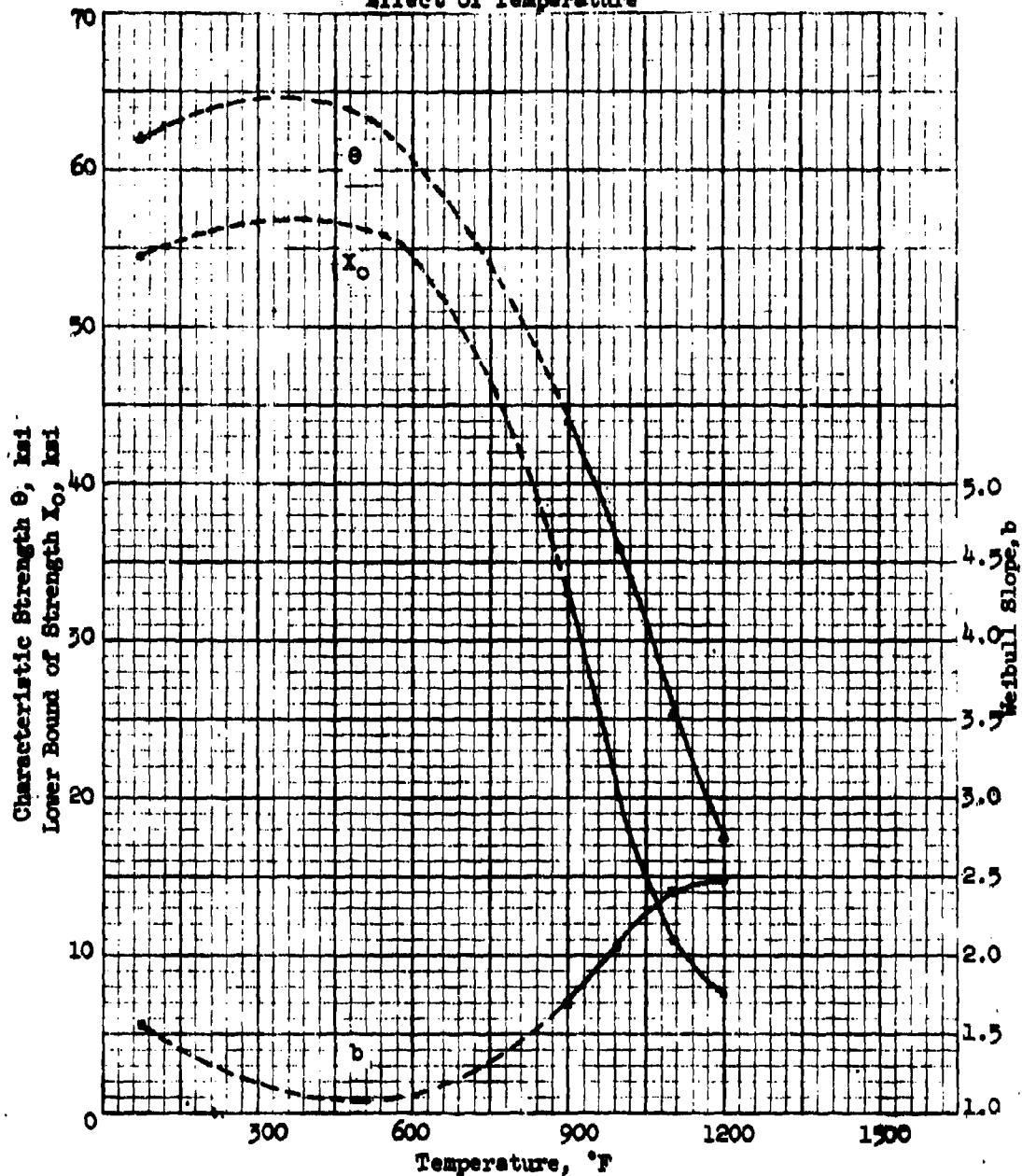
# TENSILE STRENGTH

Low Carbon, Low Alloy Steel

$S_u = 60 \text{ ksi}$

$S_y = 41 \text{ ksi}$

Effect of Temperature



Composition:

(.12-.17)%C, .55% Mn, .28% Si

Heat Treatment:

Annealed, 1550°F

Figure: 6.102 (For Tabulated Data See Page 245)

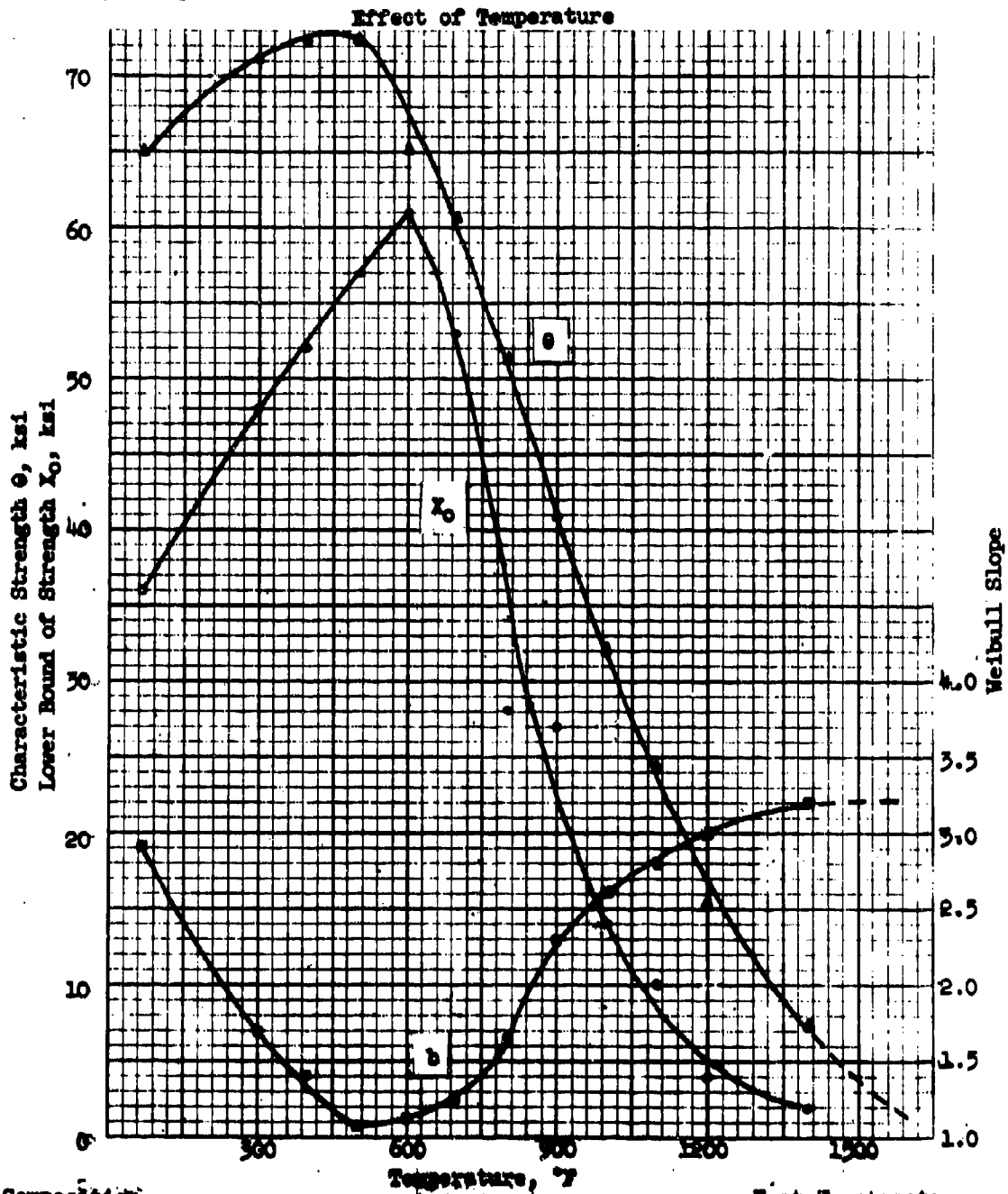


TENSILE STRENGTH

Low-Medium Carbon, Low Alloy Steel

$S_u \approx 62 \text{ ksi}$

$S_y = 42 \text{ ksi}$



Composition:

(.08-.30)% C, (.5-1.0)% Mn, .25% (max.) Si

Heat Treatment:

None Specified

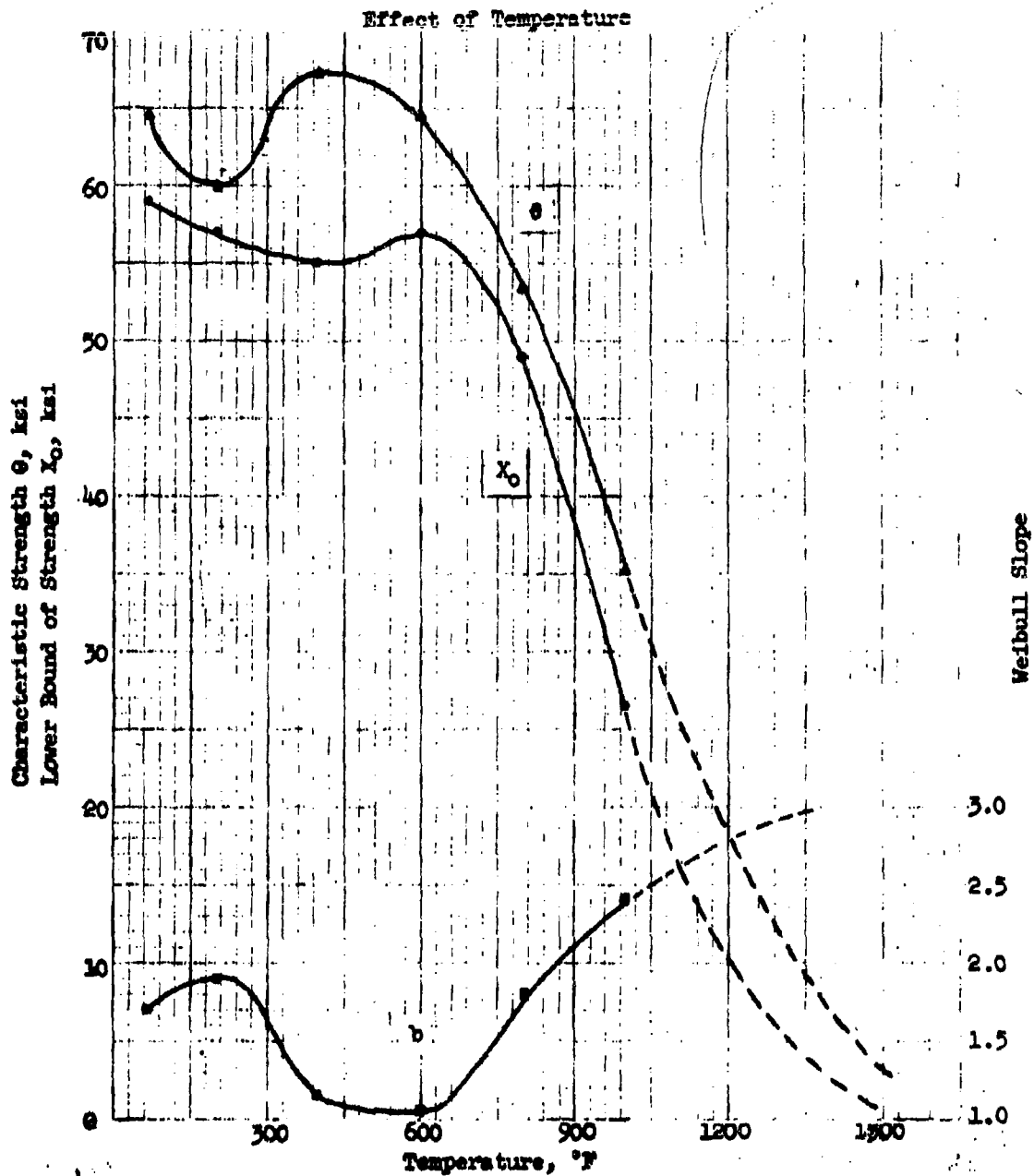
Figure: 6.103 (For Tabulated Data See Page 245)

# TENSILE STRENGTH

Killed, Low Carbon, Low Alloy Steel

$S_u = 63 \text{ ksi}$

$S_y = 37 \text{ ksi}$



Composition:

(.18-.24)% C, .86% (max.) P, .24% (max.) Si

Heat Treatment:

Stress Relieved

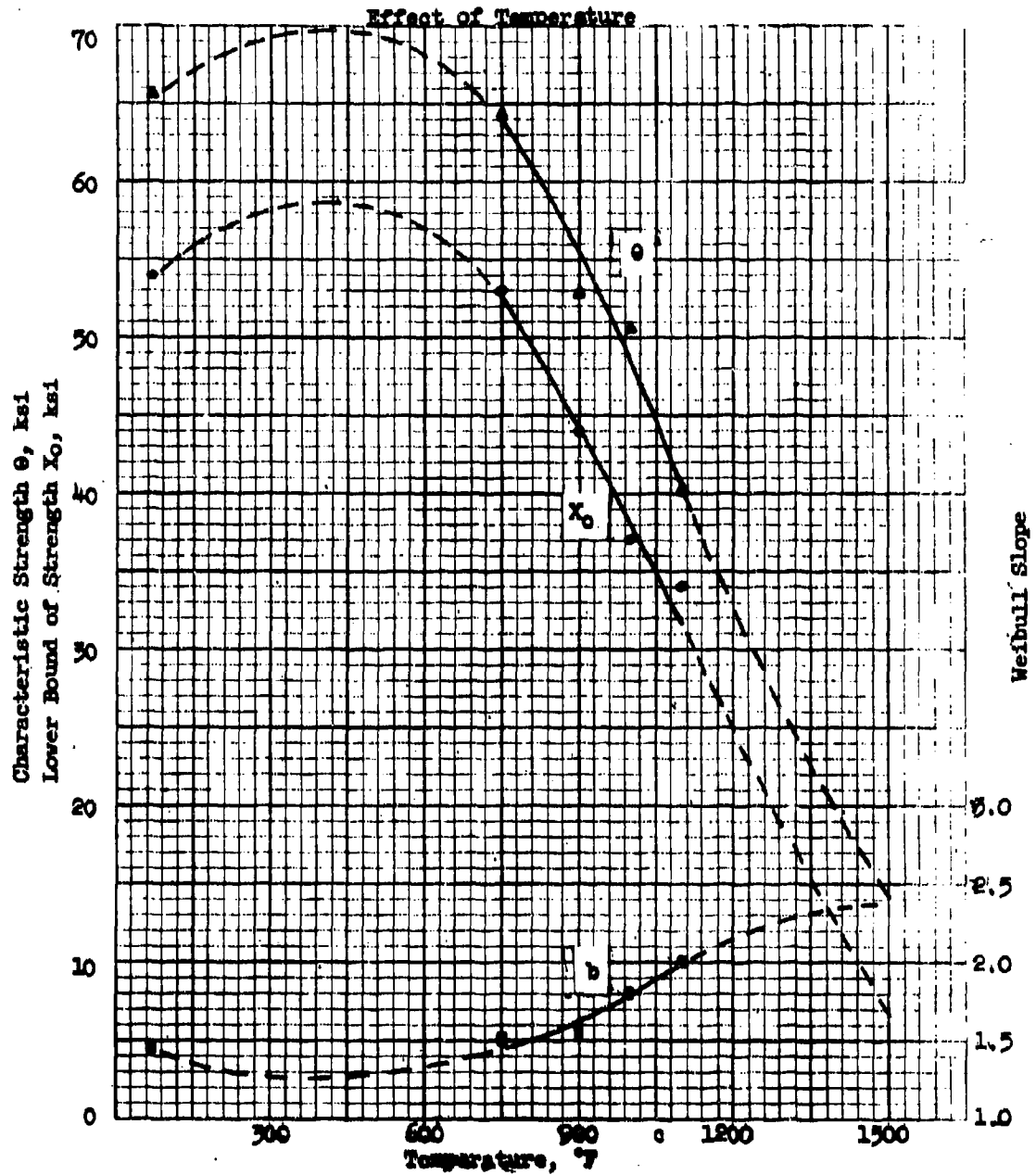
Figure: 6.104 (For Tabulated Data See Page 245)

# TENSILE STRENGTH

Low-Medium Carbon, Low Alloy Steel

$S_u = 63 \text{ ksi}$

$S_y = 40 \text{ ksi}$



Composition: (.08-.35)% C, (.3-.8)% Mn, (.1-.5)% Si, (.4-.65)% Mo

Heat Treatment: None Specified

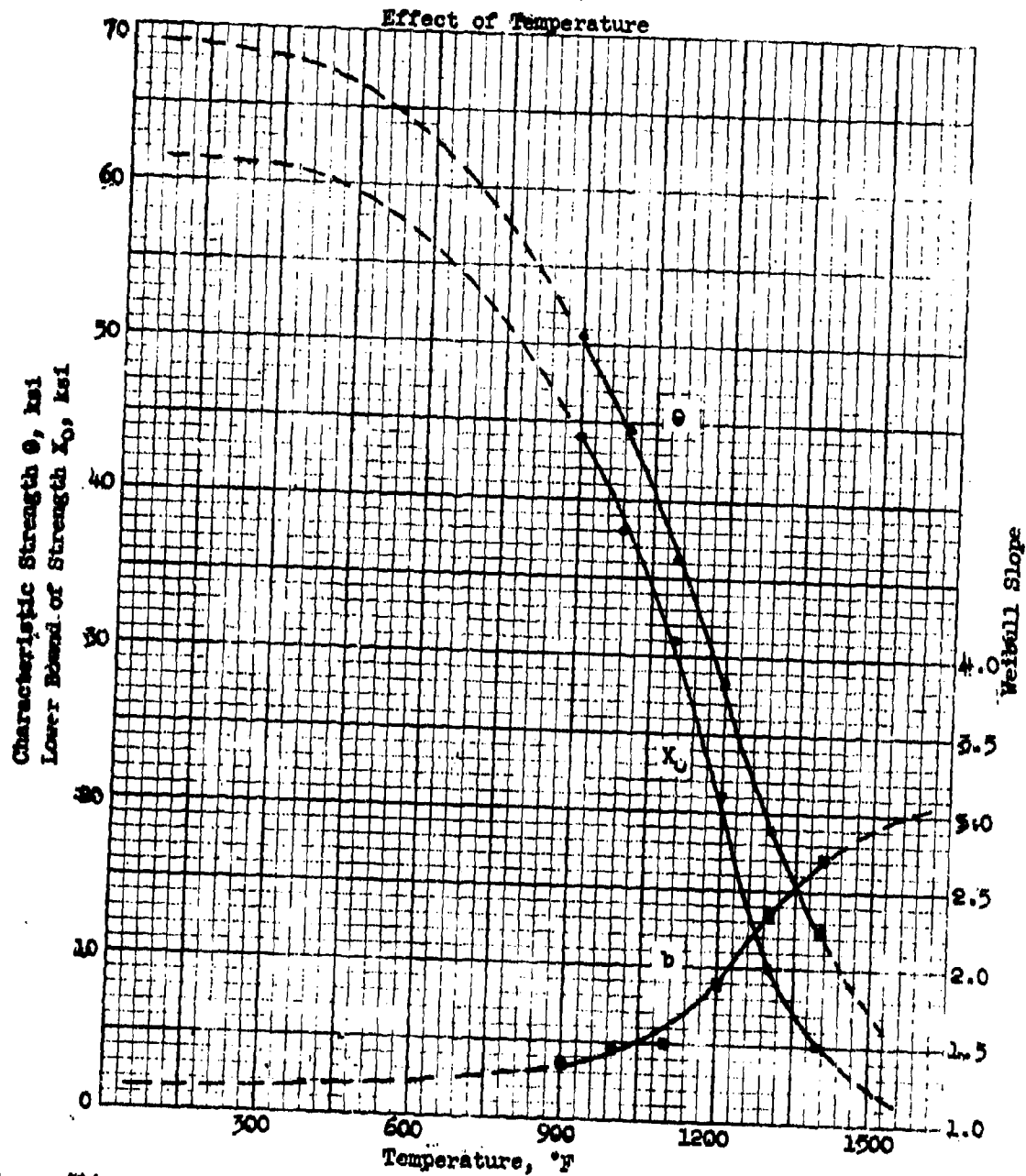
Figure: 6.105 (For Tabulated Data See Page 245)

# TENSILE STRENGTH

Low Carbon, High Alloy Steel

$S_u = 65 \text{ ksi}$

$S_y = 21 \text{ ksi}$



Composition:

.12% (max.) C, 5.0% Cr, .5% Mo, (.1-.5)% Ti

Heat Treatment:

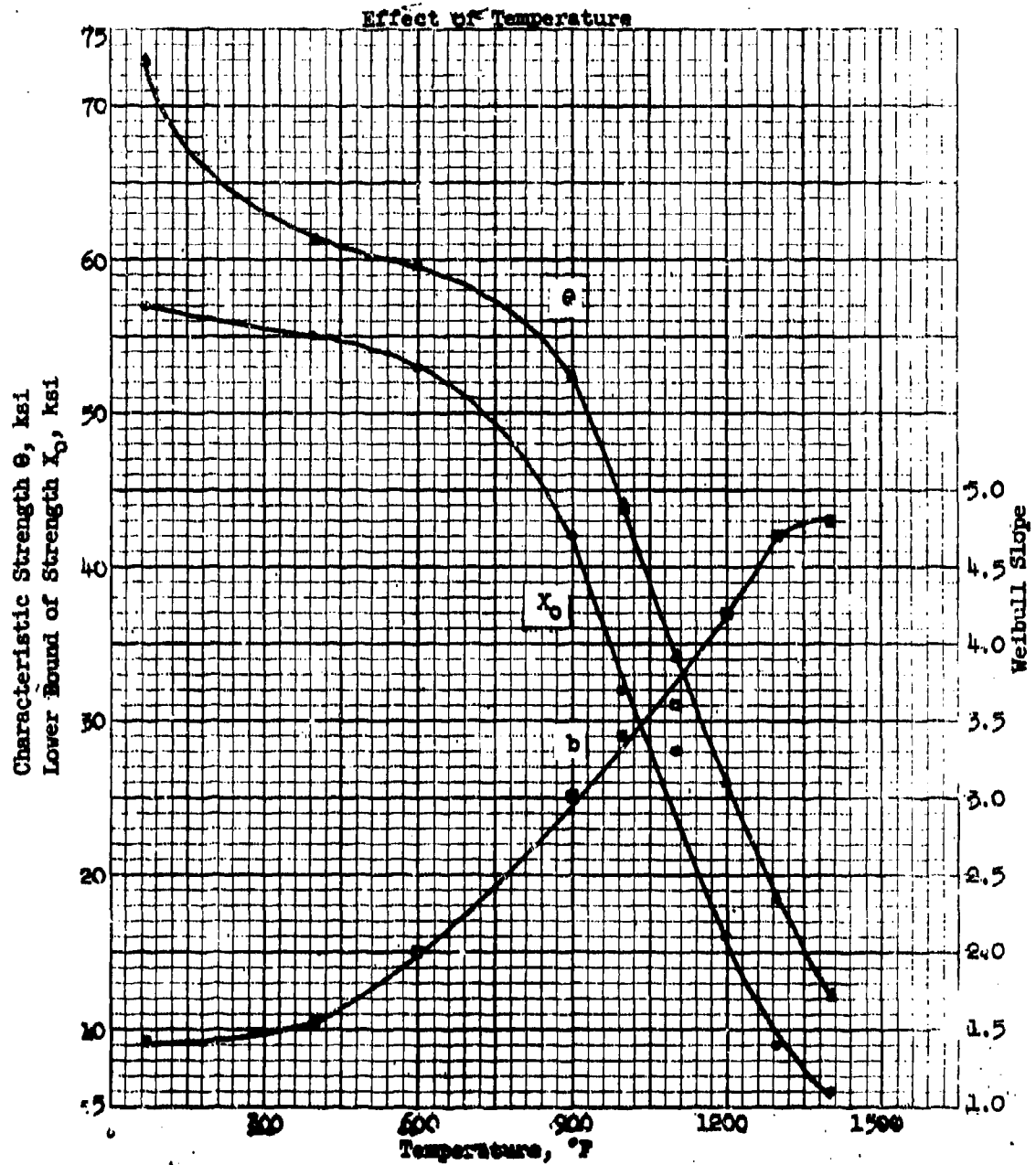
Annealed, 1550°F

Figure: 6.106 (For Tabulated Data See Page 246 )

# TENSILE STRENGTH

Low Carbon, High Alloy Steel

$S_u = 71 \text{ ksi}$   $S_y = 30 \text{ ksi}$



Composition:

.25% C, .75% Mn, .05% P, .05% S

Heat Treatment:

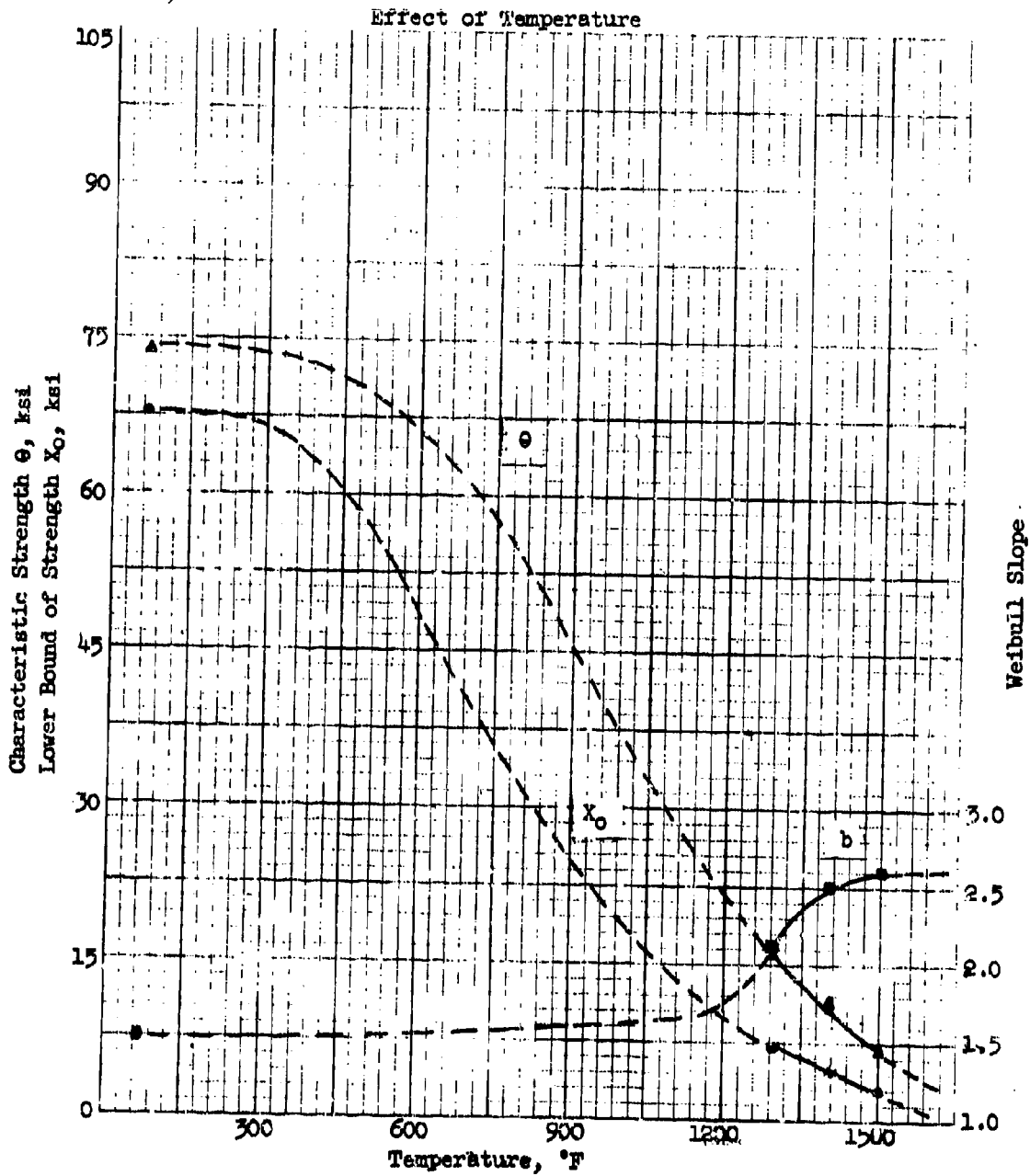
None Specified

Figure: 6.107 (For Tabulated Data See Page 244.)

# TENSILE STRENGTH

Stainless Steel

$S_u = 72 \text{ ksi}$      $S_y = 39 \text{ ksi}$



Composition:

.12% (max.) C, 17% Cr

Heat Treatment:

Annealed, 1950°F

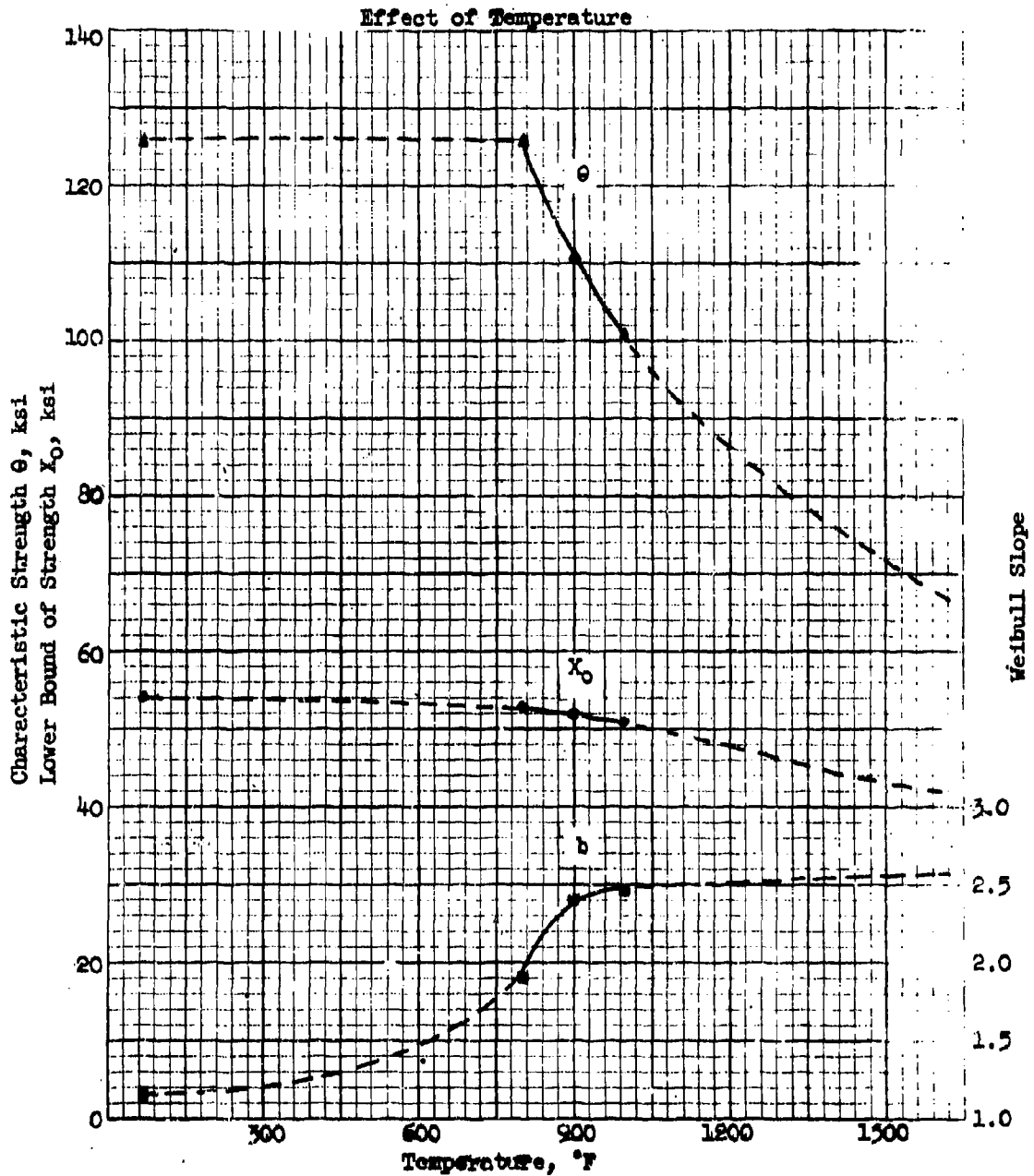
Figure: 6.108 (For Tabulated Data See Page 246)

# TENSILE STRENGTH

Low Carbon, High Alloy Steel

$S_u = 110 \text{ ksi}$

$S_y = 75 \text{ ksi}$



Composition:

.15% C, 2.25% Cr, 1.0% Mo

Heat Treatment:

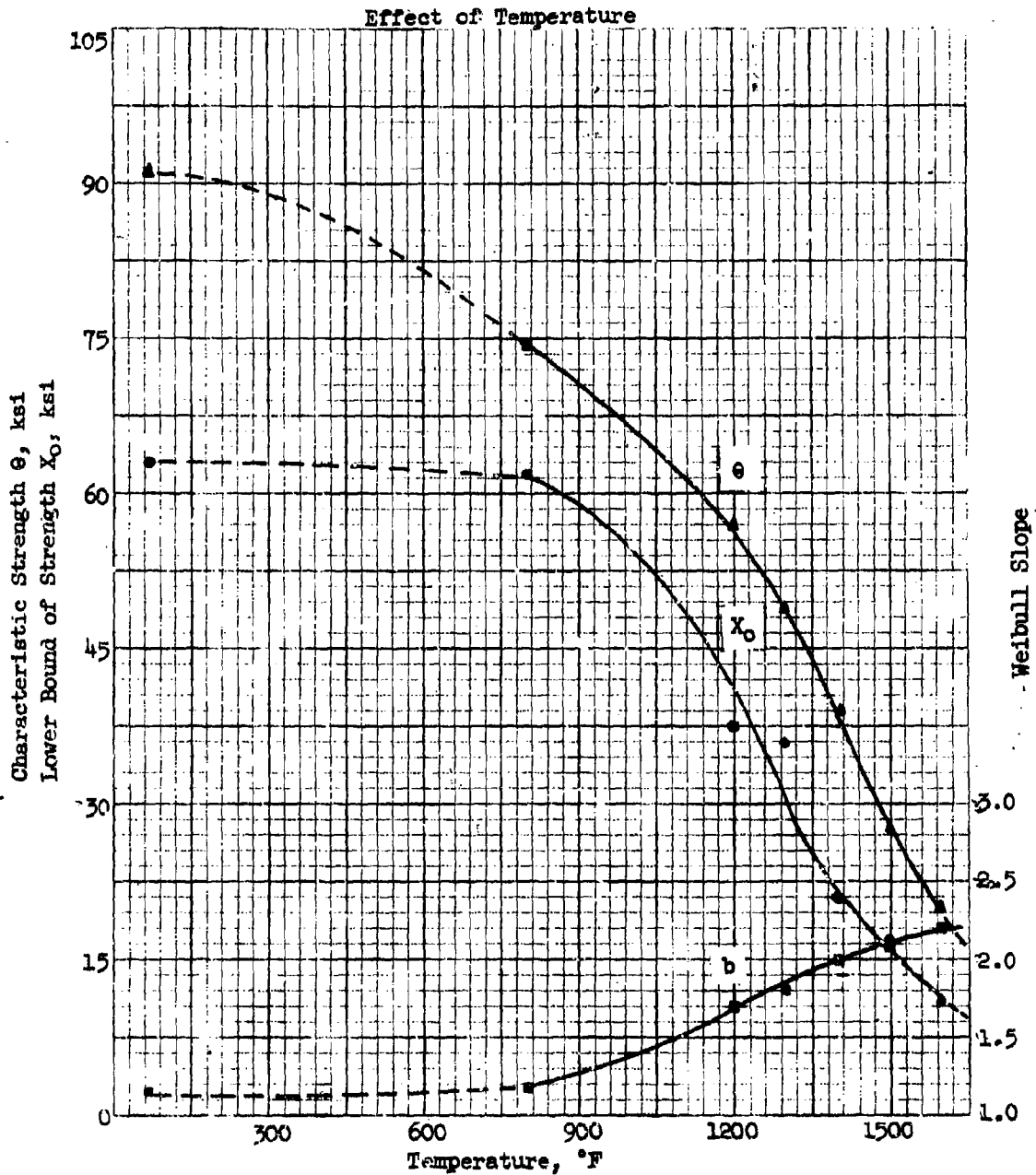
Normalized, 1650°F; Drawn, 1300°F

Figure: 6.109 (For Tabulated Data See Page 246)

# TENSILE STRENGTH

Stainless Steel

$S_u = 80 \text{ ksi}$      $S_y = 57 \text{ ksi}$



Composition:

.20% (max.) C, 25% Cr, 12% Ni

Heat Treatment:

Annealed, 2000°F

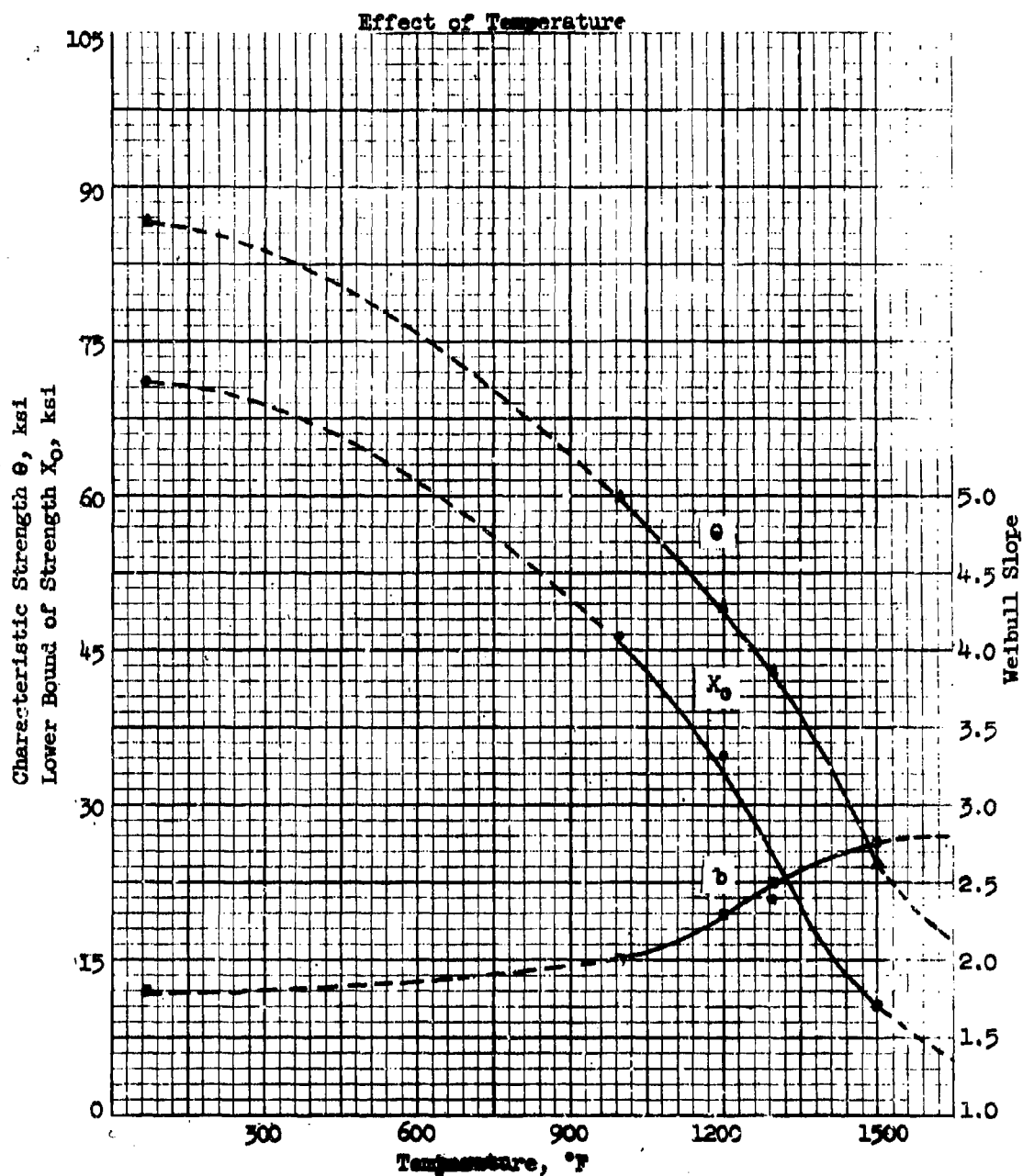
Figure: 6.110 (For Tabulated Data See Page 246 )



# TENSILE STRENGTH

Stainless Steel

$S_u = 85 \text{ ksi}$      $S_y = 35 \text{ ksi}$



Composition:

(.04-.09)% C, 18% Cr, 8% Ni, 2% Ti

Heat Treatment:

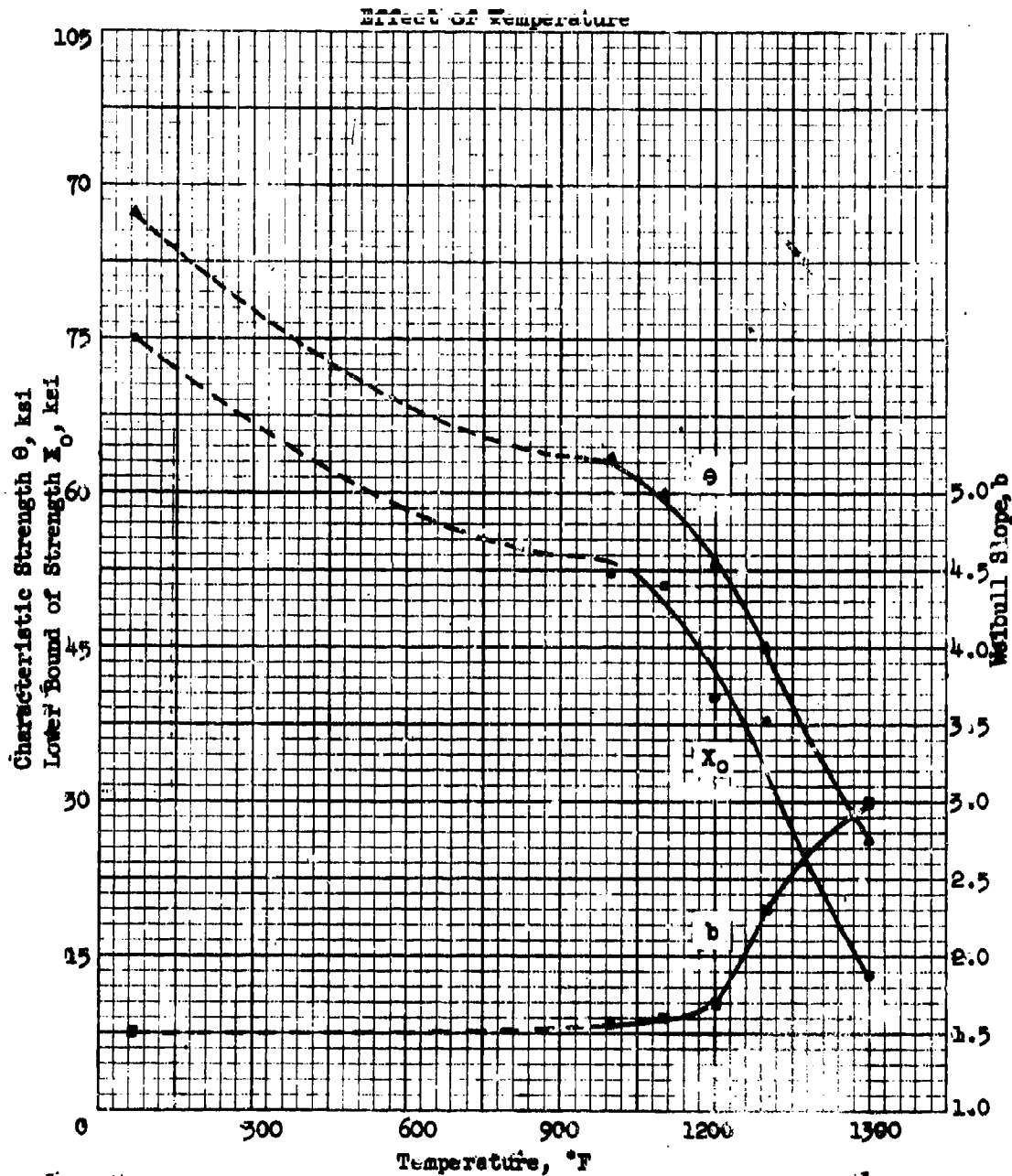
Annealed, 1950°F

Figure: 6.111 (For Tabulated Data See Page 247)

# TENSILE STRENGTH

Stainless Steel

$S_u = 85 \text{ ksi}$   $S_y = 38 \text{ ksi}$



Composition:  
 .06%C, 18%Cr, 8%Ni, .8%Cb.

Heat Treatment:  
 Annealed, 1900°F

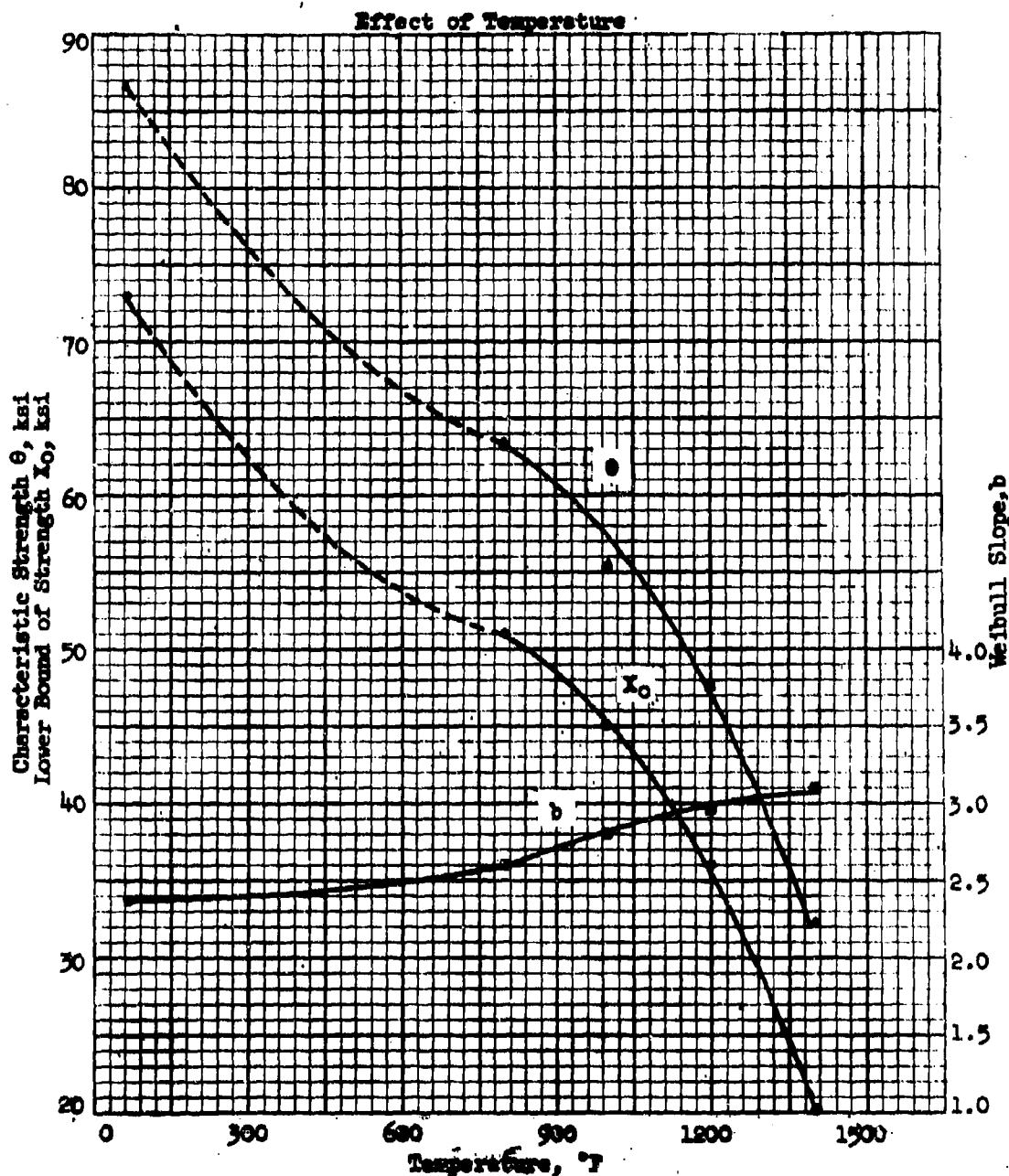
Figure: 6.112 (For Tabulated Data See Page 247)

# TENSILE STRENGTH

Stainless Steel

$S_u = 85 \text{ ksi}$

$S_y = 38 \text{ ksi}$



Composition: 18% Cr, 8% Ni  
(02-09) 90, 18% Cr, 8% Ni

Heat Treatment: Annealed 1950°F

Figure: 6.113 (For Tabulated Data See Page 743)

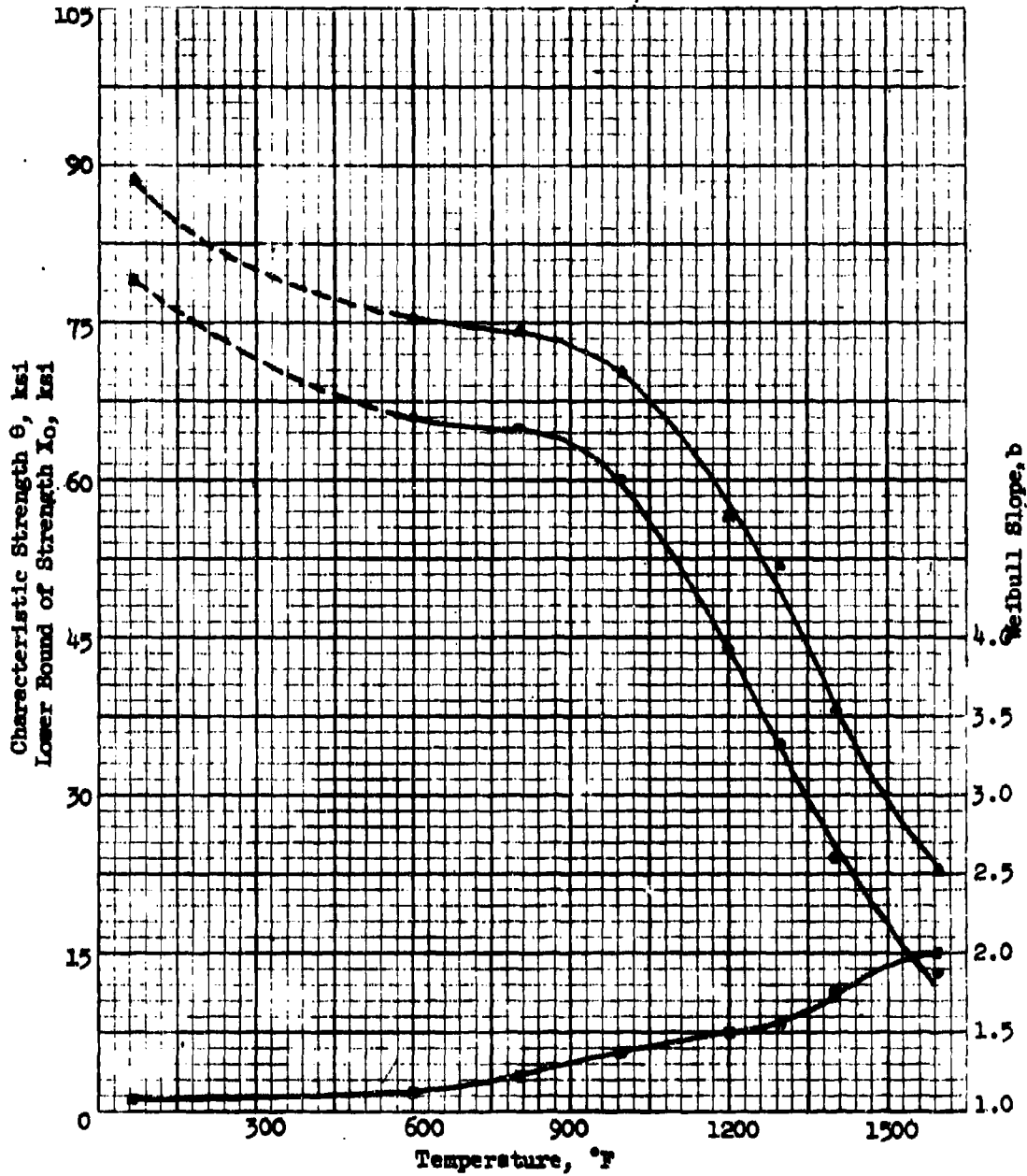
# TENSILE STRENGTH

Stainless Steel

$S_u = 88 \text{ Ksi}$

$S_y = 41 \text{ Ksi}$

## Effect of Temperature



Composition:

.08%C, 18%Cr, 12%Ni, 2%Mo

Heat Treatment:

Annealed 1950°F

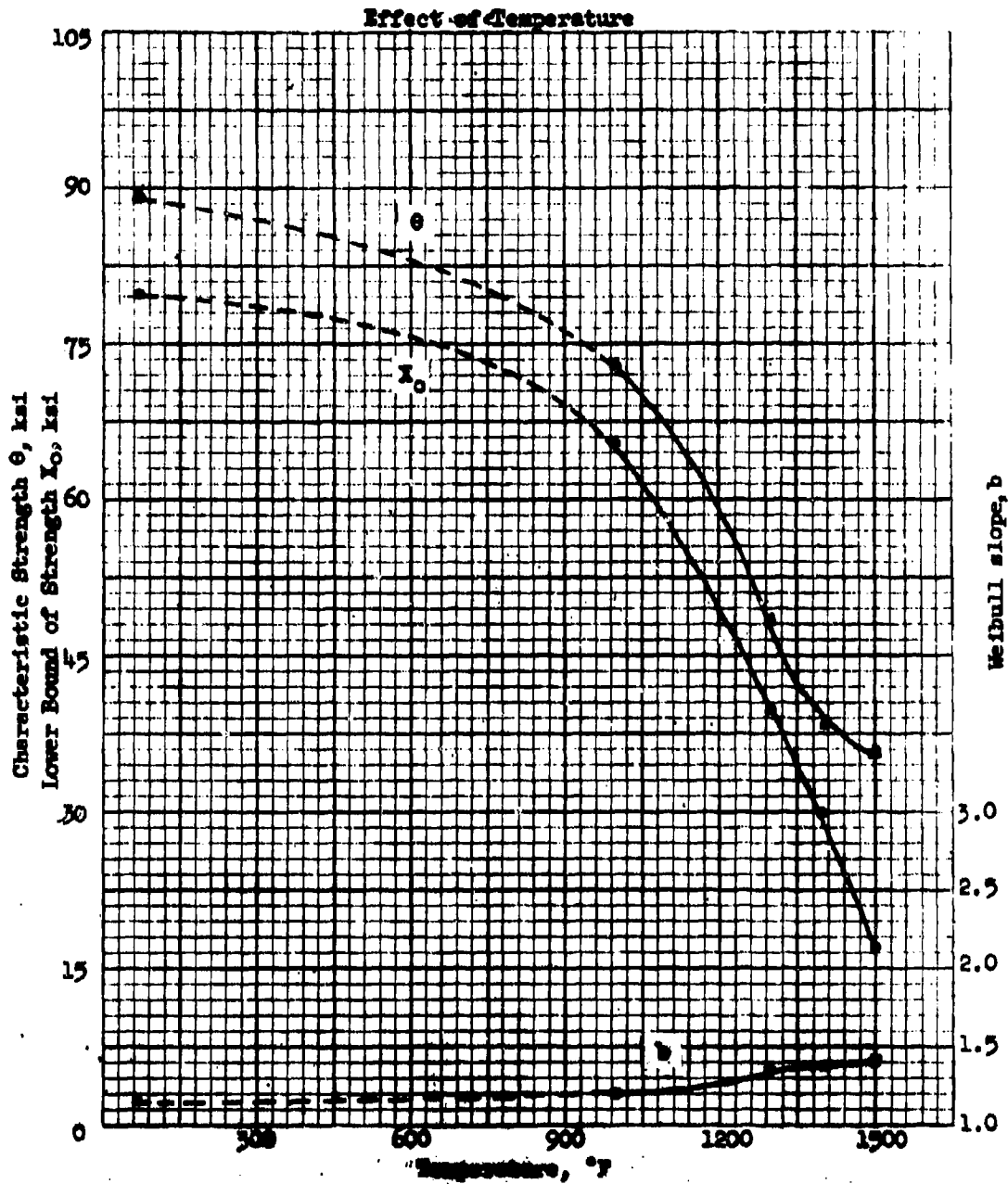
Figure: 6.114 (For Tabulated Data See Page 247)

# TENSILE STRENGTH

Stainless Steel

$S_u = 88 \text{ ksi}$

$S_y = 32 \text{ ksi}$



Composition:

.27% C, 27% Ni, 20% Mn, 20% Fe, 20% Cr

Heat Treatment:

Annealed, 2000°F

Figure 6.115 (For Tabulated Data See Page 247)

## SECTION 7 STATISTICAL DISTRIBUTION OF STRESS

### 7.1 STRESS SPECTRUM VS STRESS DISTRIBUTION

The problem of stress distribution, in the Interference Theory, appears to be much more involved than the problem of strength distribution. Consider, for example, the problem of a connecting rod in a reciprocating engine. Because of the variation in hardness, surface finish, etc, the fatigue strength will vary from one rod to another. This will result in a distribution curve, in which the strength will be plotted on the abscissa and the number of rods having a given strength (i.e. frequency of occurrence) on the ordinate.

Consider now the stress distribution in the connecting rods. The stresses in the rod result from the combined effect of gas pressure loading and inertia loading. If the attention is now focused on a single rod, then the variation in the two types of loading will produce a distribution of stresses in this particular rod. The resultant curve will be a plot of the stresses in the rod on the abscissa and the number of times that this stress occurs in this particular rod on the ordinate (Figure 7.1 (a)).

This, however, is not what is wanted in the application of the Interference Theory, because this distribution of stresses cannot be matched with the distribution of strength. In the strength distribution the ordinate gives the number of rods having a given strength. Therefore in the stress distribution the ordinate must read number of rods having a given stress (and not the number of times a given stress occurs in a single rod). This can be obtained by considering the fact that different engines will be subjected in service to different operating conditions and, therefore, the distribution of gas pressure loading and inertia loading will vary from engine to engine. As pointed out in Section 7.2 a spectrum of stresses must be converted to an equivalent stress for the purpose of Interference Theory. Therefore, if a spectrum of loading due to different service conditions varies from engine to engine, in a population of connecting rods the equivalent stress will vary from rod to rod. Thus the statistical stress distribution desired for the Interference Theory may be obtained (Figure 7.1 (b)). In this distribution the equivalent stress will be plotted on the abscissa and the number of rods (frequency of occurrence) having that stress on the ordinate. This distribution then can be compared with the strength distribution to obtain the probability of interference.

### 7.2 CONVERSION OF STRESS SPECTRUM TO AN EQUIVALENT STRESS ( $S_{equ}$ )

By definition, equivalent stress is a completely reversed stress of constant amplitude which, when imposed on a part, should cause failure

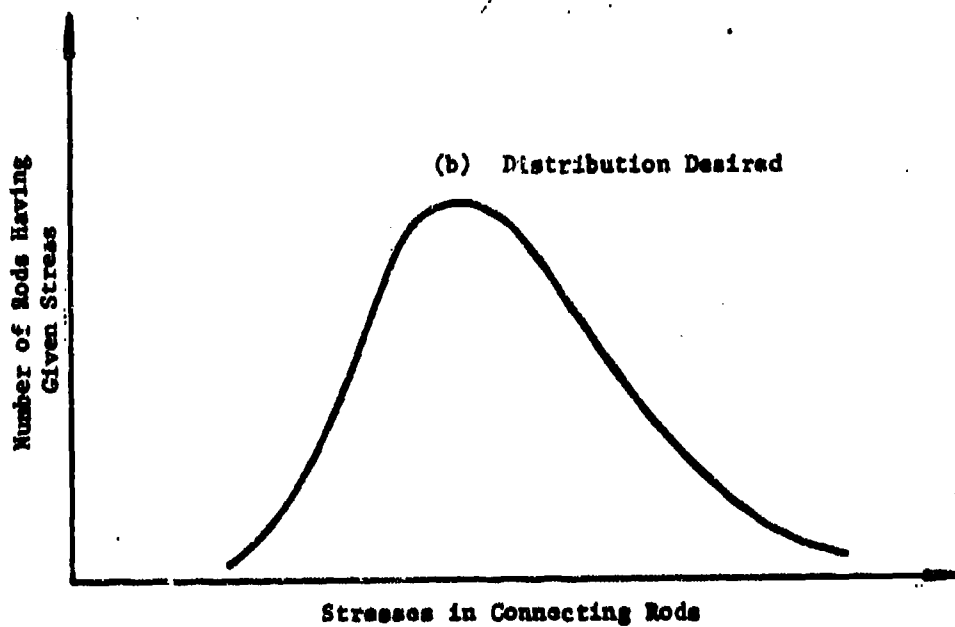
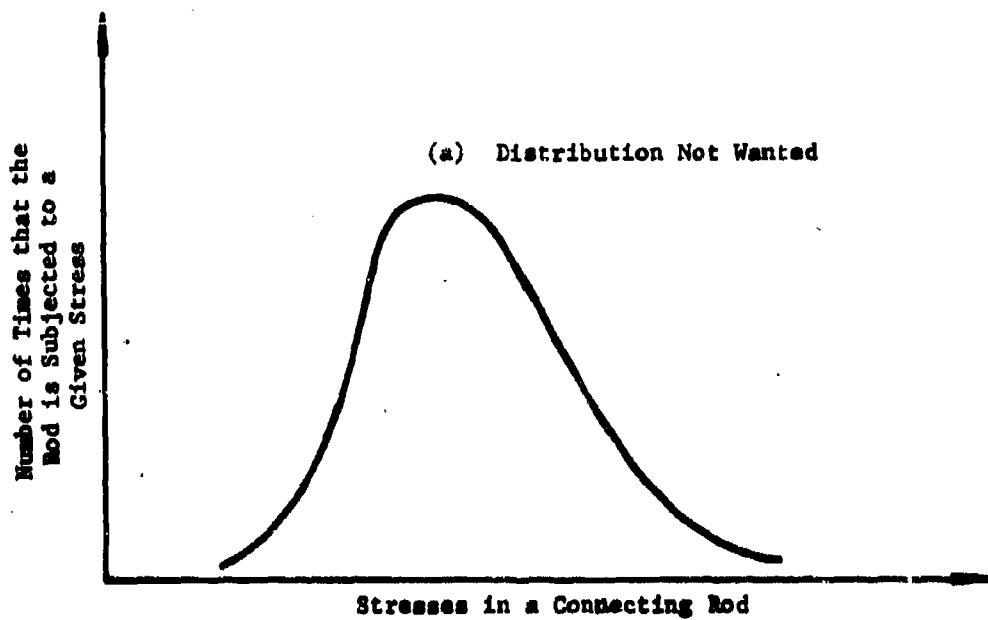


Figure 7.1 Stress Distribution for the Interference Theory

at the same life as if the stress spectrum was imposed instead. Thus, the damage accumulated at any given life, due to this equivalent stress, will be the same as if due to the spectrum of stresses.

The first step towards converting the spectrum to a single stress ( $S_{equ}$ ) is to convert the operating stresses, which may have some mean stress associated with them, to zero mean stress, that is, the completely reversed stress. (Figure 7.2). This can be done by means of the modified Goodman diagram. Draw the Goodman diagram as shown in Figure 7.3. From the spectrum of operating stresses plot each stress cycle on this diagram as shown, for example, line AB. Connect CA and CB and extend to the vertical line where mean stress is equal to zero. Hence, XY is the zero mean stress equivalent to AB. After reducing all such stress cycles to zero mean stress the stress spectrum will have all the stress cycles completely reversed. The magnitude XY will be different for different stress cycles. Therefore, the original operating stress spectrum (Figure 7.2 (a)), with various mean stress levels, is thus reduced to a stress spectrum with zero mean stress level, that is, a completely reversed stress (Figure 7.4).

This spectrum can then be reduced to a single equivalent stress of constant amplitude, by means of Miner's or Corten-Dolan's Rules.

#### 7.2.1 Miner's Rule

Miner's rule<sup>7</sup> assumes that the total life of a component can be estimated by simply adding the fraction of life consumed by each over-stress cycle. Overstress can be defined as the stress above the endurance limit of the material which, if applied, will damage the part.

This rule is expressed as:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots + \frac{n_k}{N_k} = 1$$

or

$$\sum_{i=1}^{i=k} \frac{n_i}{N_i} = 1 \quad (7.1)$$

where  $n_1, n_2, n_3 \dots n_k$  represent the number of cycles at specific overstress levels, and  $N_1, N_2, N_3 \dots N_k$  the life cycles to failure at these levels, as read from the S-N curve.

The equivalent life of a part ( $N_{equ}$ ) under a spectrum of stresses may be found by rearranging the above equation:



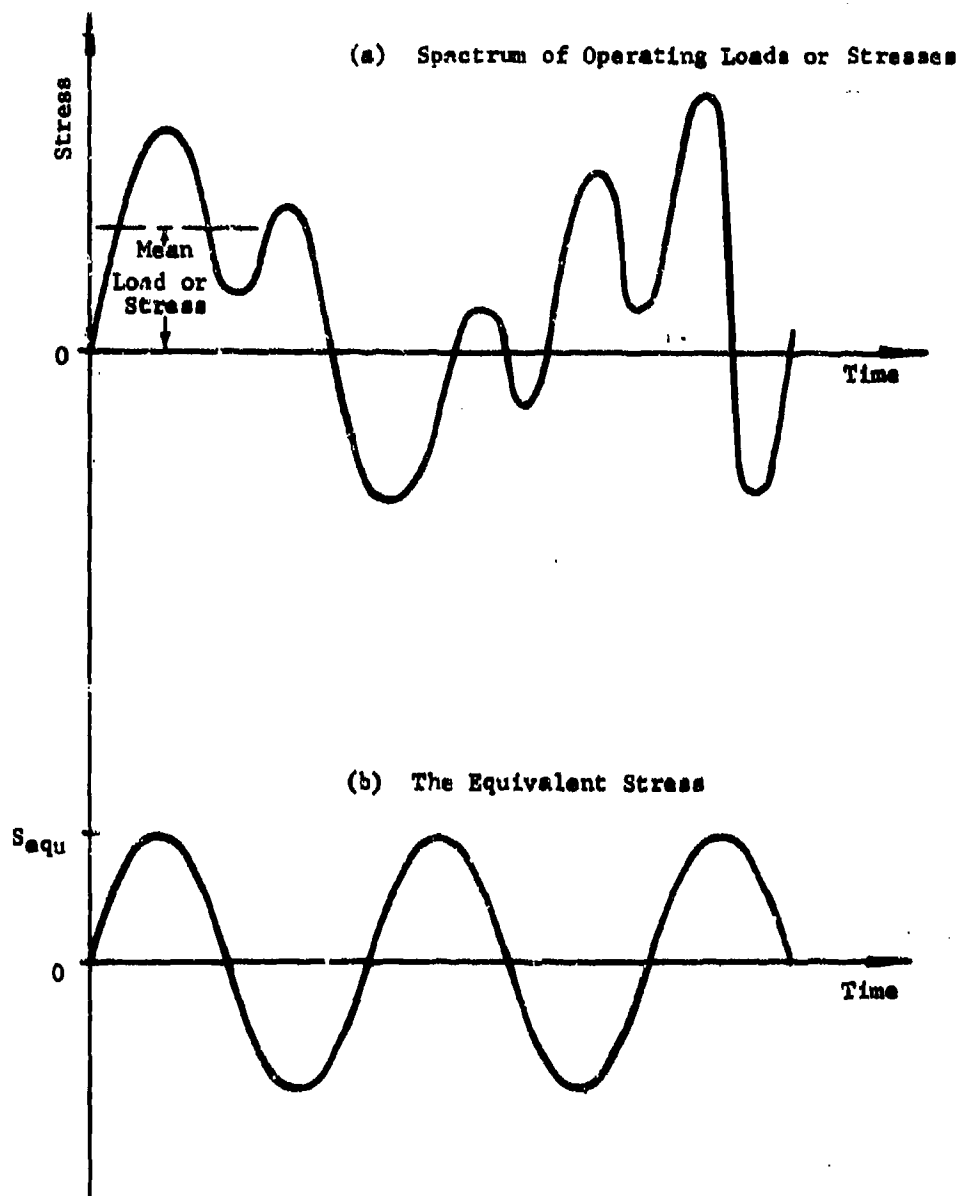


Figure 7.2 Conversion of Stress Spectrum to Equivalent Stress

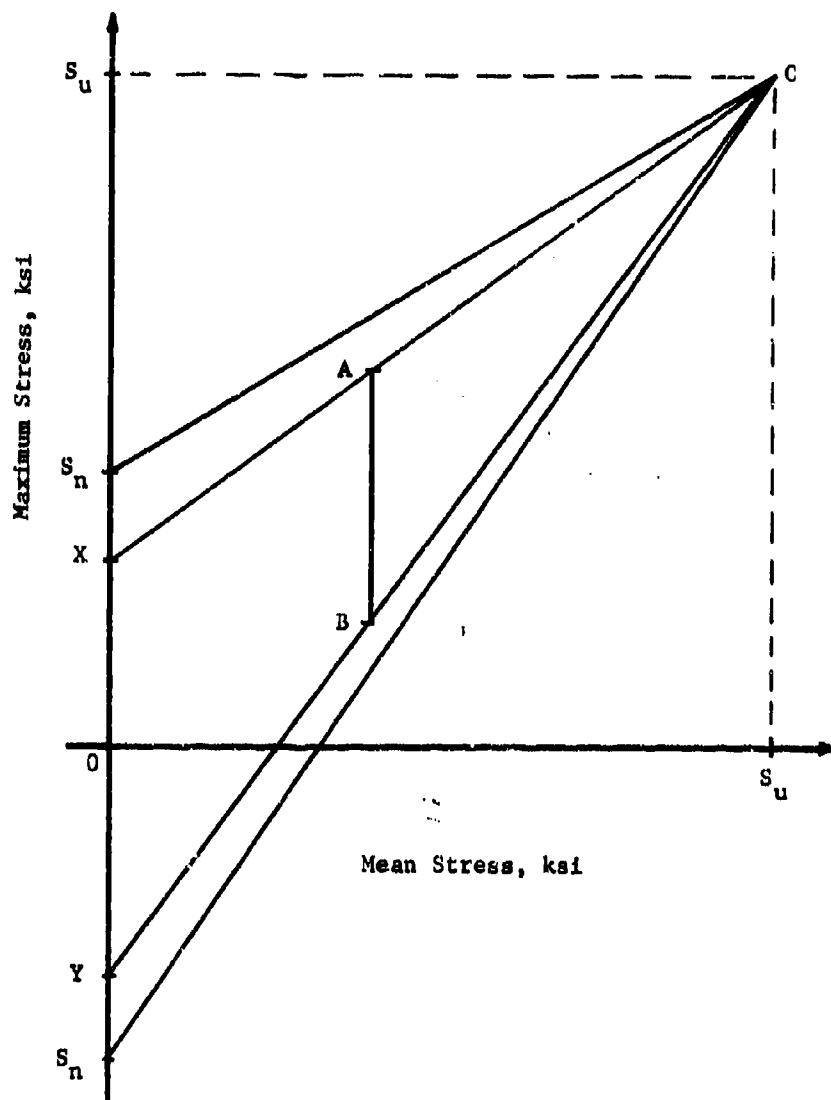


Figure 7.3 Modified Goodman Diagram

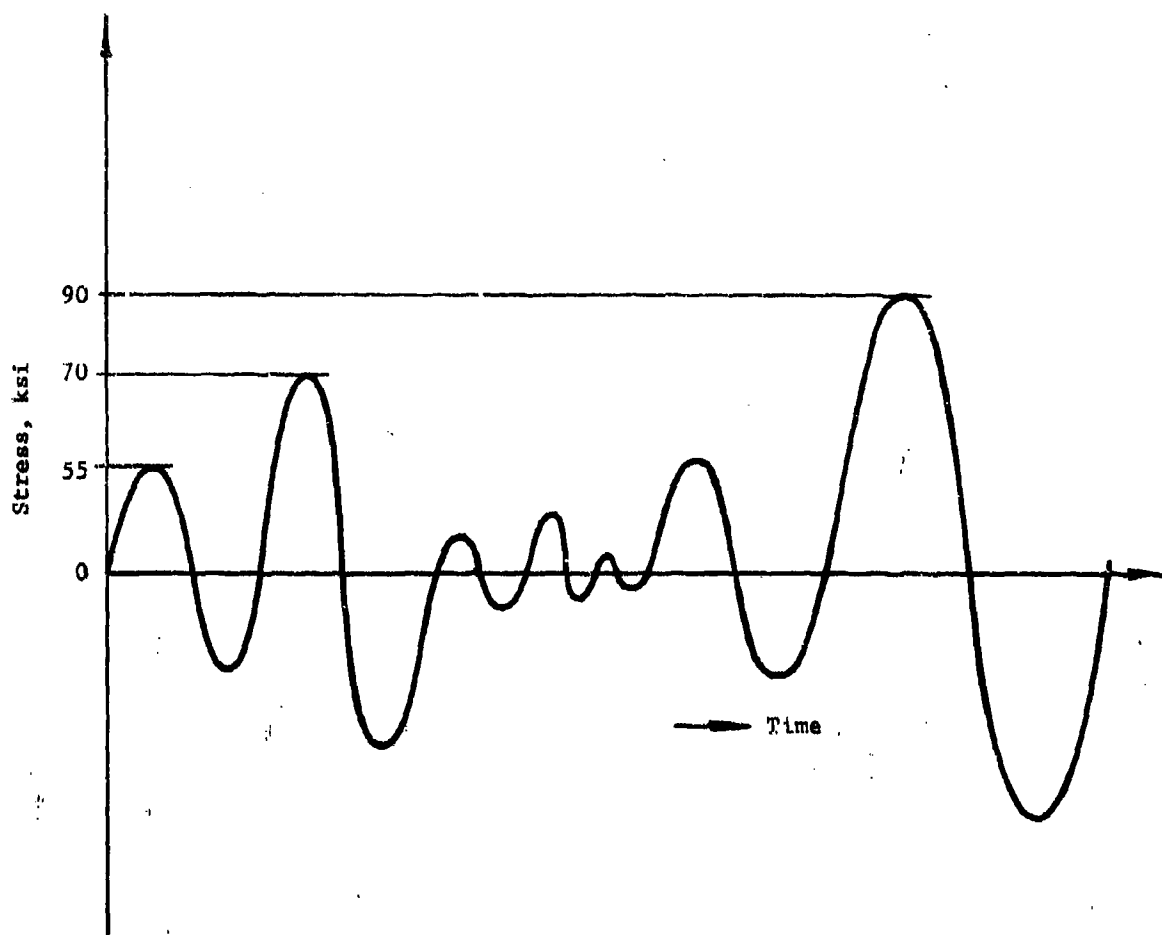


Figure 7.4 Completely Reversed Stress Reduced from the Stress Spectrum  
Through Modified Goodman Diagram

$$N_{equ} = 1 \times \frac{\sum_{i=1}^{i=k} n_i}{\sum_{i=1}^{i=k} \frac{n_i}{N_i}} \quad (7.2)$$

Suppose, for example, there are three stress levels, 90, 70, and 50 ksi, in a given spectrum. With the reference to the curve in Figure 7.5  $1/(6 \times 10^4)$  life is consumed by each 90 ksi stress cycle,  $1/(5 \times 10^5)$  by each 70 ksi cycle,  $1/(8 \times 10^5)$  by each 55 ksi cycle, etc. Using equation (7.2).

$$N_{equ} = \frac{1 + 1 + 1}{\frac{1}{6 \times 10^4} + \frac{1}{5 \times 10^5} + \frac{1}{8 \times 10^5}} = 1.5 \times 10^5 \text{ cycles}$$

Thus, the life of the part under the above spectrum of stresses will be equivalent to a life of  $1.5 \times 10^5$  cycles. The stress equivalent to this life is (from Figure 7.5) 75 ksi. Hence, the damage that the part accumulates due to the above spectrum of varying stress amplitude will be the same as if stress cycles of constant amplitude equal to  $S_{equ}$  (in this case, 75 ksi) were imposed for  $N_{equ}$  ( $1.5 \times 10^5$  cycles). Thus, the spectrum of stresses can be replaced by a single stress.

Miner's rule, as stated in equation (7.1), gives one (1.0) as the criterion for failure. Miner's original tests showed that the value for the summation in Equation 7.1 actually varied between 0.61 and 1.45. His more recent data<sup>9</sup> gives a range of 0.7 to 2.2. Other sources<sup>9</sup> quote a range as high as 0.18 to 23.0. In view of all this scatter it is generally agreed that the value of one (1.0), originally proposed by Miner, is probably the best overall estimate that can be made at this time.

### 7.2.2 Corten-Dolan's Rule

The application of this rule in converting the stress spectrum to a single equivalent stress ( $S_{equ}$ ) is identical to that of Miner's rule, except that the S-N curve used to obtain the life values  $N_1, N_2, \dots, N_k$  is modified. This modification is done, as shown in Figure 7.6, by changing the slope of the S-N curve. A line is drawn with an inverse slope  $d$  and passing through the point  $N_1$  on the S-N curve, of maximum stress amplitude (in this case,  $S_1$ ) occurring in the stress spectrum. This new line is known as the Corten-Dolan line.

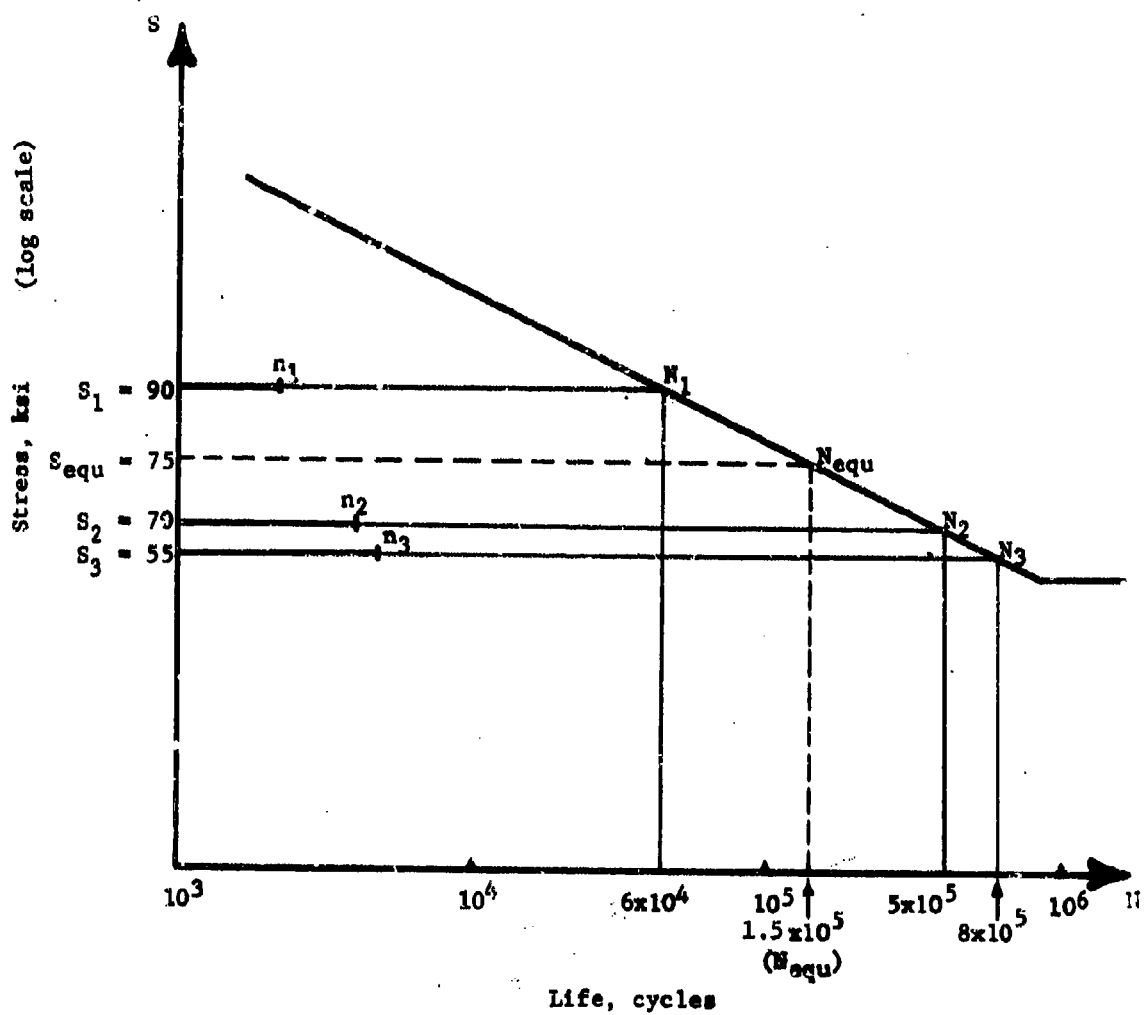


Figure 7.5 Miner's Rule

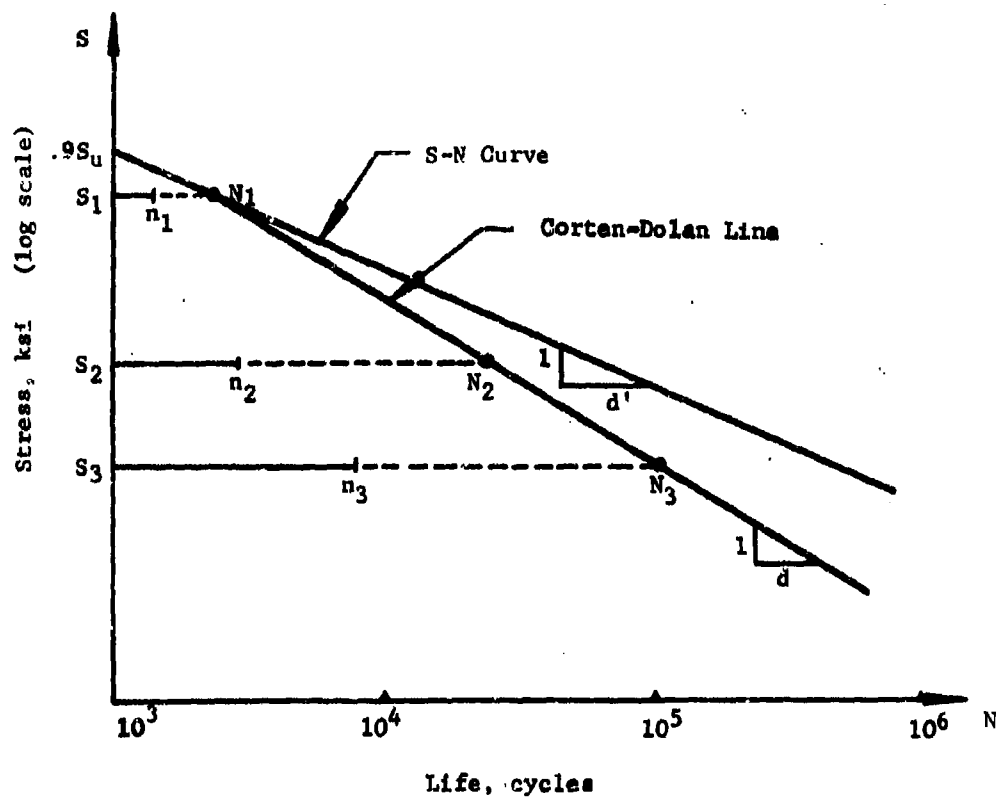


Figure 7.6 Corten-Dolan Line Vs S-N Curve

From available data<sup>10,11</sup> it appears that for structural steel specimens, having no stress concentration ( $K_f = 1$ ), the value of  $d/d' = 0.8$  is a reasonable estimate. A recent study by Harris and Lipson<sup>12</sup> indicates that when stress concentrations are present the following relationship can be used

$$d/d' = (0.73 + 0.07K_f) \quad (7.3)$$

This can be graphically expressed as in Figure 7.7. It will be noted that if  $K_f = 3.5$ ,  $d/d' \approx 1$  and this becomes equivalent to the criterion obtained from Miner's Rule.

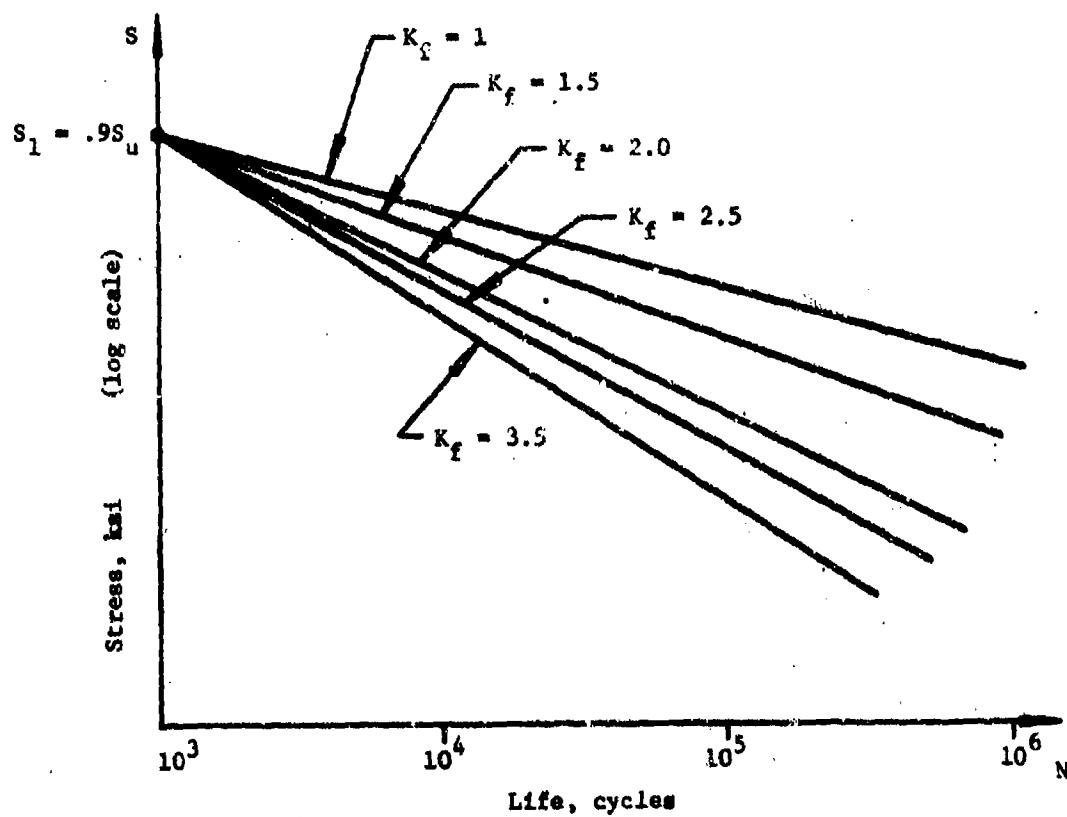


Figure 7.7 Corten-Dolan's Lines for Various Stress Concentration Factors, According to Harris and Lipson<sup>12</sup>



## SECTION 8 INTERFERENCE OF STRESS DISTRIBUTION WITH STRENGTH DISTRIBUTION

After the strength distribution and the stress distribution are determined (Sections 6 and 7 respectively) the two are compared and the percent interference is determined, as discussed in Section 2, Section 5, and in detail, in Section 9. For a given strength distribution the percent interference will depend on the distribution of the equivalent stress  $S_{equ}$ . A search through literature and other sources produced considerable amount of data leading to strength distribution but very little information on stress distribution.

In some engineering applications there is very little scatter in stresses. This leads to a stress distribution with standard deviation equal to zero. This distribution can be represented by a straight line, as in Figure 8.1, and the interference can be determined as shown.

For a given  $S_{equ}$ , interference may increase or decrease, if the life to which the components are designed is changed. This is shown in Figure 8.2, and in terms of S-N diagram in Figure 8.3. The shape of the distribution curve in Figure 8.2 is different from those in Figure 8.3 because the former are plotted on a linear scale while the latter on a log-log scale.

In those engineering applications where the scatter in stresses is appreciable the above approach will obviously not apply. On the basis of past experience, in the present investigation the stress distribution ( $S_{equ}$ ) was assumed to be normal and the range of standard deviations to be not less than  $.01\mu$  and not more than  $.10\mu$  where  $\mu$  is equal to  $S_{equ}$ . The resulting interference is represented qualitatively in Figure 8.4.

Examples of a design problem employing this method are given in Section 9.

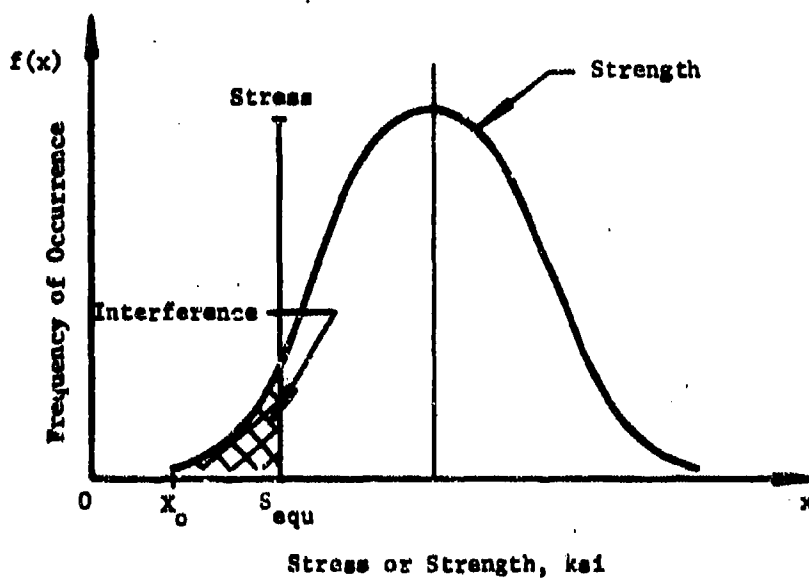


Figure 8.1 Interference with Standard Deviation of Stress equal to Zero

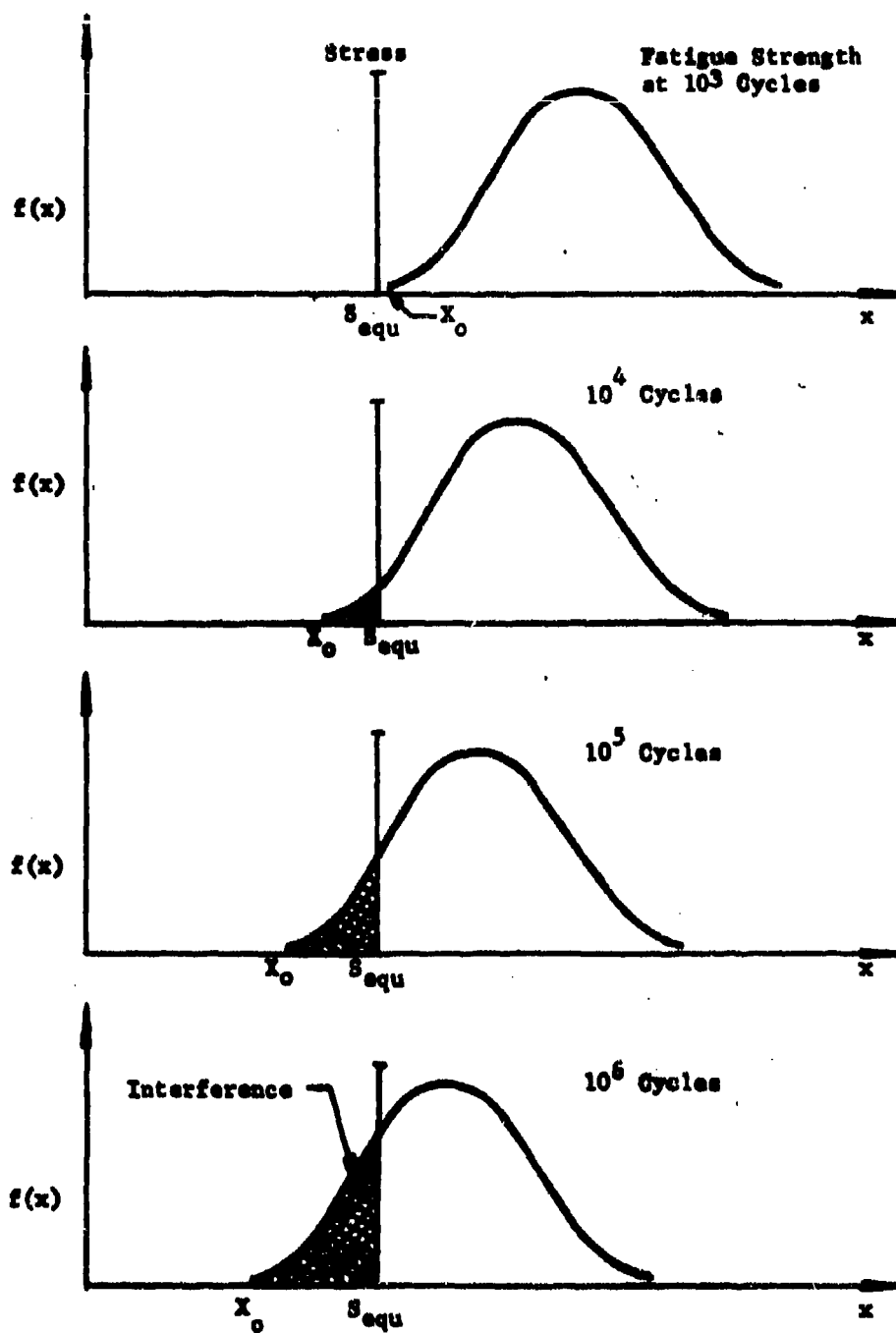


Figure 8.2 Interference of  $s_{equ}$  with Strength Distribution at Different Lives

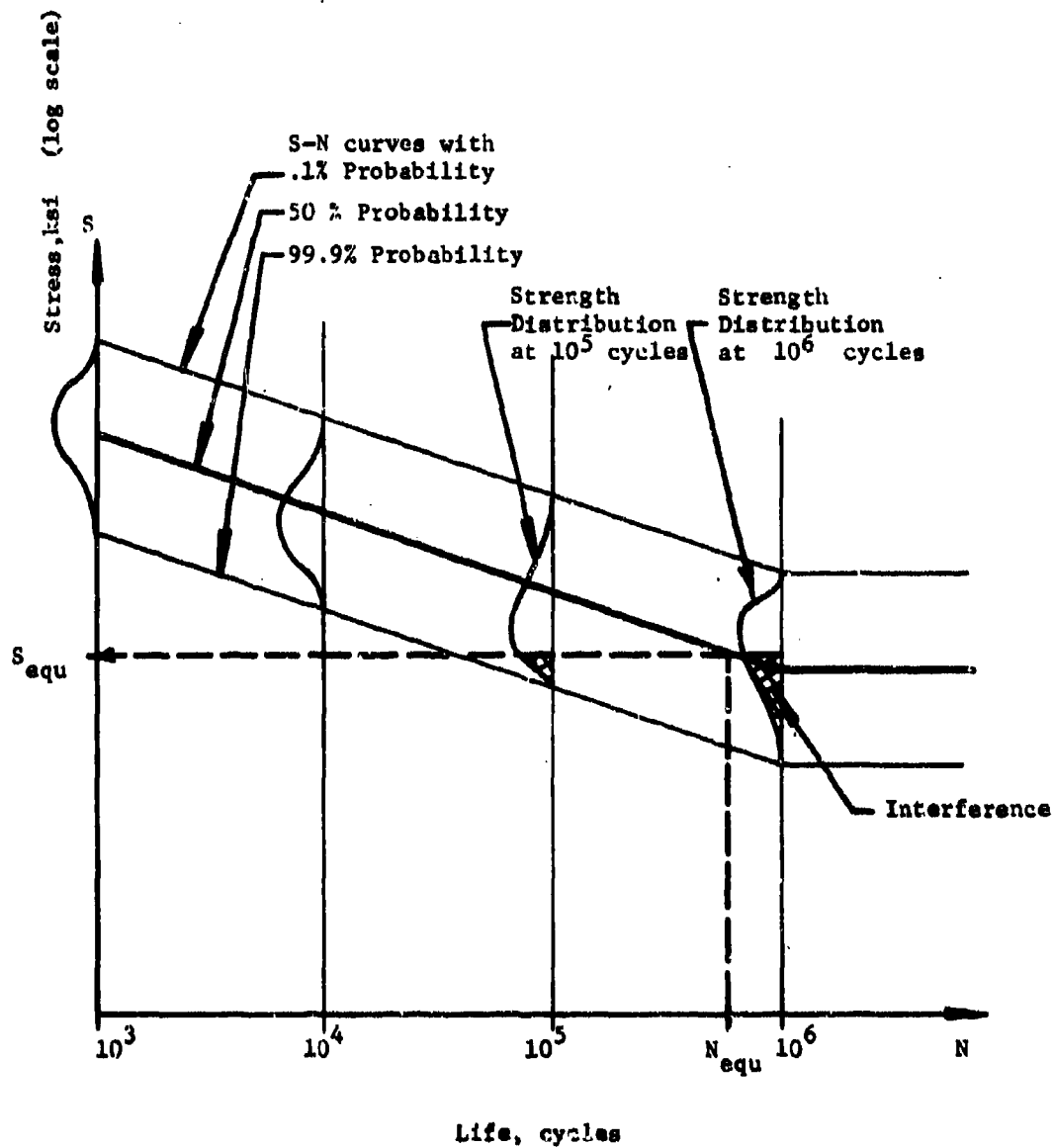


Figure 8.3 S-N Diagram Representing the Dependence of Interference on Life

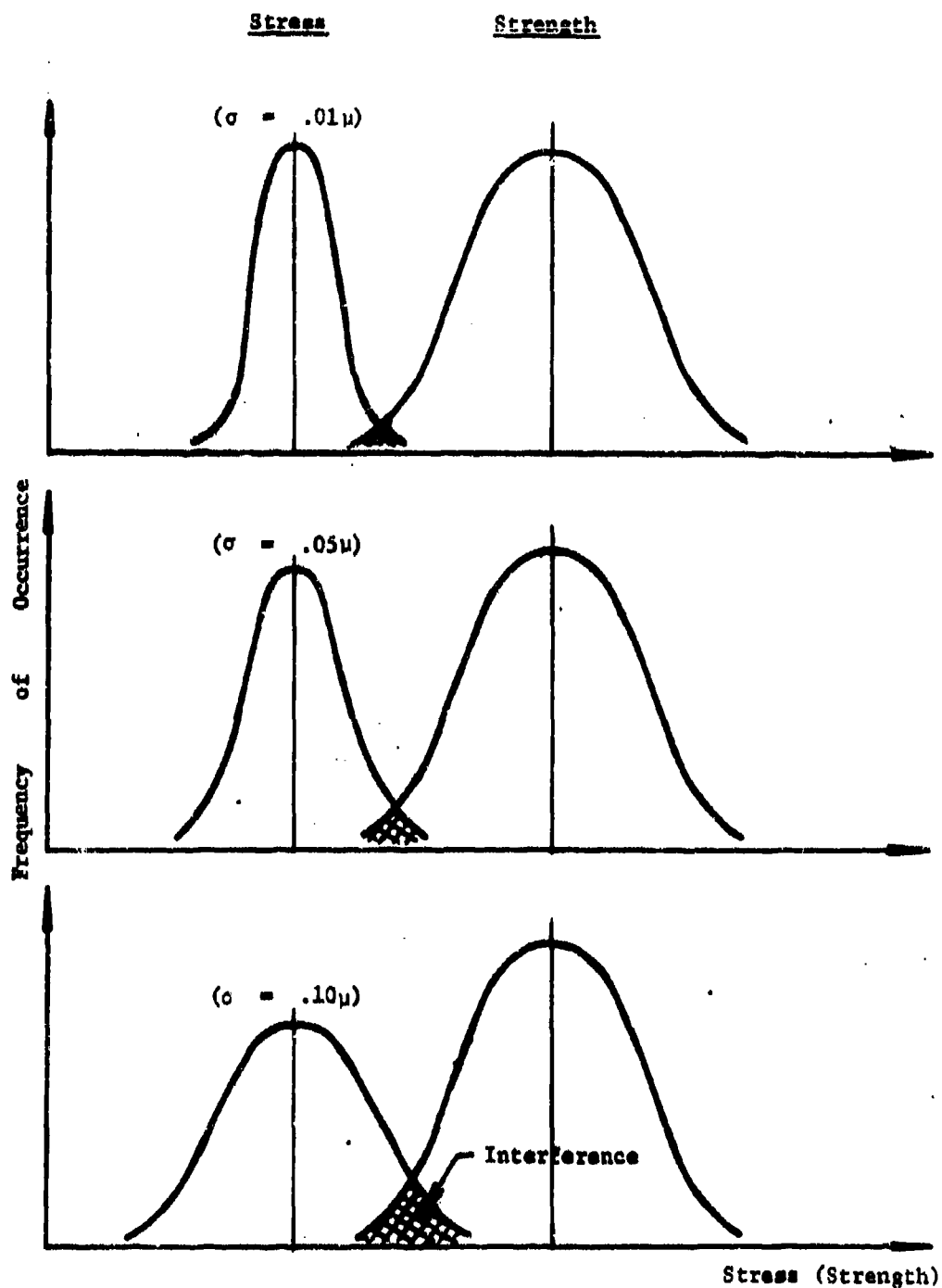


Figure 8.4 Interference of  $S_{equ}$  with Strength Distribution for Various Values of Standard Deviation

## SECTION 9 APPLICATION OF INTERFERENCE THEORY TO DESIGN PROBLEMS

Once the parameters of the strength distribution ( $X_0$ ,  $b$ ,  $\theta$ ) and stress distribution ( $\mu = S_{equ}$  and  $\sigma = k\mu$ , where  $k$  represents a fraction of the average stress) are determined, as shown in Sections 6 and 7 respectively, the percent interference can be computed with the aid of Tables on pages 258-396. Specific steps to be taken are illustrated by the following example.

A certain machine part was designed to withstand in service 10,000 overload cycles. The problem was to predict its reliability under the following conditions:

Material:  $T_1$ -6Al-4V,  $S_u = 177$  ksi,  $S_y = 166$  ksi

Design Life:  $10^4$  cycles

Type of Loading: Bending, completely reversed

Size: 0.25 in.

Surface Finish: Hot rolled

Theoretical Stress Concentration Factor:  $k_t = 1.0$

Operating Temperature:  $600^\circ F$

### 9.1 Weibull Parameters

The first step was to determine the strength distribution in terms of the Weibull parameters. From the graph on page 129 or Table on page 237. Weibull parameters corresponding to the above conditions were found to be:

$$X_0 = 50 \text{ ksi}$$

$$b = 2.65$$

$$\theta = 77.1 \text{ ksi.}$$

### 9.2 The Equivalent Stress

As to the stress distribution, the part was instrumented and the stress spectrum was recorded as shown in columns 1 and 2 of Table 9.1.

Spectrum of Stress		Miner's Rule Data	
Completely* reversed stress S, ksi	Occurrences n, cycles	Number of cycles to failure, N	$\frac{n}{N}$
1	2	3	4
52.0	200	$3.5 \times 10^5$	$5.710 \times 10^{-4}$
54.1	80	$2.4 \times 10^5$	$3.333 \times 10^{-4}$
56.5	50	$1.6 \times 10^5$	$3.125 \times 10^{-4}$
58.0	60	$1.2 \times 10^5$	$5.000 \times 10^{-4}$
59.3	20	$1.0 \times 10^5$	$2.000 \times 10^{-4}$
62.0	10	$6.6 \times 10^4$	$1.515 \times 10^{-4}$
64.8	5	$4.3 \times 10^4$	$1.162 \times 10^{-4}$
$\sum n_i = 425$		$\sum \frac{n_i}{N_i} = 21.845 \times 10^{-4}$	

Table 9.1 Stress and Life Data for Miner's Rule

\*Actually, stress was not completely reversed. It was reduced with the aid of the Goodman diagram to a completely reversed stress using the procedure given in Section 6.1.1.

In order to determine the parameters of the stress distribution ( $S_{equ} = \mu$ , and  $\sigma$ ) Miner's rule was used. From the S-N curve of the material (Figure 9.1), the number of cycles to failure,  $N$ , corresponding to stresses in Column 1, Table 9.1 were determined. This is shown in Column 3, Table 9.1. Using Miner's rule, as expressed in equation (7.2) and tabulated data in Table 9.1,  $N_{equ}$  was determined:

$$N_{equ} = 1 \times \frac{\sum n_i}{\sum \frac{n_i}{N_i}}$$

$$N_{equ} = 1 \times \frac{425}{21.845 \times 10^{-4}} = 1.945 \times 10^5 \text{ cycles.}$$

From the S-N curve (Figure 9.1), the stress corresponding to  $N_{equ} = 1.945 \times 10^5$  cycles was found to be  $S_{equ} = 55$  ksi. Hence, a completely reversed stress application of 55 ksi can be substituted for the recorded stress spectrum (Columns 1 and 2, Table 9.1).

### 9.3 Percent Interference

Once the strength and stress distribution parameters are established, percent interference can be determined.

In some engineering applications the scatter in the operating stresses is very small and, therefore, the standard deviation of the stress can be assumed to be zero. In those cases the percent interference can be determined as follows:

$$\text{Interference} = F(x) = 1 - e^{-\left(\frac{x - X_0}{\theta - X_0}\right)^b} = \text{shaded area under the curve shown in Figure 8.1}$$

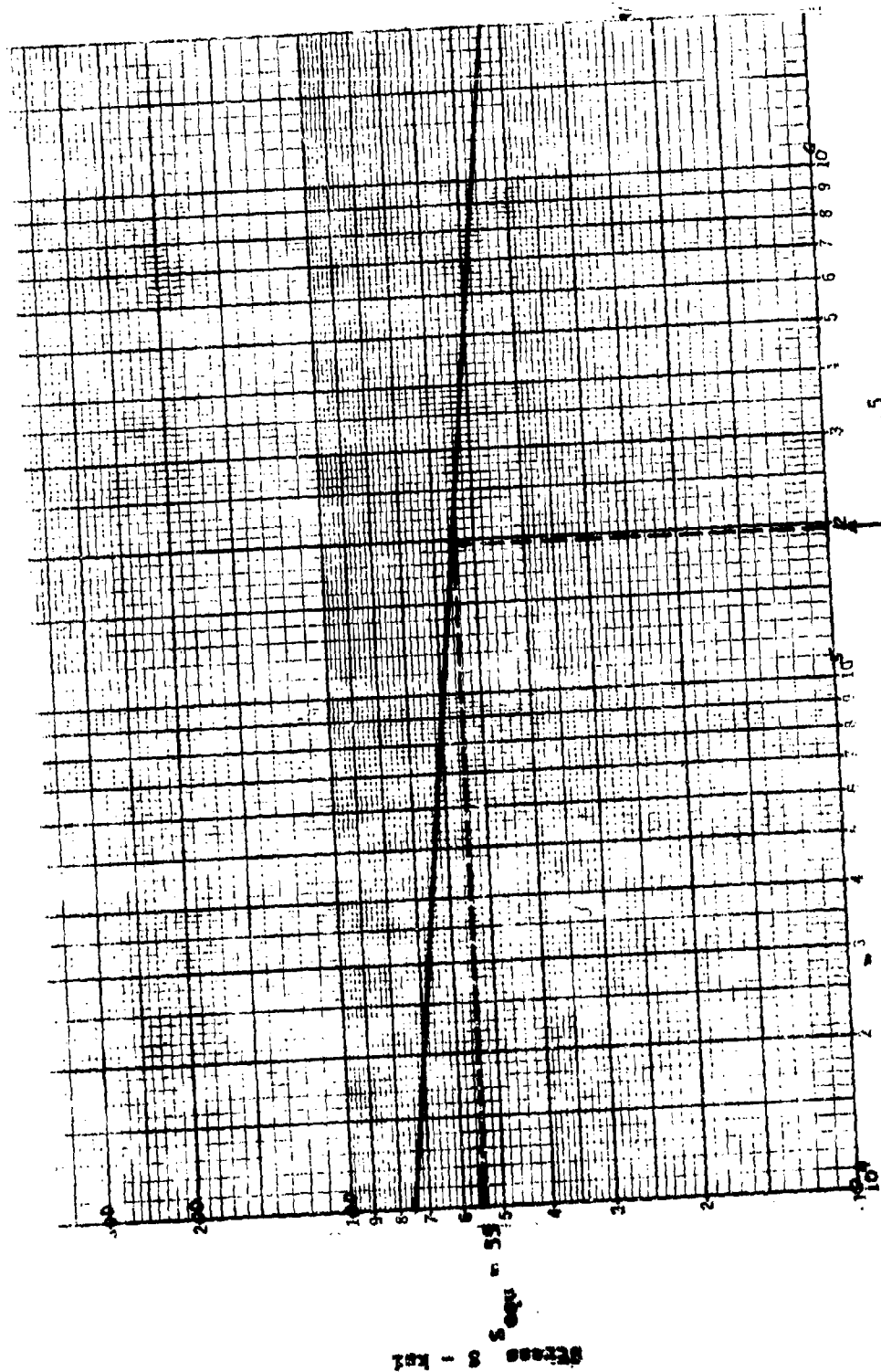
$$\text{where } x = S_{equ} = 55 \text{ ksi}$$

$$X_0 = 50 \text{ ksi}$$

$$b = 2.65$$

$$\theta = 77.1 \text{ ksi.}$$





Life  $N$  - cycles  
Figure 9.1 S-N Relationship

$$\begin{aligned}
 F(x) &= 1 - e^{-\left(\frac{55 - 50}{77.1 - 50}\right)^{2.65}} \\
 &= 1 - e^{-0.0114} \\
 &= .0113
 \end{aligned}$$

Percent Interference = 1.13%

This can also be read directly from the Table on page 262.

$$\text{Find } X = \left( \frac{x - X_0}{\theta - X_0} \right)^b = .0114$$

Corresponding to  $X = .0114$  read interference  $F(x) = .0113$ , from the above table. Therefore, Percent Interference = 1.13%.

In those engineering applications where the scatter of stress is appreciable interference may be found as follows. As pointed out before, in actual engineering practice, the standard deviation lies in the range

$$0.01 \leq \frac{\sigma}{\mu} \leq 0.10$$

In the absence of any specific information, an average value of  $\frac{\sigma}{\mu} = 0.05$  can probably be assumed. Using this value, percent interference is determined:

<u>Strength</u>	<u>Stress</u>
$X_0 = 50 \text{ ksi}$	$\mu = S_{eq} = 55 \text{ ksi}$
$b = 2.65$	$\sigma = 0.05\mu$
$\theta = 77.1 \text{ ksi}$	$= 0.05 \times 55 \text{ ksi}$
	$= 2.75 \text{ ksi}$

From the above data, parameters C, A and B(x), (for definition see page 27) to be used in the interference table, were computed:

$$C = \frac{\theta - X_o}{\sigma} = \frac{77.1 - 50}{2.75} \approx 10$$

$$A = \frac{X_o - \mu}{\sigma} = \frac{50 - 55}{2.75} = -1.82$$

$$B(x) = b = 2.65$$

The interference value corresponding to these parameters was found by interpolation between Table on page 293 (for  $B(x) = 2.0$ ) and Table on page 295 (for  $B(x) = 3.0$ ). By interpolating between these two sets of data, the interference was found to be

$$\text{Interference} = .0245$$

$$\text{or Percent Interference} = 2.45\%$$

Thus, percent interferences, that is, probabilities of failure to be expected are:

In the event of no scatter in stresses - 1.13% Failures.

For the scatter of the order of  $0.05\mu$  (2.75 ksi) - 2.45% Failures.

#### 9.4 The Effect of Design Factors

In this manner, the effect of various design factors on percent interference, can be determined. Table 9.2 shows the effect of temperature on interference for design conditions stated in the above example. Table 9.3 gives the effect of life on interference for a different set of conditions stated below:

Material: M10 Tool Steel,  $S_u = 330 \text{ ksi}$

Design Life:  $10^5$  cycles

Type of loading: Bending, completely reversed

Surface Finish: Mechanically Polished

Theoretical Stress Concentration Factor:  $k_t = 1.0$

Heat Treatment: 2A shown on the Table on page 223.

Material	Temperature of	Equivalent Stress $S_{equ}$ , ksi	Weibull Parameters of Strength			Percent Interference	
			$X_0$ , ksi	$b$	$\theta$ , ksi	$\sigma = 0$	$\sigma = 0.05\mu$
T <sub>1</sub> -6Al-4V	600	55.0	50	2.65	77.1	1.13	2.45
	30	55.7	70	3.2	96.8	0.0	0.0

Table 9.2 Effect of Temperature on Percent Interference

Material	Life, cycles	Equivalent Stress, $S_{equ}$ , ksi	Weibull Parameters of Strength			Percent Interference	
			$X_o$ , ksi	b	$\theta$ , ksi	$\sigma = 0$	$\sigma = .028\mu$
M 10 Tool Steel	$10^4$	122	127	1.89	163.5	0.0	0.0
	$10^5$		119	1.95	153.2	0.865	2.04
	$10^6$		111	2.0	143.5	10.80	12.03

Table 9.3 Effect of Life on Percent Interference

## CONCLUSIONS

1. A method was developed for employing stress-strength Interference Theory as a practical engineering tool to be used for designing and quantitatively predicting the reliability of mechanical parts and components subjected to mechanical loading.
2. This method is based on the considerable empirical data gathered (Appendix 1) and it also has sound theoretical basis (Appendix 3 and Appendix 4). This method eliminates the concept of a Factor of Safety and substitutes Percent Interference (Probability of Failure). Tables of interference values are given in Appendix 2 for a variety of stress and strength conditions.
3. Although a great deal of data were gathered and analyzed in the course of the present study, no data were found to permit the establishment of confidence intervals on the probability of interference.
4. This method can be used for three cases most commonly encountered in engineering practice:

### Stress Distribution

Normal  
Normal  
Weibull

### Strength Distribution

Normal  
Weibull  
Weibull

5. The effect of type of loading, surface finish, surface treatment, temperature, stress concentration, heat treatment etc, on the statistical distribution was also studied. These effects were expressed in terms of Weibull parameters  $X_0$ ,  $\theta$ , and  $b$  (see graphs in Section 6.4 of the body of the report and Tables pages 194-257).
6. For most of the materials studied, the lower bound of fatigue strength ( $X_0$ ) and the characteristic strength ( $\theta$ ) have a linearly decreasing relationship with life, on a log-log scale. In the case of the Weibull slope ( $b$ ) it decreases or increases linearly with life, on a log-log scale, depending on the material and the loading, surface, etc. conditions.
7. In the case of the tensile strength data were obtained to study the effect of temperature. None of the Weibull parameters showed any recognizable relationship between tensile strength and temperature, on either Cartesian or log-log scale.
8. Although the relationship between the fatigue factors (listed under item 5 above) and the statistical distribution of strength was established on an individual basis (item 6 above), no data were found which could be used to determine their combined effect. It may be safely assumed that under this condition the fatigue strength will follow a normal

distribution (a special case of Weibull). As in the present study, this distribution will probably vary with the design life.

9. As to the problem of stress distribution, the data found in literature and other sources were in the spectral form. For use in the Interference Theory they had to be converted into a distribution of equivalent stresses.

## RECOMMENDATIONS

1. In the present investigation the work involved the determination of Weibull parameters, mostly for ferrous materials. These parameters are essential for the prediction of interference. In the aircraft industry the materials are largely non-ferrous. It would be desirable, therefore, that the interference for these materials be determined too.
2. It is proposed that a computer method, instead of the currently used (Section 6) graphical method, be used for the determination of the Weibull parameters. This method has the advantage of time saving, higher accuracy, and it may allow the establishment of confidence levels associated with interference.
3. An analytical expression for the general case of interference, as a function of time (life, cycles), should be developed. At the present time, Weibull parameters of strength ( $X_0$ ,  $b$ ,  $\theta$ ) have to be specifically determined for each particular life in order to calculate interference at that life. By establishing a general expression, once the interference at one life is known, the interference at any other life can be quickly calculated.
4. The problem of stress distribution demands further work. Means of conversion from stress spectrum to stress distribution should be refined and a more exact form of the statistical distribution of the equivalent stress should be established.
5. At present, in using the tables of interference it is necessary to extrapolate and interpolate interference values in a given table or between the tables. Because of the highly non-linear behavior of these values (as discussed in Appendix 4, Section 4.1.6 and Appendix 4, Section 4.1.9) it would be desirable to have tables calculated for a finer grade of values of the parameters.
6. In the case when both interfering distributions are Weibull, percent interference will depend on six parameters ( $X_{01}$ ,  $X_{02}$ ,  $\theta_1$ ,  $\theta_2$ ,  $b_1$ ,  $b_2$ ). By appropriate grouping, these parameters can be reduced to four and percent interference calculated. In order to include a reasonable range of values for each parameter a large number of tables, cumbersome to handle, would be necessary. Hence, four or five dimensional nomographs should be prepared which would give percent interference as a function of a full range of values of the four parameters.
7. In order to verify the validity of the Interference Technique developed here it should be checked against an actual life situation. That is, percent interference should be computed for an actual engineering problem. These results then should be compared with actual service failures.



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APPENDIX 1 TABLES OF WEIBULL PARAMETERS

A-1.1 WEIBULL PARAMETERS OF FATIGUE STRENGTH

INDEX TO WEIBULL PARAMETERS  
FOR  
FATIGUE STRENGTH OF VARIOUS MATERIALS

Material

Weibull Parameters  
can be found:  
Page No.

CARBON AND ALLOY STEELS

1.	AISI 3140	207
2.	AISI 1045	208
3.	AMS 5727	209
4.	AISI 2340	210
5.	AISI 4140	212
6.	D <sub>6</sub> AC Steel (Ladish)	213
7.	H-11 Steel	214
8A.	4340 Steel	217
8B.	4340 Steel	220
9.	Thermold J	221
10.	Fe - 5.5, Mo - 2.5, Cr - .5C	222
11.	M 10 Tool Steel	223

STAINLESS STEELS

12.	321 Stainless	224
13.	A - 286 Stainless	226
14.	347 Stainless	227
15A.	Multiment N-155	228

15B.	Multiment N-155	229
16.	PH 15 - 7 Mo Stainless Steel	231
17.	17 - 7 PH Stainless Steel	323

MISCELLANEOUS BASE MATERIALS

18.	Timkin 16-25-6 (AMS 5727)	233
19.	Stainless 403	233
20.	Lapelloy 311	234
21.	S-816 (AMS 5534)	234
22.	Inco SHS 260	234
23.	GMR - 235	234
24.	S - 816 (AMS 5765)	235
25.	Udimet 500	235
26.	Ti - 140	235
27.	Duralumin	236
28.	Ti-6Al-4V	237
29.	Inconel X	240

(COMPOSITION, HEAT TREATMENT AND TENSILE STRENGTH)

1. AISI 3140 Steel: Su = 108 - 109 Ksi.  
Composition: .4% C, .8% Mn, .3% Si, 1.2% Ni, .65% Cr.  
Heat Treatment: K 3 = OQ from 1520°F, Tempered at 1300°F, Su = 108 Ksi.  
K 4 = Air blast quenched from 1520°F, Tempered at 1050°F, Su = 109 Ksi.
2. AISI 1045 Steel: Su = 105 - 120 Ksi.  
Composition: .43 - .5% C, .6 - .9% Mn, .040% P(max), .05% S(max)  
Heat Treatment: K 1 = Water quenched from 1520°F, Tempered at 1210°F, Su = 105 Ksi, Sy = 82 Ksi.  
K 2 = Oil quenched from 1520°F, Tempered at 1050°F, Su = 120 Ksi, Sy = 84 Ksi.
3. AMS 5727 Steel: Su = 120 Ksi  
Composition:  
Heat Treatment: Fleishmann hot cold-work, equalize at 1950°F, reduce crosssection 18% from 1200°F, stress relieve at 1200°F for 8 hours.
4. AISI 2340 Steel: Su = 116 - 122 Ksi.  
Composition:  
Heat Treatment: A = Oil quenched from 1450°F, Tempered at 1200°F, Su = 116 Ksi.  
B = Air Blast quenched from 1450°F, Tempered at 700°F, Su = 119 Ksi.  
C = Air Blast quenched from 1450°F, no temper; Su = 122 Ksi.
5. AISI 4140 Steel: Su = 135 Ksi.  
Composition: C = .37 - .44%, Mn = .55 - .90%, Si = .20 - .35%, Ni = 1.55 - 2.00%, Cr = .65 - .95%, Mo = .20-.30%.  
Heat Treatment: Aust 1550°F, 1 hour; OQ, Temperature 1230°F, 1 hour.
6. D6AC Steel - (Ladish): Su = 270 Ksi.  
Composition: .42 - .48% C, .6 - .9% Mn, .015% P, .015% S, .15 - .13% S, .4 - .7% Ni, .9 - 1.2% Cr, .9 - 1.1% Mo, .05 - .1% V.  
Heat Treatment: Hold at 1500°F in oxidizing atmosphere, OQ Temperature 500°F, 2 hours.

7. M-11 Steel: Su = 272 Ksi.  
 Composition: 5% Cr, 1.5% Mo, .4% V, .35% C.  
 Heat Treatment: Vacuum arc melt, pre heat 1400°F, 30 min., Aust. at 1850°F, 45 min., A.C., Temperature 2 plus 2 hours at 1050°F, A.C. + pre test exposure.
- 8A. 4340 Steel: Su = 206 - 280 Ksi.  
 Composition: .37 - .44% C, .55 - .90% Mn, .20 - .35% Si, 1.55 - 2.00% Ni, .65 - .95% Cr, .20 - .30 Mo.  
 Heat Treatment: A = normalize at 1550°F, OQ, Temper 440°F, AC.  
 B = normalize at 1550°F, Quench, Temper 775°F, AC.  
 Melt Practice: .8 = Air melt, Vacuum arc remelt.  
 9 = Vacuum Induction melt.  
 10 = Vacuum Induction melt, Vacuum arc remelt.
- 8B. 4340 Steel: - varied tensile strength. Su = 144 - 290 Ksi.  
 Composition: same as 8A.  
 Heat Treatment: A: Norm, 1600°F, 1.5 hours, AC; Aust. 1525°F, 1.5 hours, OQ, Temper 1150°F, 4 hours, AC.  
 B: Norm, 1600°F, 2 hours, AC, Aust. 1500°F, 2 hours, OQ, Temper 1150°F, 4 hours, AC.  
 C: Norm, 1600°F, 1 hour.  
 D: Aust., 1550°F, Salt Bath 20 min., OQ to 120°F to 150°F, Temper 400°F, 4 hours, Melt-practice - elect. Furnace  
 E: Aust. 1550°F, Salt Bath 20 min., OQ to 120°F to 150°F, Temper 400°F, 4 hours, Melt-practice - Vacuum furnace.
9. Thermold J Su = 294 Ksi.  
 Composition: .37 - .44% C, .55 - .90% Mn, .20 - .35% Si, 1.55 - 2.00% Ni, .65 - .95% Cr, .20 - .30% Mo.  
 Heat Treatment: Sol Treated 1825°F, A.C., Tempered 1025°F, 2 hours - A.C. Retempered 1025°F, 2 hours A.C. .
10. Fe - 5.5 Mo - 2.5 Cr - .5 C: Su = 314 Ksi.  
 Composition: designated in name  
 Heat Treatment: Preheat 1400°F, 1/2 hours, harden 1950°F, 20 min. A.C. Temper 1050°F, 2 hours, Retemper 2 hours after finish machining.

11. M 10 Tool Steel: Su = 330 Ksi.  
Composition: 4% Cr, 2% V, .85% C.  
Heat Treatment: A = Preheat 1450°F, 1/2 hour, harden 2150°F, 5 min., OQ until black, A.C. Temper 1100°F, 2 hours, A.C., Retemper 1100°F, 2 hours, A.C., after finishing operation, nitrided 975°F, 48 hours.  
B = Same as A, but instead of nitriding, stress relieve at 1000°F in protective atmosphere, Furnace cool.

#### STAINLESS STEELS

12. 321 Stainless Steel: Su = 87 Ksi.  
Composition: 18% Cr, 10% Ni, 2% Mn, 1% Si, .08% C.  
Heat Treatment: Annealed
13. A-286 Stainless Steel: Su = 90 Ksi.  
Composition: 15% Cr, 26% Ni, 1.25% Mo, 2% Ti, .25% Al  
Heat Treatment: Hot rolled, solution treated.
14. 347 Stainless Steel: Su = 92 Ksi.  
Composition: 18% Cr, 11% Ni, 2% Mn, 1% Si, .08 C.  
Heat Treatment: Annealed.
- 15A. Multiment N - 155: Su = 119 Ksi.  
Composition: 21% Cr, 20% Ni, 20% Co, 5% Si, 3% Mo, 3% W, 1.5% Mn, 1% Cu, .15% C.  
Heat Treatment: Sol. Treated 2200°F, 1 hour, W.Q., Aged 1440°F, 16 hours, A.C. .
- 15B. Multiment N - 155: Su = 114 - 126 Ksi.  
Composition: Same as 15A.  
Heat Treatment: Same as 15A, except  
Some specimens were stress relieved after the Heat Treatment
16. Ph 15 - 7 Mo Stainless: Su = 201 Ksi  
Composition: 15% Cr, 7 % Ni, 2.25% Mo, 1.15% Al.  
Heat Treatment: Condition at 1750°F, 10 hours, A.C. refrigerate at -100°F for 8 hours, age 950°F, for 1 hour, A.C. TH 105.



17. 17 - 7 PH Stainless Steel:

Su = 205 Ksi.  
 Composition: 17% Cr, 7 % Ni, 1.15% Al, .7% Mn, .4% Si, .07% C.  
 Heat Treatment: Condition at 1750°F for 10 hours, A.C., Refrigerate at -100°F for 8 hours, Age 950°F for 1 hour A.C.

MISCELLANEOUS BASE MATERIALS.

18. Timken 10-25-6: Su = 120 Ksi.  
 Composition: Not Available.
19. Stainless 403: Su = 141 Ksi (Axial test), 129 Ksi (Rotary test)  
 Composition: .15% C(max), 1.0% Mn(max), .5% Si(max), 11.5% - 13.0% Cr.
20. Lapelloy 311: Su = 136 Ksi (Axial), 129 Ksi (Rotary)  
 Composition: Not Available.
21. S - 816 (AMS 5534): Su = 147 Ksi.  
 Composition: Not Available.
22. Inco SHS 260: Su = 260 Ksi (Axial), 129 - 132 (Rotary)  
 Composition: Not Available.
23. GMR - 235: Su = Not Available  
 Composition: 65% Ni, 15% Cr, 10% Fe, 5% Mo, 3% Al, 2% Ti.
24. S-816 (AMS 5765): Su = 147 Ksi.  
 Composition: 42% Co, 20% Cr, 20% Ni, 4% Mo, 4% W, 4% Cb, 4% Fe.
25. Udimet 500: Su = Not Available  
 Composition: Ni base, .1% C, 19% Cr, 19% Co, 4% Mo, 3% Ti, 2.9% Al, 4% Fe.
26. Ti - 140 (AMS 4923): Su = 130 - 150 Ksi.  
 Composition: 5.5 - 6.75% Al, 3.5 - 4.5% V, .07% Ni, .1% H(max), .4% Fe(max).
27. Duralumin: Su = unknown  
 Composition: Al, Cu, Mn, Mg  
 Heat Treatment: unknown

28. Ti - 6Al - 4V: Su = 177 Ksi.  
Composition: Ti base, 6% Al, 4% V, .07% Ni(max), .1% C(max),  
.015% H(max), .4% Fe(max), .3% O(max).  
Heat Treatment: A = Sol treated 1690°F, 12 min, W.Q., Aged  
900°F, 4 hours, A.C. .  
B = Sol Treated 1675°F, 20 min, W.Q., Aged  
900°F, 4 hours, A.C. .
29. Inconel X: Su = 225 Ksi.  
Composition: 73% Ni, 15% Cr, 7% Fe, 2.5% Ti, 1% Cu, .7% Mn,  
.04% C.  
Heat Treatment: Aged 1350°F, 16 hours, A.C. .

## REFERENCE DATA FOR WEIBULL PARAMETERS

### (SPECIMEN CONDITIONS)

#### CARBON AND ALLOY STEELS.

1. AISI 3140 Steel:  
Tested at Room Temperature.
2. AISI 1045 Steel:  
Hot rolled, lathed; tested at Room Temperature.
3. AMS 5727 Steel:  
Ground and lapped, 10 RMS.
4. AISI 2340 Steel:  
Hot rolled, lathe turned, hand polished, tested at Room Temperature.
5. AISI 4140 Steel:  
Hot rolled, longitudinal machining (mechanical), tested at Room Temperature.
6. D6AC Steel (Ladish):  
Vacuum furnace melt, hot rolled, machine polished with 600 grit belt.
7. H-11 Steel:  
Hot rolled, lathed, grain direction is transverse to lengthwise axis.
- 8A. AISI 4340 Steel:  
Lathe turned, mechanical polish.
- 8B. AISI 4340 Steel:  
for specimens with  $S_u = 144, 158, 171$  Ksi, preparation = hot rolled and lathed, for specimens with  $S_u = 275, 290$  Ksi, preparation = forged and ground. All tested at Room Temperature.
9. Thermold J:  
Tested at Room Temperature.
10. Fe - 5.5 Mo - 2.5 Cr. - .5C  
Forged and Swaged - 5 RMS.
11. M 10 Tool Steel:  
Forged and Swaged, lathe turned, 5 RMS, tested at Room Temperature.

#### STAINLESS STEELS.

12. 321 Stainless Steel:  
Hot rolled, mechanical polish.  
Some specimens T.I.G. (Tungston Inert Gas) welded.
13. A-286 Stainless Steel:  
Hot rolled, mechanical polish.  
Some specimens T.I.G. welded.
14. 347 Stainless Steel:  
Hot rolled.
- 15A. Multiment N-155:  
Hot rolled, lathed, mechanical polish. Tested at Room Temperature.
- 15B. Multiment N-155:  
Hot rolled, lathed, mechanical polish. Tested at Room Temperature.  
Surface preparation code:  
A. Stress relieved after surface finishing.  
B. Surface finished, stress relieved, refinished.  
C. Heat treated after surface finishing.
16. Ph 15-7Mo, Stainless Steel:  
Hot rolled, milled edges.
17. 17-7 PH, Stainless Steel:  
Hot rolled, hand polished, ground edges.

#### MISCELLANEOUS BASE MATERIALS.

18. Timkin 16-25-6:  
Completely reversed test, unnotched specimen.
19. Stainless 403:  
Completely reversed test, unnotched specimen.
20. Lapelloy 311:  
Completely reversed test, unnotched specimen.
21. S-816 (AMS 5534):  
Completely reversed test, unnotched specimen.

22. Inco SMS 260:  
Completely reversed test, unnotched specimen.
23. GMR - 235:  
Completely reversed test, unnotched specimen.
24. S-816 (AMS 5765):  
Completely reversed test, unnotched specimen.
25. Udimet 500:  
Completely reversed test, unnotched specimen.
26. Ti - 140 (AMS 493):  
Completely reversed test, unnotched specimen.
27. Duralumin:  
Non corroded and corroded in saltwater.
28. Ti - 6Al - 4V:  
Hot rolled.
29. Inconel X:  
Hot rolled.

**TABLES**

# AISI 3140 STEEL

$S_u = 108, 109 \text{ ksi}$   
 $S_y = 87, 75 \text{ ksi}$

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Conditions<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

## ROTARY BENDING

H.T.	Spec	$X_0$ ksi	$b$						$\theta$ ksi					
Life, Cycles		$10^3$	$10^4$	$10^5$	$10^6$	$10^3$	$10^4$	$10^5$	$10^3$	$10^4$	$10^5$	$10^6$		
Effect of Stress Concentration														
$K_5$	Mo-N	89	74	66	57	3.7	5.2	5	5.5	100.4	87.7	76.7	67.2	
$K_3$	V-N		30	19	20		5.6	6.1	5.1		62.2	47.3	36	
$K_4$	Mo-N	85	75	66.5	59.0	2.2	2.2	2.4	2.2	87.9	77.87	69.08	61.07	
$K_4$	V-N		53	36.5	25		5	3.2	3.5		70.2	44.25	29.53	

## Effect of Heat Treatment

$K_5$	Mo-N	89	74	66	57	3.7	5.2	5	5.5	100.4	87.7	76.7	67.2	
$K_4$	Mo-N	85	75	66.5	59.0	2.2	2.2	2.4	2.2	87.9	77.87	69.08	61.07	
$K_3$	V-N		30	19	20		5.6	6.1	5.1		62.2	47.3	36	
$K_4$	V-N		53	36.5	25		5	3.2	3.5		70.2	44.25	29.53	

1 Composition - see page 19C, Item 1

2 Heat Treatment

A:  $K_5$  - OQ from 1520°F, Tempered at 1300°F,  $S_u = 108 \text{ ksi}$ ,  $S_y = 87 \text{ ksi}$

B:  $K_4$  - Air Blast quenched from 1520°F, Temp. at 1050°F,  $S_u = 109 \text{ ksi}$ ,  $S_y = 75 \text{ ksi}$

3 Specimen condition - see page 203

4 Meaning of Symbols - see page 46

# AISI 1045 STEEL

$S_u = 105, 120 \text{ ksi}$

## ROTARY BENDING

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Conditions<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

H.T.	$S_u$	Spec	$X_0 \text{ ksi}$			$b$			$\theta \text{ ksi}$		
Life, Cycles			$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$
Effect of Heat Treatment											
$K_1$	105	V-N	56.0	39.0	27.0	1.67	2.25	2.75	67.3	47.3	33.4
$K_2$	120	V-N	54.0	36.0	24.2	2.72	3.1	3.25	65.3	44.25	29.9
Effect of Stress Concentration											
$K_1$	105	No-N	79.0	67.0	56.7	2.6	2.75	2.85	86.2	73.0	61.65
$K_1$	105	V-N	56.0	39.0	27.0	1.67	2.25	2.75	67.3	47.3	33.4

1 For Composition - see page 198, Item 2

2 Heat Treatment

A:  $K_1$  WQ for 1520°F Tempered at 1210°F,  $S_u = 105 \text{ ksi}$  and  $S_y = 82 \text{ ksi}$

B:  $K_2$  Oil Quenched from 1520°F Tempered at 1050°F,  $S_u = 120 \text{ ksi}$ ,  $S_y = 84 \text{ ksi}$

3 For Specimen Condition - see page 203

4 For Meaning of Symbols - see page 46



AMS 5727 STEEL

$S_u = 120 \text{ ksi}$   
 $S_y = 30 \text{ ksi}$

AXIAL LOAD  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Conditions<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

T, °F	K <sub>t</sub>	X <sub>0</sub> , ksi			b			σ, ksi		
Life, cycles		10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
Effect of Temperature										
80	1.0	68	61	55	2.5	2.75	2.8	74	67	61
1200	1.0	51	47	44	2.85	2.85	3.0	58	54	50
80	2.4	59	42	30	2.75	2.95	2.98	66	47	34
1200	2.4	29	24	20	2.21	2.3	2.32	36	31	26
80	3.4	48	31	19	2.15	2.15	2.33	61	38	25
1200	3.4	27	20	15	2.62	2.75	3.02	33	25	19
Effect of Stress Concentration										
80	1.0	68	61	55	2.5	2.75	2.8	74	67	61
80	2.4	59	42	30	2.75	2.95	2.98	66	47	34
80	3.4	48	31	19	2.15	2.15	2.33	61	38	25
1200	1.0	51	47	44	2.85	2.85	3.0	58	54	50
1200	2.4	29	24	20	2.21	2.3	2.32	36	31	26
1200	3.4	27	20	15	2.62	2.75	3.02	33	25	19

- 1 For Composition - see page 198 Item 3
- 2 For Heat Treatment - see page 198 Item 3
- 3 For Specimen Conditions - see page 203 Item 3
- 4 For Meaning of Symbols - see page 46

AISI 2340 STEEL

$S_u = 116-122 \text{ ksi}$   
 $S_y = 76-96 \text{ ksi}$

ROTARY BENDING

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Condition<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

H.T. <sup>2</sup> (Code)	Spec Life, cycles	$S_y$	$S_u$	$X_o \text{ ksi}$		$b$		$\sigma \text{ ksi}$	
				$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$

Effect of Heat Treatment

A	NO-N	96	116	84.0	74.0	61.0	4.3	3.4	4.1	101.5	85.4	72.0
B	NO-N	79	119	81.0	71.0	62.0	2.8	2.8	2.8	88.7	77.6	67.8
C	NO-N	76	122	87.0	75.0	64.0	4.4	4.4	4.9	94.4	81.55	70.0
A	V-N	96	116	48.0	36.0	27.5	2.7	2.6	2.3	59.1	44.8	32.3
B	V-N	79	119	57.0	37.0	24.0	2.2	2.4	2.6	62.5	40.36	26.26
C	V-N	76	122	47.0	30.0	20.0	5.3	5.3	5.2	70.4	46.2	30.6

Effect of Stress Concentration

A	NO-N	96	116	84.0	74.0	61.0	4.3	3.4	4.1	101.5	85.4	72.0
A	V-N	96	116	48.0	36.0	27.5	2.7	2.6	2.3	59.1	44.8	32.3
B	NO-N	79	119	81.0	71.0	62.0	2.8	2.8	2.8	88.7	77.6	67.8
B	V-N	79	119	57.0	37.0	24.0	2.2	2.4	2.6	62.5	40.36	26.26
C	NO-N	76	122	87.0	75.0	64.0	4.4	4.4	4.9	94.4	81.55	70.5
C	V-N	76	122	47.0	30.0	20.0	5.3	5.3	5.2	70.4	46.2	30.6

1. For Composition - see page 198, Item 4
2. Heat Treatment Code
  - A: Oil Quenched from 1450°F, Tempered at 1200°F
  - B: Air Blast Quenched from 1450°F, Tempered at 700°F
  - C: Air Blast Quenched from 1450°F, no Temper
3. For Specimen Conditions - see page 203, Item 4
4. For Meaning of Symbols - see page 46

# 4140 STEEL

$S_u = 135 \text{ ksi}$   
 $S_y = 122 \text{ ksi}$

## ROTARY BENDING

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Condition<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

Type of Specimen	$X_o$ ksi	b						θ ksi		
Life, cycles	$10^4$	$10^5$	$10^6$	$10^7$	$10^4$	$10^5$	$10^6$	$10^7$	$10^4$	$10^6$ $10^7$
Effect of Stress Concentration										
unnotched	74	68	63	58	2.4	2.5	2.7	2.8	90.8	83.5 76.8 70.4
V-notched Flank Angle = 60°	36.0	28	23	18	3.2	3.45	3.6	3.8	56	46.4 37.6 31.8

- 1 For Composition - see page 198, Item 5
- 2 For Heat Treatment - see page 198, Item 5
- 3 For Specimen Condition - see page 203, Item 5
- 4 For Meaning of Symbols - see page 46

# D6AC STEEL

$S_u = 270 \text{ ksi}$   
 $S_y = 237 \text{ ksi}$

AXIAL LOAD  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Conditions<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

T	$S_m$	$K_t$	$X_o$			b			$\sigma$		
Life, cycles			$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$

## Effect of Temperature

80	000	1.0	160	90	50	2.8	2.9	3.0	191	106	60
450	000	1.0	145	115	90	3.15	3.3	4.6	162	129	102
550	000	1.0	125	100	78	3.7	3.8	4.0	161	125	98
80	000	3.0	52	40	30	3.3	3.4	3.8	82	66	53
450	000	3.0	73	41	34	2.1	2.54	3.4	81	46	40
550	000	3.0	63	44	35	2.75	3.0	3.4	70	51	40
80	30-50	3.0	55	38	26	2.7	3.1	3.25	66	46	32
450	30-50	3.0	39	34	29	3.8	4.1	4.7	48	42	37
550	30-50	3.0	43	35	29	4.0	4.5	4.7	51	42	34

## Effect of Stress Concentration

80	000	1.0	160	90	50	2.8	2.9	3.0	191	106	60
80	000	3.0	52	40	30	3.3	3.4	3.8	82	66	53
450	000	1.0	145	115	90	3.15	3.3	4.6	162	129	102
450	000	3.0	73	41	34	2.1	2.45	3.4	81	46	40
550	000	1.0	125	100	78	3.7	3.8	4.0	161	125	98
550	000	3.0	63	44	35	2.75	3.0	3.4	70	51	40

## Miscellaneous Results

80	100-135	1.0	98	81	65	3.6	4.0	4.2	119	96	78
450	85-131	1.0	75	60	48	3.8	4.0	4.2	115	91	72
550	90-122	1.0	110	91	82	2.6	3.25	4.0	122	101	90

- 1 For Composition - see page 198, Item 6
- 2 For Heat Treatment - see page 198, Item 6
- 3 For Specimen Conditions - see page 203, Item 6
- 4 For Meaning of Symbols - see page 46

# K-11 STEEL

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Condition<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

## ROTARY BENDING

S<sub>u</sub> = 272 ksi  
S<sub>y</sub> = 228 ksi

Surf Cond	Sec	Remarks <sup>5</sup>	Life, cycles	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>8</sup>	10 <sup>9</sup>	10 <sup>10</sup>	10 <sup>11</sup>	10 <sup>12</sup>	10 <sup>13</sup>	10 <sup>14</sup>	10 <sup>15</sup>	10 <sup>16</sup>	10 <sup>17</sup>	10 <sup>18</sup>
Effect of Surface Treatment																		
S P	C-B	00	116	100	86	74	2.7	2.8	3.0	3.2	152	131.4	113.6	91.6	74.8	62.4	51.2	41.2
C B	S P	00	135	105	82	80	3.4	3.5	3.7	3.9	166	131	102.6	100.2	91.6	80.8	70.8	60.8
MP	MP	00	118	100	85	76	2.8	2.9	3.0	3.1	147.1	126.5	109	97.6	86.8	76.8	66.8	56.8
SP	MP	00	137	112	92	82	3.1	3.4	3.6	3.8	160	131.4	109.6	97.7	86.8	76.8	66.8	56.8
SP	C-B	00	116	100	86	74	2.7	2.8	3.0	3.2	152	131.4	113.6	91.6	74.8	62.4	51.2	41.2
MP	MP	49	150	117	92	87	3.4	3.6	3.8	4.0	166.1	130.4	102.1	91.8	80.8	70.8	60.8	50.8
SP	C-B	49	130	106	88	74	2.95	3.2	3.4	3.5	163	135.6	112.2	95.2	84.8	74.8	64.8	54.8
MP	MP	50	113	90	80	67	3.1	3.4	3.5	3.8	150	122.8	108.8	92.8	81.8	71.8	61.8	51.8
SP	MP	50	133	102	78	70	2.95	3.1	3.3	3.45	175.5	136	104.4	96.8	85.8	75.8	65.8	55.8
SP	C-B	50	118	100	85	73	3.2	3.35	3.5	3.7	154.5	131.2	112	95.8	84.8	74.8	64.8	54.8
MP	MP	51	128	105	86.5	80	1.75	1.9	1.99	2.1	143	118.4	97.9	81.2	70.8	60.8	50.8	40.8
SP	MP	51	127	106	89	75	2.03	2.1	2.2	2.22	156.5	132.9	112.7	95.5	84.8	74.8	64.8	54.8
SP	C-B	51	165.5	130.7	102.6	84.8	1.8	1.85	1.9	2.05	173.65	137.4	108.45	89.5	78.8	68.8	58.8	48.8

# Effect of Surface Treatment

MP	52	148	124.3	104.5	88	2.03	2.03	2.1	2.18	161.2	136.0	114.4	95.7
SP	52		155	113.3	83		2.02	2.05	2.35		166.7	121.9	90.3
SP	52	147	117	94	76	1.9	2.06	2.12	2.23	176.5	143	115.5	92.9

# Effect of Heat Treatment

MP	00	118	100	85	76	2.8	2.9	3.0	3.1	147.1	126.5	109	97.6
MP	49	150	117	92	87	3.4	3.6	3.8	4.	166.1	130.4	102.1	93.8
MP	50	113	90	80	67	3.1	3.4	3.5	3.8	150	122.8	108.8	92.8
MP	51	128	105	86.5	80	1.75	1.9	1.99	2.1	143	118.4	97.9	91.2
MP	52	148	124.3	104.5	88	2.03	2.03	2.1	2.18	161.2	136.0	114.4	95.7
SP	00	137	112	92	82	3.1	3.4	3.6	3.8	160	131.4	109.6	97.7
SP	50	133	102	78	70	2.95	3.1	3.3	3.45	175.5	136	104.4	96.8
SP	51	127	106	89	75	2.03	2.1	2.2	2.22	156.5	132.9	112.7	95.5
SP	52		155	113.3	83		2.02	2.05	2.35		166.7	121.9	90.3
CP	00	83	59	40	38	2.4	2.55	2.9	3.1	121.8	88.5	61.2	57.2
CP	50	138	78	52	41	2.7	2.9	3.2	3.4	166.5	84.4	65.4	54.3
CP	51	141	102.5	76.5	76.3	1.98	2.04	2.12	2.18	151.7	110.7	82.15	81.15
CP	52	123	88.5	66	66	2.0	2.1	2.12	2.12	162	118.5	90	90
SP	00	116	100	86	4	2.7	2.8	3.0	3.2	152	131.4	113.6	98
SP	49	150	106	88	74	2.95	3.2	3.4	3.5	163	135.6	112.2	96.2
SP	50	118	100	85	73	3.2	3.35	3.5	3.7	154.5	131.2	112	95.8
SP	51	165.5	130.7	102.6	84.8	1.8	1.85	1.9	2.05	173.65	137.4	108.45	89.5
SP	52	147	117	94	76	1.9	2.06	2.12	2.23	176.5	143	115.5	92.9

- 1 For Composition - see page 199, Item 7
- 2 For Heat Treatment - see page 199, Item 7
- 3 For Specimen Condition - see page 203, Item 7
- 4 For Meaning of Symbols - see page 46
- 5 Remarks

00 - No Preparation  
49 - exposed 4 hr at 375°F before test  
50 - exposed 4 hr at 500°F before test  
51 - exposed 4 hr at 750°F before test  
52 - exposed 4 hr at 1000°F before test  
53 - exposed 4 hr at 1250°F before test



4340 STEEL

S <sub>u</sub> = 256-280 ksi		ROTARY BENDING										Composition <sup>1</sup> Specimen Condition <sup>3</sup> Meaning of Symbols <sup>4</sup>		
H.T. <sup>2</sup>	Melt <sup>5</sup>	Spec	S <sub>u</sub>	X <sub>0</sub> ksi		b		b		b		ksi		
Life, cycles				10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup> 10 <sup>7</sup>
Effect of Heat Treatment														
A	8	V-N	246	78	34	15	13	2.4	3.4	3.6	4.4	90.8	40.1	18.5 17.4
B	8	V-N	222	46	25	12	6.0	2.4	2.6	3.05	3.5	77.5	42.8	23 12.7
A	8	Mo-N	246	75	64	57	49	3.05	3.3	3.5	3.8	128.5	114	101.5 90
B	8	Mo-N	222	77	65	55	45	2.75	2.85	3.0	3.2	113.5	94.5	79.4 66.2
A	9	V-N	264	45	27	16	9	2.6	2.8	3.3	3.9	73.5	46.8	29.8 19.1
B	9	V-N	206	33	27	24	18	2.2	2.5	2.75	3	49.2	41.2	37.4 30.2
A	9	Mo-N	264	65	60	55	50	2.7	2.9	3.3	3.5	108	96	91 82.6
B	9	Mo-N	206	54	30	46.0	43	2.7	2.9	3.1	3.3	94	86	81.5 76.0
A	10	V-N	280	40	30	22	18	2.75	3.0	3.4	3.7	67	51.6	39.5 32.6
B	10	V-N	206	38	32	26	22	2.7	2.8	2.95	3.1	64.5	54	45.8 38.6

# Effect of Melt Practice

A	8	V-N	246	78	34	15	13	2.4	3.4	3.6	4.4	90.8	40.1	18.5	17.4
A	9	V-N	268	40	20	15	12	2.4	2.4	2.75	2.9	67	40	33.2	28
A	10	V-N	280	40	30	22	18	2.75	3.0	3.4	3.7	67	51.6	39.5	32.6
B	8	V-N	222	46	25	12	6.0	2.4	2.6	3.05	3.5	77.5	42.8	23	12.7
B	9	V-N	207	27	23	17	10	2.8	3.0	3.5	3.8	55.5	42.5	34	29.2
B	10	V-N	206	38	32	26	22	2.7	2.8	2.95	3.1	64.5	54	45.8	38.6
A	8	Mo-N	246	75	64	57	49	3.05	3.3	3.5	3.8	128.5	114	101.5	90
A	9	Mo-N	268	108	90	76	63	2.7	2.8	2.9	3.2	144	120	100	83.2
A	8	Mo-N	222	77	65	55	45	2.75	2.85	3.0	3.2	113.5	94.5	79.4	66.2
A	9	Mo-N	207	75	68	63	56	3.0	3.1	3.2	3.4	112.5	102.5	94	85.5

# Effect of Stress Concentration

A	8	V-N	246	78	34	15	13	2.4	3.4	3.6	4.4	90.8	40.1	18.5	17.4
A	8	Mo-N	246	75	64	57	49	3.05	3.3	3.5	3.8	128.5	114	101.5	90
B	8	V-N	222	46	25	12	6.0	2.4	2.6	3.05	3.5	77.5	42.8	23	12.7
B	8	Mo-N	222	77	65	55	45	2.75	2.85	3.0	3.2	113.5	94.5	79.4	66.2
A	9	V-N	268	108	20	15	12	2.7	2.8	2.9	2.9	144	40	33.2	28
A	9	Mo-N	268	108	90	76	63	2.7	2.8	2.9	3.2	144	120	100	83.2
B	9	V-N	207	27	23	17	10	2.8	3.0	3.5	3.8	55.5	42.5	34	29.2
B	9	Mo-N	207	75	68	63	56	3.0	3.1	3.2	3.4	112.5	102.5	94	85.5

A	9	V-N	264	45	27	16	9	2.6	2.8	3.3	3.9	73.5	46.8	29.8	19.1
A	9	Mo-N	264	65	60	55	50	2.7	2.9	3.3	3.5	108	98	91	82.6
B	9	V-N	206	33	27	24	18	2.2	2.5	2.75	3	49.2	41.2	37.4	30.2
B	9	Mo-N	206	54	50	46.0	43	2.7	2.9	3.1	3.3	94	88	81.5	76.0

- 1 For Composition - see page 199, Item 84
- 2 For Heat Treatment code
  - A: normalize 1550°F, Oil, Temper 400°F AC
  - B: normalize 1550°F, Quench, Temper at 775°F AC
- 3 For Specimen Condition - see page 203, Item 8A
- 4 For Meaning of Symbols - see page 46
- 5 Melt Practice code
  - 8 Air Melt, Vacuum arc reselt
  - 9 Vacuum Induction Melt
  - 10 Vacuum Induction Melt, vacuum arc Reselt

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Condition<sup>3</sup>  
Mean of Symbols<sup>4</sup>

44 For Meaning of Symbols - see page 46

# THERMOLD J

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Condition<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

AXIAL LOAD  
Completely Reversed

S<sub>u</sub> = 294 ksi

9 ksi

b

X<sub>0</sub> ksi

K<sub>t</sub> Spec

Life, cycles 10<sup>3</sup> 10<sup>4</sup> 10<sup>5</sup> 10<sup>6</sup> 10<sup>7</sup> 10<sup>3</sup> 10<sup>4</sup> 10<sup>5</sup> 10<sup>6</sup> 10<sup>7</sup>

## Effect of Stress Concentration

1.0	Mo-N	100	102	90	4.4	2.4	2.1	143	124.3	108.3
2.0	V-N	120	85	62	59	5.2	5.9	72.7	69.1	
3.0	V-N	98	73	54		5.2	5.5	82.8	61.8	

- 1 For Composition - see page 199, Item 9
- 2 For Heat Treatment - see page 199, Item 9
- 3 For Specimen Condition - see page 203, Item 9
- 4 For Meaning of Symbols - see page 46

Fe -5.5 Mo -2.5 Cr -.5C

ROTARY BENDING  
Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Conditions<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

S<sub>u</sub> = 314 ksi  
S<sub>y</sub> = 267 ksi

K <sub>t</sub>	Spec	X <sub>o</sub> ksi	b					σ ksi				
Life, cycles			10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>8</sup>	10 <sup>9</sup>	10 <sup>10</sup>	10 <sup>11</sup>	10 <sup>12</sup>	10 <sup>13</sup>
1.0	Mo-N	160	150	140	130	120	110	100	90	80	70	60
2.6	V-N	72.0	69.0	66.0	63.0	60.0	57.0	54.0	51.0	48.0	45.0	42.0

Effect of Stress Concentration

- 1 For Composition - see page 19<sup>a</sup>, Item 10
- 2 For Heat Treatment - see page 199, Item 10
- 3 For Specimen Conditions - see page 203, Item 10
- 4 For Meaning of Symbols - see page 46

# M10 TOOL STEEL

$S_u = 330$  ksi

ROTARY BENDING

Composition<sup>1</sup>  
Specimen Condition<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

H.T. <sup>2</sup>	K <sub>t</sub>	Spec	X <sub>0</sub> ksi			b			θ ksi		
Life, cycles			10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
Effect of Heat Treatment											
A	1.0	No-N	152	133	117	2.67	2.7	2.73	185.7	163.7	144
B	1.0	No-N	127	119	111	1.89	1.95	2.0	163.5	153.2	143.5
Miscellaneous Results											
B	2.6	V-N	71	65	59.5	3.37	3.5	3.62	94.4	86.8	79.8

1 For Composition - see page 200, Item 11

2 Heat Treatment

A: Preheat 1450°F 1/2 hr, harden 2150°F 5 min, OQ until black, AC, Temp. 1100°F 2 hrs, A.C. Retemp 1100°F 2 hrs, AC, after finishing op. Nitrided 975°F 48 hrs.

B: Same as A but instead of nitriding stress relieve at 1000°F in protective atmosphere F.C.

3 For Specimen Condition - see page 203, Item 11

4 For Meaning of Symbols - see page 46

# 321 STAINLESS STEEL

$S_y = 86 \text{ ksi}$   
 $S_u = 38 \text{ ksi}$

AXIAL LOAD  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Condition<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

$K_t$	$T, ^\circ F$	Spec.*	$\sigma_0$ ksi						$\theta$ ksi					
			$10^3$	$10^4$	$10^5$	$10^6$	$10^3$	$10^4$	$10^5$	$10^6$	$10^3$	$10^4$	$10^5$	$10^6$
Effect of Stress Concentration														
1.0	80	A	44.0	39.6	32.4	27.9	1.5	1.4	1.4	1.4	47.15	42.5	34.8	29.9
3.5	80	B	38.0	29.0	21.6	16.0	1.8	2.0	2.2	2.5	45.2	34.8	26.3	19.85
1.0	80	C		39.0	29.5	22.2		2.56	2.74	2.85		46.85	35.5	26.95
1.0	-320	A		87.0	58.3	40.3		2.5	3.42	3.6		91.35	61.32	42.33
3.5	-320	B		53.3	31.0	17.5		1.48	1.60	2.15		58.4	34.45	20.25
1.0	-320	C		41.0	27.0	18.0		2.33	2.38	2.45		70.6	46.3	29.6
1.0	-423	A		93.0	62.0	41.0		2.92	2.94	3.0		115.3	78.9	53.6
3.5	-423	B		39.0	23.0	12.5		2.35	2.37	2.4		67.8	40.2	24.0
1.0	-423	C	120.0	64.0	35.0	18.0	2.6	2.8	3.4	4.3	161.5	92.4	53.6	30.8

## Effect of Temperature

1.0	80	A	44.0	39.6	32.4	27.9	1.5	1.4	1.4	1.4	47.15	42.5	34.8	29.9
1.0	-320	A		87.0	58.3	40.3		2.5	3.42	3.6		91.35	61.32	42.33
1.0	-423	A		39.0	23.0	12.5		2.35	2.37	2.4		67.8	40.2	24.0
3.5	80	B	38.0	29.0	21.6	16.0	1.8	2.0	2.2	2.5	45.2	34.8	26.3	19.85
3.5	-320	B		53.3	31.0	17.5		1.48	1.6	2.15		58.4	34.45	20.25
3.5	-423	B		39.0	23.0	12.5		2.35	2.37	2.4		67.8	40.2	24.0



# Effect of Temperature

1.0	80	C	39.0	29.5	22.2	2.56	2.74	2.85	46.85	35.5	26.95
1.0	-320	C	41.0	27.0	18.0	2.33	2.38	2.45	70.6	46.3	29.6
1.0	-423	C	120.0	64.0	18.0	2.6	3.4	4.3	92.4	53.6	30.8
									161.5		

## \* Specimen

- 1 For Composition - see page 200, Item 12
- 2 For Heat Treatment - see page 200, Item 12
- 3 For Specimen Conditions - see page 204, Item 12
- 4 For Meaning of Symbols - see page 46

- A - unnotched
- B - edge notch
- C - TIG welded

# A-286 STAINLESS STEEL

$S_u = 90 \text{ ksi}$   
 $S_y = 46 \text{ ksi}$

AXIAL LOAD  
 Completely Reversed

<sup>1</sup> Composition  
<sup>2</sup> Heat Treatment  
<sup>3</sup> Specimen Conditions  
<sup>4</sup> Meaning of Symbols

T, °F	K <sub>t</sub>	X <sub>0</sub> ksi			b			σ ksi		
		10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
Effect of Temperature										
80	1.0 *	40	31	24	1.84	2.1	2.7	54	43	34
-320	1.0 *	61	58	54	2.3	2.34	2.45	78	74	70
-423	1.0 *	73	67	62	2.12	2.3	2.33	84	78	72
80	3.5 *	30	22		2.7	2.9		36	27	
-320	3.5 *	35	25	17	1.58	1.58	1.8	56	39	27
-423	3.5 *	30	12	5	2.15	2.25	2.37	86	36	15
80	1.0 **	29	18	12	2.27	2.49	2.56	40	25	16
-320	1.0 **	27	15	8	3.05	3.12	3.25	66	37	21
-423	1.0 **	41	28	19	2.48	2.51	2.52	70	47	32

## Effect of Stress Concentration

80	1.0*	40	31	24	1.84	2.1	2.2	54	43	34
80	3.5*	30	22		2.7	2.9		36	27	
-320	1.0*	61	58	54	2.3	2.34	2.45	78	74	70
-320	3.5*	35	25	17	1.58	1.58	1.8	56	39	27
-423	1.0*	73	67	62	2.12	2.3	2.33	84	78	72
-423	3.5*	30	12	5	2.15	2.25	2.37	86	36	15

## Effect of Process

80	1.0*	40	31	24	1.84	2.1	2.2	54	43	34
80	1.0**	29	18	12	2.27	2.49	2.56	40	25	16
-320	1.0*	61	58	54	2.3	2.34	2.45	78	74	70
-320	1.0**	27	15	8	3.05	3.12	3.25	66	37	21
-423	1.0*	73	67	62	2.12	2.3	2.33	84	78	72
-423	1.0**	41	28	19	2.48	2.51	2.52	70	47	32

1 For Composition - see page 200, Item 13

2 For Heat Treatment - see page 200, Item 13\* Mechanically Polished

3 For Specimen Conditions - see page 204, Item 13\*\* Tungsten Inert Gas Welded

4 For Meaning of Symbols - see page 46

# 347 STAINLESS STEEL

$S_u = 92 \text{ ksi}$   
 $S_y = 46 \text{ ksi}$

AXIAL  
LOAD

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Conditions<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

$K_t$	$S_m$	$X_0 \text{ ksi}$				$b$				$\sigma \text{ ksi}$			
		$10^3$	$10^4$	$10^5$	$10^6$	$10^3$	$10^4$	$10^5$	$10^6$	$10^3$	$10^4$	$10^5$	$10^6$
Life, cycles													

## Effect of Stress Concentration

1.0	25-35			28	26			1.47	1.55			32	30
2.0	25-35	54	35	23		3.9	5.2	3.6		58	38	30	
4.0	25-35	32	22	15		2.9	2.7	2.25		36	25	17	
1.0	38-45	38	36	35		4.3	4.7	4.9		43	41	39	
2.0	38-45	42	34	28		4.9	5.5	3.8		47	39	32	

## Miscellaneous Results

1.0	32-37	39	31	24		3.9	4.0	4.8		44	35	28
2.0	18-22		18	17			3.6	3.9			22	21
4.0	10-15		9	8			3.0	3.8			16	14

- 1 For Composition - see page 200, Item 14
- 2 For Heat Treatment - see page 200, Item 14
- 3 For Specimen Conditions - see page 204, Item 14
- 4 For Meaning of Symbols - see page 46

# MULTIMENT N-155

$S_u = 119 \text{ ksi}$   
 $S_y = 60 \text{ ksi}$

ROTARY, PLATE  
 AND AXIAL LOADING  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Condition<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

T of L	T, °F	$X_0$ ksi			b			$\sigma$ ksi		
Life, Cycles		$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$
Effect of Test Temperature										
Axial	1200	48.0	45.0	42.5	2.25	2.4	2.52	54.7	51.95	49.0
Axial	1350	43.0	40.5	38.0	2.55	2.73	2.85	49.85	46.8	43.95
Axial	1500	27.0	24.3	22.0	2.83	2.91	3.0	41.6	37.1	33.1
Rotary	1200	44.5	43.0	41.6	2.77	2.9	3.0	51.85	49.85	47.95
Rotary	1350	41.0	35.0	32.5	2.28	2.28	2.35	55.5	47.3	44.1
Rotary	1360	39.5	38.0	36.7	2.77	2.8	2.9	44.05	42.53	41.1
Rotary	1500	39.0	33.7	29.3	2.27	2.37	2.45	48.5	42.0	36.15

## Effect of Surface Treatment

Axial A	1350	43.0	40.5	38.0	2.55	2.73	2.85	49.85	46.8	43.95
Axial B	1350	35.0	34.0	33.0	2.23	2.27	2.32	43.2	42.1	40.9

## Effect of Type of Loading

Rotary	1200	44.5	43.0	41.6	2.77	2.9	3.0	51.85	49.85	47.95
Axial	1200	48.0	45.0	42.5	2.25	2.4	2.52	54.7	51.95	49.0
Rotary	1350	41.0	35.0	32.5	2.28	2.28	2.35	55.5	47.3	44.1
Axial	1350	43.0	40.5	38.0	2.55	2.73	2.85	49.85	46.8	43.95
Rotary	1500	39.0	33.7	29.3	2.27	2.37	2.45	48.5	42.0	36.15
Axial	1500	27.0	24.3	22.0	2.83	2.91	3.0	41.6	37.1	33.1

- 1 For Composition - see page 200, Item 15A
  - 2 For Heat Treatment - see page 200, Item 15A
  - 3 For Specimen Condition - see page 204, Item 15A
  - 4 For Meaning of Symbols - see page 46
- A Lathed  
 B Milled

# MULTIPLY 1-175 STAINLESS STEEL

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Condition<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

ROTARY, PLATE BENDING  
AND AXIAL LOADING  
Completely Reversed

S<sub>u</sub> = 114-126 ksi  
S<sub>y</sub> = 60-73 ksi

Surf Finish (KAS)	T of L	Surf. Prep.	S <sub>u</sub>	X <sub>0</sub> ksi					b					θ ksi				
				10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>7</sup>
Effect of Surface Finish																		
4	A	M.P.	119	52	48	45	4	4.2	4.8	61.9	57.8	53.8						
22	A	M.P.	119	52	46	41	2.6	2.8	3.05	60.5	54.5	48.8						
75	A	M.P.	119	59	55	51	2.8	2.95	3.2	64.9	60.8	57						
23	P	Ground	126	64	44	29	4.2	4.8	5.0	78.6	53.2	35.4						
75	P	Scratched	126	61	55	50	2.8	2.9	3.1	67.3	60.9	55.1						
5	P	M.P.	119	62	50	41	2.7	3.0	3.2	73	60.6	50.7						
75	P	Scratched	119	57	53	50	3.6	3.8	4.0	63.9	60.2	56.5						

## Miscellaneous Results

5	R	M.P.	119	59	54	47	3.4	3.5	3.7	68	62.4	54.9						
23	P	Ground <sup>A</sup>	114	59	50	50	3.8	4		63.3	53.9							
5	P	M.P. <sup>B</sup>	119	63	56	50	2.5	2.8	3.05	66.9	60	53.7						
5	P	M.P. <sup>A</sup>	126	56	47	40	3.1	3.2	3.6	66.5	57.1	48.8						

# Miscellaneous Results (Continued)

5	A	Ground <sup>C</sup>	119	55	50.2	45.8	3.7	4.2	4.8	29.1	53.9	49.1
5	A	M.P. C	119	60	53	46	4.1	4.6	4.9	69.8	61.8	53.8
5	A	M.P.	119	62	47	35	4.4	4.8	5.0	76	58.8	43.6

- 1 For Composition - see page 200, Item 15B
- 2 For Heat Treatment - see page 260, Item 15B
- 3 For Specimen Condition - see page 204, Item 15B
- 4 For Meaning of Symbols - see page 46

## Surface Preparation Remarks

- A: Stress relieved after surface finishing
- B: Surface finished, stress relieved, refinished
- C: Heat Treated after surface finishing

PH 15-7 MO STAINLESS STEEL

$S_u$  = 201 ksi  
 $S_y$  = 196 ksi

AXIAL LOAD  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Conditions<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

$T, ^\circ F$	$K_t$	$X_o$ ksi			$b$			$\theta$ ksi		
Life, cycles		$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$

Effect of Temperature

80	1.0	145	104	75	1.68	1.73	1.78	169	119	86
500	1.0	120	73		2.8	2.85		129	79	
80	4.0	42	31	23	2.35	2.4	2.43	56	41	30
500	4.0	36	32	29	2.55	2.58	2.7	41	37	32

Effect of Stress Concentration

80	1.0	145	104	75	1.68	1.73	1.78	169	119	86
80	4.0	42	31	23	2.35	2.4	2.43	56	41	30
500	1.0	120	73		2.8	2.85		129	79	
500	4.0	36	32	29	2.55	2.58	2.7	41	37	32

- 1 For Composition - see page 200, Item 16
- 2 For Heat Treatment - see page 200, Item 16
- 3 For Specimen Conditions - see page 204, Item 16
- 4 For Meaning of Symbols - see page 46

17-7PH

$S_u = 205 \text{ ksi}$   
 $S_y = 195 \text{ ksi}$

AXIAL LOAD  
 Completely Reversed

Composition<sup>1</sup>  
 Heat Treatment<sup>2</sup>  
 Specimen Condition<sup>3</sup>  
 Meaning of Symbols<sup>4</sup>

$K_t$	Spec	$X_o \text{ ksi}$			$b$			$\theta \text{ ksi}$		
		$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$	$10^4$	$10^5$	$10^6$
1	No-N	90.0	82.0	63.0	8.3	6.0	6.0	153.6	126.6	88.6
2.3	H-N	76.0	56.0	40.5	3.5	3.9	4.2	80.24	58.82	43.15
4.0	H-N	44.2	31.0	21.7	4.8	4.4	4.3	47.4	33.5	23.4
5.0	H-N	39.2	28.5	20.8	3.8	3.4	3.1	41.59	30.43	21.74

- 1 For Composition - see page 201, Item F.
- 2 For Heat Treatment - see page 201, Item 17
- 3 For Specimen Condition - see page 204, Item 17
- 4 For Meaning of Symbols - see page 46



Type of Loading	Temp. °F	X <sub>0</sub> , ksi			b			θ, ksi		
		10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
<u>TIMKEN 16-25-6</u>										
AXIAL LOAD								Composition <sup>1</sup>		
Completely Reversed								Heat Treatment <sup>2</sup>		
								Meaning of Symbols <sup>3</sup>		
Effect of Temperature										
Axial	Room	63.0	58.0	53.0	4.8	5.0	5.2	71.1	65.4	60.1
Axial	1200	47.0	43.0	40.0	4.4	5.0	5.2	56.9	53.3	50.0

<u>STAINLESS 403</u>										
AXIAL AND PLATE LOADING Completely Reversed								Composition <sup>1</sup> Heat Treatment <sup>2</sup> Meaning of Symbols <sup>3</sup>		
Effect of Temperature										
Axial	Room	75.0	69.0	65.0	1.9	2.5	2.65	78.4	73.1	68.0
Axial	500	59.0	54.0	50.0	3.0	4.3	4.8	65.6	61.7	58.2
Axial	700	58.0	53.0	49.0	4.7	4.82	5.1	62.2	59.4	53.0
Axial	900	50.0	45.5	41.6	3.2	3.8	4.1	57.2	46.6	42.68
Rotary	Room	95.0	76.0	60.0	3.2	3.5	3.7	106.0	85.4	69.9
Rotary	700	80.0	62.0	49.0	3.8	5.1	5.9	86.5	67.2	52.6
Rotary	900	71.0	58.0	47.5	3.45	3.55	4.8	74.2	60.5	49.7
Effect of Type of Loading										
Axial	Room	75.0	69.0	65.0	1.9	2.5	2.65	78.4	73.1	68.0
Rotary	Room	95.0	76.0	60.0	3.2	3.5	3.7	106.0	85.4	69.9
Axial	700	58.0	53.0	49.0	4.7	4.82	5.1	62.2	59.4	53.0
Rotary	700	80.0	62.0	49.0	3.8	5.1	5.9	86.5	67.2	52.6
Axial	900	50.0	45.5	41.6	3.2	3.8	4.1	57.2	46.6	42.68
Rotary	900	71.0	58.0	47.5	3.45	3.55	4.8	74.2	60.5	49.7

1 For Composition - see page 201, Items 10, 19

2 Heat Treatment Unknown

3 For Meaning of Symbols - see page 46

Type of Loading	Temp. °F	X <sub>0</sub> , ksi			b			e, ksi		
		10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
<u>LAPELLOY 311</u>										
AXIAL LOAD								Composition <sup>1</sup>		
Completely Reversed								Heat Treatment <sup>2</sup>		
Miscellaneous Results								Meaning of Symbols <sup>3</sup>		
Axial	1100	35.0	33.0	31.0	4.5	4.8	5.0	42.2	39.9	37.7

<u>S-816 (AMS 5534)</u>										
ROTARY BENDING								Composition <sup>1</sup>		
Completely Reversed								Heat Treatment <sup>2</sup>		
Effect of Temperature								Meaning of Symbols <sup>3</sup>		
Rotary	Room	100.0	81.0	65.0	2.8	2.9	3.2	122.5	99.5	82.1
Rotary	1350	58.0	50.0	44.0	3.5	4.1	4.6	70.2	61.8	51.4
Rotary	1650	15.0	13.0	12.0	3.3	3.4	3.5	42.0	39.0	36.6

<u>INCO SHS 260</u>										
AXIAL LOAD								Composition <sup>1</sup>		
Completely Reversed								Heat Treatment <sup>2</sup>		
Effect of Temperature								Meaning of Symbols <sup>3</sup>		
Axial	525	110.0	80.0	60.0	3.8	4.1	4.3	190.0	136.0	102.0
Axial	800	100.0	75.0	55.0	5.8	6.0	6.5	160.0	119.0	89.0

<u>GMR-235</u>										
AXIAL LOAD								Composition <sup>1</sup>		
Completely Reversed								Heat Treatment <sup>2</sup>		
Effect of Temperature								Meaning of Symbols <sup>3</sup>		
Axial	Room	74.0	56.0	42.0	2.4	2.7	2.8	79.9	60.81	46.05
Axial	1200	43.0	39.0	35.5	3.9	4.0	4.1	53.4	48.43	44.18
Axial	1650	26.5	24.5	23.0	2.43	2.7	2.8	30.95	29.2	27.53

- 1 For Composition - see page 201, Items 20, 21, 22, 23  
2 Heat Treatment Unknown  
3 For Meaning of Symbols - see page 46

Type of Loading	Temp. °F	X <sub>0</sub> , ksi			b			θ, ksi		
		10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>6</sup>
<u>S-816 (AMS 5765)</u>										
AXIAL, PLATE, AND ROTARY BENDING Completely Reversed								Composition <sup>1</sup> Heat Treatment <sup>2</sup> Meaning of Symbols <sup>3</sup>		
Effect of Temperature										
Axial	Room	55.0	54.0	52.0	3.7	4.1	4.6	69.2	67.1	65.2
Axial	1500	39.0	37.0	36.0	3.1	4.1	4.8	42.8	41.2	39.6
Axial	1650	37.0	32.0	28.0	4.6	5.2	5.6	44.0	38.6	33.9
Plate	Room	75.0	60.0	50.0	4.5	5.0	5.3	110.0	93.1	79.2
Plate	1200	76.6	72.4	68.8	2.05	2.48	2.95	78.88	75.04	71.22
Effect of Type of Loading										
Plate	1200	76.6	72.4	68.8	2.05	2.48	2.95	78.88	75.04	71.22
Rotary	1200	62.0	57.0	54.0	2.42	3.0	3.4	79.2	76.8	75.3

<u>UDIMET 500</u>										
AXIAL LOAD Completely Reversed								Composition <sup>1</sup> Heat Treatment <sup>2</sup> Meaning of Symbols <sup>3</sup>		
Effect of Temperature										
Axial	Room	116.0	92.0	76.0	2.35	2.5	2.7	136.5	112.7	92.4
Axial	1200	84.0	78.0	71.0	3.1	3.13	3.25	96.4	87.65	80.45

<u>Ti-140 (AMS 493)</u>										
ROTARY BENDING Completely Reversed								Composition <sup>1</sup> Heat Treatment <sup>2</sup> Meaning of Symbols <sup>3</sup>		
Effect of Temperature										
Rotary	Room	83.0	62.0	45.0	2.4	3.22	3.5	113.9	87.3	67.2
Rotary	600	85.0	61.5	45.0	2.35	2.6	2.85	92.1	67.8	50.28

1 For Composition - see page 201, Items 24, 25, 26

2 Heat Treatment Unknown

3 For Meaning of Symbols - see page 46

# DURALUMIN

Specimen Condition	S <sub>u</sub> S <sub>y</sub>	unknown	ROTARY BENDING		Composition Al, Cu, Mn, Mg	
			K <sub>0</sub> ksi	b	0	ksi
Life, cycles			10 <sup>5</sup> 10 <sup>6</sup> 10 <sup>7</sup> 10 <sup>8</sup>	10 <sup>5</sup> 10 <sup>6</sup> 10 <sup>7</sup> 10 <sup>8</sup>	10 <sup>5</sup> 10 <sup>6</sup> 10 <sup>7</sup> 10 <sup>8</sup>	
non-corroded			22.0 17.0 13.0 10.0	4.4 4.5 5.2 5.6	30.9 24.8 20.2 16.4	
corroded in salt water			12.0 7.0 4.0 2.0	2.8 3.2 3.75 4.7	25.0 16.4 10.8 7.2	

Ti-6Al-4V

ROTARY BENDING

Composition<sup>1</sup>  
Specimen Condition<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

S<sub>u</sub> = 177 ksi  
S<sub>y</sub> = 166 ksi

T °F	H.T. <sup>2</sup>	S <sub>E</sub> ksi	X <sub>0</sub> ksi	b	0 ksi
			10 <sup>4</sup> 10 <sup>5</sup> 10 <sup>6</sup>	10 <sup>4</sup> 10 <sup>5</sup> 10 <sup>6</sup>	10 <sup>4</sup> 10 <sup>5</sup> 10 <sup>6</sup>
Effect of Temperature					
80	A	0	67.0	57.0	50.0
400	A	0	75.0	56.0	41.0
600	A	0	50.0	40.0	33.0
800	A	0	43.0	34.0	26.0
900	A	0	45.0	34.0	25.0
80	B	0	70.0	55.0	46.0
400	B	0	45.0	35.0	25.0
600	B	0	42.0	32.0	22.0
800	B	0	40.0	30.0	20.0
Effect of Heat Treatment					
80	A	0	70.0	40.0	26.0
80	B	0	70.0	55.0	46.0

# Effect of Heat Treatment

400	A	0	64.0	48.0	36.0	1.75	1.94	2.49	81.5	62.3	47.4
400	B	0	45.0	35.0	25.0	1.9	2.2	2.7	78.8	57.5	44.0
600	A	0	50.0	40.0	33.0	2.65	2.7	3.0	77.1	60.5	47.0
600	B	0	42.0	32.0	22.0	3.15	3.2	3.6	79.2	59.9	44.5
800	A	0	43.0	34.0	26.0	2.9	3.1	3.25	71.0	55.0	44.0
800	B	0	40.0	30.0	20.0	3.25	3.5	4.1	71.7	50.4	36.6
80	A	82-107	35.0	24.0	16	2.85	2.85	2.95	49.8	34.2	23.55
80	B	82-107	35.0	25.0	17.5	2.8	2.95	3.3	46.5	33.6	24.2
400	A	30-45	40.0	28.0	19.0	3.4	3.75	3.9	66.8	44.5	29.7
400	B	30-45	22.0	20.0	18.0	2.95	3.4	3.8	40.6	38.4	37.5
400	A	77-100	30.0	26.5	23.6	3.5	3.7	3.97	32.24	28.96	25.92
400	B	77-100	22.0	20.0	18.0	3.5	3.7	3.6	32.3	29.2	26.35
800	A	40-64	52.0	43.0	36.0	3.35	3.5	3.7	58.5	43.75	40.85
800	B	40-64	52.0	29.7		2.45	2.75		54.45	31.25	

## Miscellaneous Results

80	A	40-55	55.0	45.0	38.0	3.7	4.0	4.15	70.5	58.9	49.2
80	A	40-78	62.0	40.0	30.0	5.0	5.1	5.5	55.0	81.2	41.2
80	A	60-80	60.0	50.0	43.0	3.65	4.0	4.3	74.4	63.5	54.0
80	A	85-100	27.0	25.0	23.0	3.7	4.0	4.2	32.0	29.6	27.4
80	A	100-135	34.0	26.0	20.5	3.2	3.5	4.0	43.5	33.6	25.75

# Miscellaneous Results

400	A	35-40	36.0	34.0	33.0	2.9	3.45	3.5	42.17	40.6	39.1
400	A	45-70	36.0	27.0	20.0	4.2	4.6	5.0	67.3	50.9	38.5
400	A	85-115	35.0	26.0	19.5	2.6	2.95	3.25	39.12	29.76	22.64
600	B	30-65	40.0	28.0	19.0	3.6	3.7	3.9	66.3	45.8	31.5
600	B	82-112	34.0	28.25	23.3	3.4	3.6	3.8	34.75	28.89	24.0
800	B	58-77	22.0	19.6	17.6	3.3	3.5	3.75	23.95	21.44	19.14
800	A	40-60	47.0	34.0	34.0	2.3	2.5	2.6	54.8	42.14	40.85
800	A	62-85	22.0	20.0	18.0	2.0	2.18	2.5	25.8	23.6	21.4
900	A	25-35	35.0	30.0	24.0	3.05	3.2	3.6	47.8	40.5	33.5
900	A	35-61	53.0	38.0	30.0	2.7	3.3	3.75	63.3	50.2	38.8
900	A	38-58	42.0	33.0	26.0	2.95	3.05	3.3	48.2	38.17	30.24
900	B	30-52	33.0	28.0	25.0	3.1	3.2	3.45	44.1	39.0	33.8

1 For Composition - see page 202, Item 23

2 For Heat Treatment code

A: Sol Treated 1690°F, 12 min, WQ, Aged 900°F, 4 hrs, Air Cooled.

B: Sol Treated 1675°F, 20 min, WQ, Aged 900°F, 4 hrs, Air Cooled.

3 For Specimen Condition - see page 205, Item 28

4 For Meaning of Symbols - see page 46

# INCOCEL X

$S_u = 225 \text{ ksi}$

$S_y = 194 \text{ ksi}$

## AXIAL AND ROTARY LOAD

Composition<sup>1</sup>  
Heat Treatment<sup>2</sup>  
Specimen Conditions<sup>3</sup>  
Meaning of Symbols<sup>4</sup>

T, °F	Type of Load	$K_0 \text{ ksi}$			$b$			$\sigma \text{ ksi}$		
		Life, cycles	$10^3$	$10^4$	$10^5$	$10^3$	$10^4$	$10^5$	$10^3$	$10^4$

### Effect of Temperature

200	Axial	50	40	30	2.6	2.75	3.2	73	57	47
400	Axial	54	44	35	4.1	4.6	6.1	59	48	39
600	Axial	59	49	42	1.5	2.3	2.4	65	56	47
1200	Axial	29	27	24	2.2	3.05	3.15	51	29	27
1500	Axial	17	14	12	3.4	3.7	4.0	23	20	17

### Miscellaneous Results

80	Rotary	69	50	38	1.0	1.12	1.54	71	53	39
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- 1 For Composition - see page 202, Item 29
- 2 For Heat Treatment - see page 202, Item 29
- 3 For Specimen Conditions - see page 204, Item 29
- 4 For Meaning of Symbols - see page 46



## REFERENCES FOR FATIGUE STRENGTH DATA

### Materials:

### Source:

1 through 7	Mechanical Properties Data Center (MPDC) Traverse City, Michigan.
8A	Aeronautical Systems Division (ASD), Tech. note 61-117, Part III, Mechanical Properties Information Processing System. "Fatigue of Metals, Low Alloy Steels" Sec. 1. Feb. 1962, Belfour Eng. Co., Suttons Bay, Michigan.
8B through 17	MPDC (See Above)
18 through 26	ASD Tech. Note 61-117 Part II, "Fatigue of Metals - Corrosion and Heat Resistant Metals," Nov. 1961. Belfour Eng. Co., Suttons Bay, Michigan.
27	L. R. Jackson, H. J. Groover, R. C. McMaster, Advisory Report on the "Fatigue Properties of Aircraft Materials and Structures." OSRD, Rep. #6600.
28 and 29	MPDC (See Above)

#### A-1.2 WEIBULL PARAMETERS FOR TENSILE STRENGTH

INDEX TO WEIBULL PARAMETERS  
FOR  
TENSILE STRENGTH OF VARIOUS MATERIALS

<u>Material</u>	<u>Page Number</u> where Weibull Parameters for a given Material can be located
1. (.12 - .17)% C Steel.	245
2. (.08 - .30)% C Steel.	245
3. (.18 - .24)% C Steel.	245
4. (.08 - .35)% C Steel.	245
5. 5% Cr, .5% Mn, .5% Ti, .12% C Steel.	246
6. 5% Cr, .5% Mo, .2% C Steel.	246
7. 17% Cr, .12% C Stainless Steel.	246
8. 2.25% Cr, 1% Mo, .15% C Steel.	246
9. 1.25% Cr, .5% Mo, (.07 - .15)% C Steel.	246
10. 25% Cr, 12% Ni Stainless Steel.	246
11. 18% Cr, 8% Ni + Ti Stainless Steel.	247
12. 18% Cr, 8% Ni + Co Stainless Steel.	247
13. 18% Cr, 8% Ni Stainless Steel.	247
14. 18% Cr, 12% Ni, 2% Mo Stainless Steel.	247
15. 25% Cr, 20% Ni Stainless Steel.	247
16. SAE 4340 Steel.	247
17. SAE 4140 Steel.	248
18. 1.25% Cr, .5% Mo, .25% V, .4% C Steel.	248
19. 1% Cr, .35% Mo, .25% V, .4% C Steel.	248
20. D <sub>6</sub> AC Steel. (Ladish).	248

**TABLES**

**Effect of Temperature on Tensile Strength  
of Various Commercially Available Materials**

Material	Heat Treatment	Temp. °F	S <sub>y</sub> ksi	S <sub>u</sub> ksi	X <sub>o</sub> ksi	b	θ ksi
Killed Carbon Steel	Normalized	70	41.0	60.0	52.0	1.42	61.8
1. (.12-.17)% C	at 1725°F	1000	17.0	31.0	23.0	1.90	35.4
.55% (max) Mn	Drawn at						
.09% (max) P	1200°F for						
.06% (max) S	1 hr						
.28% (max) Si		70	41.0	60.0	54.5	1.55	62.2
	Annealed	900	20.0	41.0	33.0	1.70	44.0
Low Carbon	at 1550°F	1000	17.0	31.0	20.0	2.05	36.0
Low Alloy Steel	for 1 hr	1100	14.0	23.0	11.0	2.40	25.4
		1200	10.0	16.0	7.5	2.49	17.4
<hr/>							
2. (.08-.30)% C		70	42.0	62.0	36.0	2.90	65.0
1.0% (max) Mn		300	37.0	66.0	48.0	1.70	71.2
.050% (max) P		400	35.0	67.0	52.0	1.40	72.5
.060% (max) S		500	32.0	66.0	57.0	1.07	72.5
.25% (max) Si		600	29.0	63.0	61.0	1.12	65.2
Low Carbon	None	700	27.0	55.0	53.0	1.23	60.9
Low Alloy Steel	Specified	800	23.0	45.0	28.0	1.65	51.5
		900	20.0	35.0	27.0	2.30	41.0
		1000	17.0	27.0	14.0	2.60	32.0
		1100	13.0	20.0	10.0	2.80	24.5
		1200	10.0	14.0	4.0	3.00	15.5
		1400	4.0	7.5	2.0	3.20	7.6
<hr/>							
3. Killed Carbon Steel		70	37.0	63.0	59.0	1.70	64.8
(.18-.24)% C		200	35.0	59.0	57.0	1.90	60.0
.86% (max) P	Stress	400	32.0	66.0	55.0	1.14	67.3
.032% (max) Mn	Relieved	600	28.0	63.0	57.0	1.05	64.8
.043% (max) S		800	25.0	52.0	49.0	1.80	53.7
.24% (max) Si		1000	20.0	34.0	26.5	2.40	35.3
Low Carbon							
Low Alloy Steel							
<hr/>							
4. (.08-.35)% C		70	40.0	63.0	54.0	1.45	65.5
(.30-.80)% Mn		750	30.0	62.5	53.0	1.50	64.5
(.10-.50)% Si	None	900	26.5	54.0	44.0	1.55	53.0
.04% (max) P	Specified	1000	23.0	46.0	37.0	1.80	51.8
.05% (max) S		1100	19.0	38.0	34.0	2.00	40.2
(.40-.65) Mo							
Low-Medium Carbon							
Low Alloy Steel							

5. Material	Heat Treatment	Temp. °F	S <sub>y</sub>	S <sub>u</sub>	X <sub>o</sub>	b	g
5.0% Cr	Annealed at 1550°F	900	20.0	50.0	44.0	1.36	50.9
0.5% Mo (1-.5)% Ti		1000	17.5	44.0	38.0	1.43	44.8
.12% C		1100	15.0	37.0	31.0	1.50	36.1
Low Carbon		1200	13.0	28.0	21.0	1.90	28.4
High Alloy Steel		1300	10.0	19.0	10.0	2.35	19.0
		1400	7.5	13.0	5.0	2.70	12.6
6. 5.0% Cr	None Specified	70	30.0	71.0	57.0	1.42	73.0
0.5% Mo		400	26.0	68.0	55.0	1.55	61.5
0.2% C		600	24.0	58.0	53.0	2.00	59.9
		900	20.0	51.0	42.0	3.00	52.3
		1000	18.0	43.0	32.0	3.40	44.0
Low Carbon		1100	16.0	34.0	28.0	3.60	34.2
High Alloy Steel		1200	14.0	25.0	16.0	4.20	26.1
		1300	10.0	17.5	9.0	4.70	18.9
		1400	8.0	12.0	6.0	4.80	12.1
7. Stainless Steel	Annealed at 1950°F	70	39.0	72.0	68.0	1.50	74.0
17% Cr		1300	8.0	15.0	7.0	2.10	16.2
.12% (max) C		1400	6.0	10.0	5.0	2.50	11.5
		1500	4.0	6.0	3.0	2.60	6.95
8. 2.25% Cr	Annealed	70	41.0	74.0	65.0	1.35	71.9
1.0% Mo	at 1550°F	1100	30.0	54.0	36.0	1.50	43.5
0.15% C	Cast	70	65.0	92.0	65.0	1.80	93.5
		1000	40.0	58.0	46.0	1.90	60.1
Low Carbon	Normalized	70	75.0	110.0	54.0	1.15	126.0
Low Alloy Steel	at 1650°F	800	75.0	110.0	53.0	1.90	126.0
	Drawn	900	70.0	107.0	52.0	2.40	111.0
	at 1300°F	1000	60.0	95.0	51.0	2.45	101.0
9. 1.25% Cr-0.5% Mo	Normalized at 1700°F Drawn at 1300°F	70	55.0	75.0	64.0	1.90	78.8
0.07-.15% C							
Low Carbon							
Low Alloy Steel							
10. Stainless Steel	Annealed at 2000°F	70	57.0	80.0	63.0	1.15	91.0
25% Cr-12% Ni		800	41.0	69.0	62.0	1.19	74.5
.20% (max) C		1200	31.0	52.0	37.5	1.17	56.9
		1300	28.0	44.0	36.0	1.80	49.3
		1400	26.0	36.0	21.0	2.00	38.9
		1500	25.0	27.0	17.0	2.10	28.2
		1600	20.0	20.0	11.0	2.20	21.5

11.	Material	Heat Treatment	Temp. °F	S <sub>y</sub>	S <sub>u</sub>	X <sub>o</sub>	b	θ
	Stainless Steel		70	35.0	85.0	71.0	1.80	87.0
	18% Cr-8% Ni .0% Ti		1000	30.0	55.0	46.0	2.00	60.4
	(.04-.09)% C	Annealed	1200	28.0	45.0	35.0	2.30	49.2
		at 1950°F	1300	25.0	41.0	21.0	2.50	43.0
			1500	18.0	22.0	11.0	2.75	24.5
12.	Stainless Steel		70	38.0	85.0	75.0	1.50	87.5
	18% Cr-8% Ni +.8% Cb		1000	29.0	60.0	52.0	1.55	63.3
	.06% C		1100	27.0	58.0	51.0	1.60	60.0
		Annealed	1200	25.0	47.0	40.0	1.70	48.7
		at 1900°F	1300	22.0	44.0	40.0	1.70	45.2
			1500	18.0	25.0	13.0	3.00	26.7
13.	Stainless Steel		70	45.0	85.0	73.0	2.40	86.9
	18% Cr-8% Ni		800	25.0	60.0	51.0	2.60	63.5
	(.02-.09)% C	Annealed	1000	20.0	54.0	45.0	2.80	55.6
		at 1950°F	1200	18.0	45.0	36.0	2.95	47.8
			1400	14.0	31.0	20.0	3.10	32.2
14.	Stainless Steel		70	41.0	88.0	79.0	1.08	89.0
	18% Cr-12% Ni-2% Mo		600	31.0	73.0	66.0	1.12	75.8
	.08% C		800	27.0	72.0	65.0	1.23	74.6
		Annealed	1000	24.0	69.0	60.0	1.39	70.3
		at 1950°F	1200	21.0	55.0	44.0	1.50	56.8
			1300	19.0	47.0	35.0	1.55	52.3
			1400	18.0	35.0	24.0	1.75	38.2
			1600	17.0	21.0	13.0	2.00	22.8
15.	Stainless Steel		70	32.0	88.0	80.0	1.17	89.2
	25% Cr-20% Ni		1000	24.0	70.0	65.5	1.20	73.2
	.25% (max) C	Annealed	1300	20.0	47.0	40.0	1.33	48.5
		at 2000°F	1400	19.0	37.0	30.0	1.38	38.8
			1500	17.0	34.0	17.0	1.40	36.0
16.	SAE 4340 Steel		70	90.0	120.0	101.0	1.09	124.5
	.4% Cr-.4% Ni-	Normalized	850	65.0	90.0	72.0	1.13	100.5
	.4% Mo	at 1600°F	950	50.0	78.0	55.0	1.59	92.5
	(.35-.50)% C	Tempered	1000	50.0	70.0	53.0	1.90	79.8
	Medium Carbon	at 1200°F						
	Low Alloy Steel							

Material	Heat Treatment	Temp. °F	S <sub>y</sub>	S <sub>u</sub>	X <sub>o</sub>	b	θ
17.							
SAE 4140 Steel	Quenched	70	115.0	125.0	113.0	3.20	130.5
.92% Cr-.6% Mn-	and	1000	56.0	75.0	50.0	4.00	81.5
.25% Si .4% C	Tempered						
Medium Carbon	at 1200° F						
Low Alloy Steel							
18.							
1.25% Cr-.5% Mo	Normalized	70	122.0	145.0	115.0	1.65	149.5
-.5% Mn-.6% Si-	at 1725° F	1000	75.0	95.0	70.0	2.05	99.5
.25% V	Drawn						
Medium Carbon	at 1200° F						
Low Alloy Steel							
19.							
1.0% Cr-.35% Mo	Normalized	70	125.0	146.0	133.0	1.80	150.5
-.25% V .4% C	at 1700° F	900	90.0	109.0	88.0	2.40	117.5
Medium Carbon	Tempered						
Low Alloy Steel	at 1200° F						
20.							
D6AC	Tempered						
	at 1150° F	70			119.0	2.00	194.0
Unnotched Steel	1200° F	70			139.0	2.30	145.4
	1250° F	70			101.0	3.30	124.5



# REFERENCES FOR TENSILE STRENGTH DATA

<u>Materials:</u>	<u>Source:</u>
1.	ASTM STP # 180 (1955)
2.	ASTM STP # 100 (1950)
3.	ASTM STP # 180 (1955)
4.	ASTM STP # 100 (1950)
5.	ASTM STP # 100 (1950)
6.	ASTM STP # 100 (1950)
7.	Scainless Steel ASTM STP # 100 (1950)
8.	ASTM STP # 151 (1953)
9.	ASTM STP # 151 (1953)
10.	Stainless Steel ASTM STP # 100 (1950)
11.	Stainless Steel ASTM STP # 124 (1952)
12.	Stainless Steel ASTM STP # 124 (1952)
13.	Stainless Steel ASTM STP # 124 (1952)
14.	Stainless Steel ASTM STP # 100 (1950)
15.	ASTM STP # 100 (1950)
16.	ASTM STP # 199 (1957)
18.	ASTM STP # 151 (1953)
19.	ASTM STP # 199 (1957)
20.	Mechanical Properties Data Center Search # 1333

### A-1.3 WEIBULL PARAMETERS FOR RUPTURE STRENGTH

INDEX TO WEIBULL PARAMETERS  
FOR  
RUPTURE STRENGTH OF VARIOUS MATERIALS

<u>Material</u>	<u>Page Number</u> where Weibull Parameters for a given Material can be located
1. 12 Cr - 2.75 Mo-V	253
2. 12 Cr Steel	253
3. 12 Cr - Cb Steel	253
4. 12 Cr - 2.5 W-V	253
5. 12 Cr - 5 Co - 3 W-V	253
6. 13 Cr - W - Mo - V	254
7. 17 - 22 - A - "S" Steel	254
8. .12 - .17 C Steel	254
9. .18 - .24 C Steel	254
10. ASTM A - 201 - B Steel	254
11. 17 - 22 A-S	254
12. .5 Cr - .5 Mo Steel	254
13. .5 Mo Steel	254
14. 5.0 Cr - .5 Mo - Ti	255
15. 18 Cr - 8 Ni + Mo + Cb	255
16. 18 Cr - 8 Ni + Cb	255
17. 18 Cr - 8 Ni	255
18. 4340 Steel	255
19. C - .5% Mo Steel.	255
20. 5% Cr - .5% Mo - Ti	255
21. 18% Cr - 8 % Ni.	256

**TABLES**

# RUPTURE STRENGTH OF HIGH ALLOY STEELS

Effect of Temperature, Time, and Heat Treatment on the Weibull Parameters

Time to Rupture in hours		X <sub>0</sub> , ksi		b					, ksi				
Temp. °F	N.T.*	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>
		(1) 12 Cr-2.75 Mo-V											
1000	A	30.0	15.0	13.0		2.5	2.72	2.8		59.0	47.0	33.4	
1100	A	15.0	14.0			2.7	3.05			36.3	24.9		
		(2) 12 Cr Steel											
1000	B	18.0	19.0	5.0		2.7	2.98	3.4		29.3	24.65	18.3	
900	B	37.0	29.5	20.0		2.05	2.48	3.25		43.5	34.3	28.9	
		(3) 12 Cr-Cb Steel											
1000	C	37.0	25.0			2.3	2.7			55.6	43.9		
900	C	56.5	48.0	33.0		2.6	2.8	2.95		68.9	62.1	56.2	
		(4) 12 Cr-2.5 W-V											
1000	D	34.0	23.0	16.0		2.45	2.7	2.9		52.8	42.4	29.1	
1100	D	17.0	8.0	7.0		2.85	3.2	3.55		31.0	17.5	10.9	
900	D		46.0	33.0			2.3	2.53			55.45	52.8	
		(5) 12 Cr-5 Co-3 W-V											
1000	D	39.0	30.0	16.0		2.4	2.6	2.9		67.4	55.6	44.7	
1100	D	18.0	13.0	7.5		2.45	2.6	2.85		41.8	25.4	13.9	
1200	D	7.0	10.0			1.75	2.45			29.5	24.5		
900	D		46.0	36.0			2.95	3.2			72.4	65.5	

	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	(6)	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>
					<u>13 Cr-0.4 Mn-V</u>								
1000 A	30.0	30.0	32.0	22.0	2.85	3.1	3.25	3.55	68.8	57.4	54.3	30.05	
1200 A	17.0	10.0	13.0		2.6	2.85	2.95		59.0	36.1	22.1		
					(7)	<u>17-22-A-"B" Steel</u>							
1000 A	28.0	24.0			2.12	2.45			68.5	52.1			
1100 A	16.0	10.0			2.5	2.8			37.4	18.65			
1200 A	30.0	2.0			2.32	2.63			15.5	7.15			
					(8)	<u>.12-.17 C Steel</u>							
800 Killed	19.0	13.5	8.5		2.12	2.5	2.62		27.4	20.1	15.4		
					(9)	<u>.18-.24 C Steel</u>							
1000 Killed	12.0	7.5	5.0		2.7	3.2	3.3		14.08	10.29	6.68		
					(10)	<u>ASTM A-201-B</u>							
1000 Str. Bol.	12.2	6.6	5.7	4.4	2.35	2.6	2.9	3.1	14.17	9.96	7.2	5.14	
					(11)	<u>17-22 A-B</u>							
1000 E		22.0			2.6					53.6			
					(12)	<u>.5 Cr-.5 Mn</u>							
1000 Cast	21.0				2.03				30.2				
1200 Cast	6.5				2.52				10.04				
1300 F		5.0	2.7		2.55	2.8			6.88	4.31			
					(13)	<u>.5 Mn Steel</u>							
1000 G		16.5	11.0		2.75	2.75			19.35	14.38			

	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	10 <sup>2</sup>	10 <sup>3</sup>	10 <sup>4</sup>	10 <sup>5</sup>	
					(14)	<u>5.0 Cr-.5 Mo-Ti</u>							
1000	H	25.0	18.0	11.5	5.5	2.1	2.35	2.55	2.85	32.55	22.25	13.95	8.46
1200	H	5.0	3.7	2.2	1.1	2.15	2.35	2.56	2.87	8.3	5.82	3.7	2.67
1200	I	5.5	7.0			2.66	2.15			9.23	8.76		
					(15)	<u>18 Cr-8 Ni + Mo + Cb</u>							
1100	Cast	22.0	21.0	19.0	19.0	2.06	2.5	2.6	3.25	35.6	28.9	23.33	20.41
1200	Cast	20.0	19.5	14.2	5.5	2.35	2.62	3.03	3.12	33.2	24.41	17.75	14.4
1500	Cast	3.5	1.2			2.25	2.3			9.87	7.15		
					(16)	<u>18 Cr-8 Ni + Cb</u>							
1100	H	30.0	23.0	13.0		2.3	2.65	1.92		44.2	36.9	29.7	
1200	H	19.0	12.0	7.0		2.4	2.85	2.12		31.7	25.8	19.5	
1300	H	8.5	1.0	.5		2.65	3.1	2.19		23.4	17.8	11.8	
					(17)	<u>18 Cr-8 Ni</u>							
1200	H	13.0	8.0	4.0	1.0	2.05	2.2	2.88	2.53	26.7	17.35	11.3	7.9
					(18)	<u>4340 Steel</u>							
1000	I	14.0	6.0	8.0		2.04	2.23	3.2		42.0	20.3	15.8	
					(19)	<u>C-.5% Mo Steel</u>							
900	-		32.5	21.0			2.18	2.5					
1000	-		15.5	6.5			2.38	2.65		46.7	38.6	30.1	18.4
					(20)	<u>5% Cr-.5% Mo-Ti</u>							
1000	-		18.0	6.0			2.5	2.87		21.98	15.6		
1200	-		4.4	2.0			2.78	3.25		5.52	4.05		

	$10^2$	$10^3$	$10^4$	$10^5$	$10^2$	$10^3$	$10^4$	$10^5$
1300	-	7.7			2.48			9.41

(21) 10% Cr-0% Ni

**Heat Treatment Code:**

- A - aust. temp. at 1300°F
- B - finished rolled at 1450°F temp.
- C - norm. temp. at 1200°F
- D - norm. temp.
- E - norm. and drawn
- F - annealed bar stock
- G - norm. and drawn
- H - annealed bar stock
- I - aust. and transformed --no temp.



# REFERENCES FOR RUPTURE STRENGTH DATA

## Materials:

## Source:

1 through 6	ASTM STP # 228 (1958)
7	ASTM STP # 199 (1957)
8 through 10	ASTM STP # 180 (1955)
11 through 14	ASTM STP # 151 (1953)
15 through 17	ASTM STP # 124 (1952)
18	ASTM STP # 199 (1957)
19 through 21	ASTM STP # 100 (1950)

APPENDIX 2 TABLES OF INTERFERENCE VALUES

A-2.1 STRESS DISTRIBUTION - NORMAL

STRENGTH DISTRIBUTION - WEIBULL

A-2.1.1 STRESS STANDARD DEVIATION = 0

# TABLES OF INTERFERENCE $[F(x)]$

Stress Distribution: Normal

Strength Distribution: Weibull

$\mu = S_{\text{equ}}$  (Equivalent Stress)

$x_0$

Depend on the material

$\sigma = 0$

$b$

and the operating conditions

$\theta$

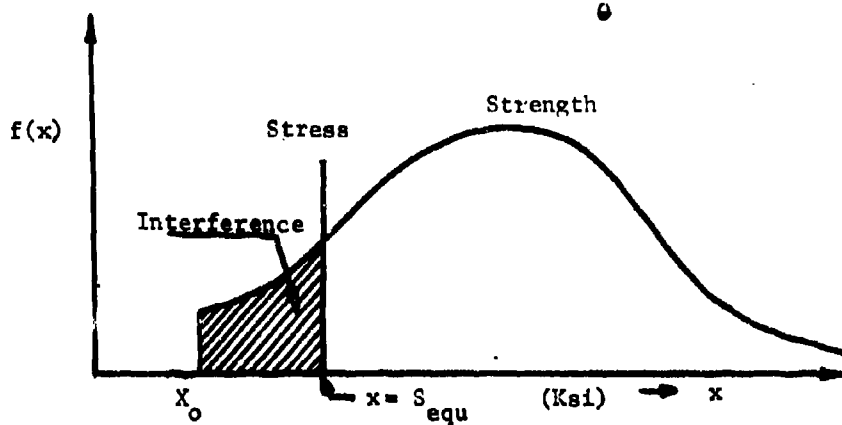


Figure A-2.1 Interference with Standard Deviation of Stress Equal to Zero

$$f(x) = \frac{b}{\theta - x_0} \left( \frac{x - x_0}{\theta - x_0} \right)^{b-1} e^{-\left( \frac{x - x_0}{\theta - x_0} \right)^b}$$

$$F(x) = \int_{x_0}^{\infty} f(x) dx$$

$$\text{Let } y = \left( \frac{x - x_0}{\theta - x_0} \right)^b, \quad \therefore dy = \frac{b}{\theta - x_0} \left( \frac{x - x_0}{\theta - x_0} \right)^{b-1} dx$$

$$F(x) = \int_{x_0}^{\infty} f(x) dx = \int_0^{\left( \frac{x - x_0}{\theta - x_0} \right)^b} e^{-y} dy = -e^{-y} \Big|_0^{\left( \frac{x - x_0}{\theta - x_0} \right)^b} = 1 - e^{-X}$$

$$\text{where } X = \left( \frac{x - x_0}{\theta - x_0} \right)^b$$

## F(X)

X	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
0.00	0	.001	.002	.003	.00399	.00498	.00598	.00697	.00796	.00896
0.01	.00995	.0109	.0119	.0129	.0139	.0149	.0158	.0168	.0178	.0188
0.02	.0198	.0208	.0217	.0227	.0237	.0246	.0256	.0266	.0276	.0286
0.03	.0295	.0304	.0314	.0324	.0334	.0344	.0353	.0363	.0372	.0382
0.04	.0392	.0401	.0411	.0420	.0430	.0440	.0449	.0458	.0468	.0477
0.05	.0487	.0496	.0506	.0515	.0525	.0535	.0544	.0553	.0562	.0572
0.06	.0581	.0591	.0600	.0610	.0619	.0628	.0637	.0646	.0656	.0665
0.07	.0675	.0685	.0694	.0703	.0712	.0721	.0730	.0740	.0749	.0759
0.08	.0768	.0776	.0786	.0795	.0805	.0814	.0823	.0832	.0841	.0850
0.09	.0860	.0869	.0878	.0887	.0896	.0905	.0914	.0923	.0932	.0941
X	.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	.0952	.1042	.1131	.1219	.1306	.1393	.1479	.1563	.1647	.1730
0.2	.1813	.1894	.1975	.2055	.2134	.2212	.2289	.2366	.2442	.2517
0.3	.2592	.2666	.2739	.2811	.2882	.2953	.3023	.3093	.3161	.3229
0.4	.3297	.3363	.3430	.3494	.3560	.3624	.3687	.3750	.3812	.3874
0.5	.3935	.3995	.4055	.4114	.4173	.4231	.4288	.4345	.4401	.4457
0.6	.4512	.4566	.4621	.4674	.4727	.4780	.4831	.4883	.4934	.4984
0.7	.5034	.5084	.5132	.5181	.5229	.5276	.5323	.5370	.5416	.5462
0.8	.5507	.5551	.5596	.5640	.5683	.5726	.5768	.5810	.5852	.5893
0.9	.5934	.5975	.6015	.6054	.6094	.6133	.6171	.6209	.6247	.6284

x	F(x)																
	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90							
	0.05	.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95							
1.0	.6321	.6671	.6988	.7275	.7534	.7769	.7981	.8173	.8341	.8504							
	.6501	.6834	.7135	.7408	.7654	.7878	.8080	.8262	.8428	.8577							
2.0	.8647	.8775	.8892	.8997	.9093	.9179	.9257	.9328	.9392	.9450							
	.8713	.8835	.8946	.9046	.9137	.9219	.9293	.9361	.9422	.9477							
3.0	.9502	.9550	.9592	.9631	.9666	.9698	.9727	.9753	.9776	.9798							
4.0	.9817	.9834	.9850	.9864	.9877	.9889	.9899	.9909	.9918	.9926							
5.0	.99326	.99391	.99448	.99501	.99549	.99592	.99630	.99665	.99697	.99726							
6.0	.99752	.99776	.99797	.99816	.99836	.99850	.99864	.99877	.99889	.99899							
7.0	.999088	.999175	.999253	.999324	.999389	.999447	.999499	.999547	.999590	.999629							
8.0	.999665	.999696	.999725	.999751	.999775	.999796	.999816	.999833	.999849	.999864							
9.0	.999877	.999888	.999899	.999909	.999917	.999925	.999932	.999939	.999944	.999950							
10.0	.999955																

A-2.1.2 STRESS STANDARD DEVIATION  $\neq 0$



# TABLES OF INTERFERENCE

Stress Distribution: Normal

Strength Distribution: Weibull

$\mu = S_{\text{equ}}$  (Equivalent Stress)

$X_0$

Depend on the material and the operating conditions

$b$

$\sigma \neq 0$

$\theta$

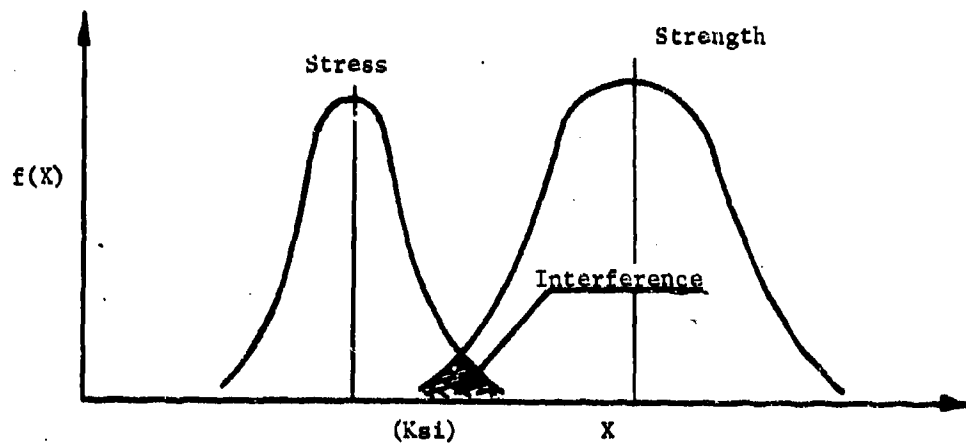


Figure A-22 Interference with Standard Deviation of Stress Not Equal to Zero

$B(x) = 1.00$

$C = \frac{0 - \bar{x}_0}{s}, A = \frac{x_0 - \mu}{s}$

A	10	15	20	25	30	35	40	45	50	55
1.0	.0080	.0054	.0041	.0033	.0027	.0023	.0021	.0018	.0017	.0015
1.2	.0054	.0036	.0027	.0022	.0018	.0016	.0014	.0012	.0011	.0010
1.4	.0035	.0024	.0018	.0014	.0012	.0010	.0009	.0008	.0007	.0007
1.6	.0022	.0015	.0011	.0009	.0008	.0007	.0006	.0005	.0005	.0004
1.8	.0014	.0009	.0007	.0006	.0005	.0004	.0004	.0003	.0003	.0003
2.0	.0008	.0006	.0004	.0003	.0003	.0002	.0002	.0002	.0002	.0002
2.2	.0005	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001	.0001
2.4	.0003	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
2.6	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.8	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$B(x) = 1.00$									
* * * C									
A	60	65	70	75	80	85	90	95	100
	*****	*****	*****	*****	*****	*****	*****	*****	*****
1.0	.0014	.0013	.0012	.0011	.0010	.0009	.0009	.0009	.0008
1.2	.0009	.0009	.0008	.0007	.0007	.0006	.0006	.0006	.0006
1.4	.0006	.0006	.0005	.0005	.0005	.0004	.0004	.0004	.0004
1.6	.0004	.0004	.0003	.0003	.0003	.0003	.0003	.0002	.0002
1.8	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
2.0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
2.2	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
2.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$R(x) = 1.10 \quad C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* \* \*

A	10	15	20	25	30	35	40	45	50	55
1.0	.0062	.0049	.0029	.0023	.0019	.0016	.0014	.0012	.0011	.0010
1.2	.0042	.0027	.0020	.0015	.0013	.0011	.0009	.0008	.0007	.0007
1.4	.0027	.0018	.0013	.0010	.0008	.0007	.0006	.0005	.0005	.0004
1.6	.0017	.0011	.0008	.0006	.0005	.0004	.0004	.0003	.0003	.0003
1.8	.0010	.0007	.0005	.0004	.0003	.0003	.0002	.0002	.0002	.0002
2.0	.0006	.0004	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001
2.2	.0004	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001
2.4	.0002	.0001	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000
2.6	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.8	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 1.10$$

		C = $\frac{\theta - x_0}{\sigma}$ , A = $\frac{x_0 - \mu}{\sigma}$									
		60	65	70	75	80	85	90	95	100	
		*****									
A	*	60	65	70	75	80	85	90	95	100	*****
1.0	*	.0009	.0008	.0008	.0007	.0007	.0006	.0006	.0005	.0005	
1.2	*	.0026	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0003	
1.4	*	.0004	.0004	.0003	.0003	.0003	.0003	.0002	.0002	.0002	
1.6	*	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001	
1.8	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	
2.0	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	
2.2	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.4	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.6	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.8	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$B(X) = 1.2^n$

$C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$

A	10	15	20	25	30	35	40	45	50	55
1.0	.0048	.0039	.0021	.0016	.0013	.0011	.0009	.0008	.0007	.0006
1.2	.0032	.0020	.0014	.0011	.0009	.0007	.0006	.0005	.0005	.0004
1.4	.0021	.0013	.0009	.0007	.0006	.0005	.0004	.0004	.0003	.0003
1.6	.0013	.0008	.0006	.0004	.0004	.0003	.0003	.0002	.0002	.0002
1.8	.0008	.0005	.0003	.0003	.0002	.0002	.0002	.0001	.0001	.0001
2.0	.0005	.0003	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001
2.2	.0003	.0002	.0001	.0001	.0001	.0001	.0001	.0000	.0000	.0000
2.4	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.6	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 1.20$$

		C = $\frac{\theta - x_0}{\sigma}$ , A = $\frac{x_0 - \mu}{\sigma}$									
		*****									
A	*	60	65	70	75	80	85	90	95	100	
1.0	*	.0006	.0005	.0005	.0004	.0004	.0004	.0004	.0003	.0003	
1.2	*	.0004	.0004	.0003	.0003	.0003	.0003	.0002	.0002	.0002	
1.4	*	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001	
1.6	*	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	
1.8	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	
2.0	*	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.2	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.4	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.6	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$R(x) = 1.3$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{1 - R}{\sigma}$$

A	*	10	15	20	25	30	35	40	45	50	55
1.0	*	.0038	.0023	.0016	.0012	.0009	.0008	.0006	.0005	.0005	.0004
1.2	*	.0025	.0015	.0010	.0008	.0006	.0005	.0004	.0004	.0003	.0003
1.4	*	.0016	.0010	.0007	.0005	.0004	.0003	.0003	.0002	.0002	.0002
1.6	*	.0010	.0006	.0004	.0003	.0002	.0002	.0002	.0001	.0001	.0001
1.8	*	.0006	.0004	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001
2.0	*	.0004	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0000	.0000
2.2	*	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.4	*	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.6	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull



$B(X) = 1.30$

		$C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$									
		60	65	70	75	80	85	90	95	100	
A	*	*	*	*	*	*	*	*	*	*	*****
1.0	*	.0004	.0003	.0003	.0003	.0003	.0002	.0002	.0002	.0002	.0002
1.2	*	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001	.0001	.0001
1.4	*	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.6	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.8	*	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.2	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.4	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.6	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

B(X) = 1.4)

$$C = \frac{\theta - X_0}{\sigma}, \quad A = \frac{X_C - \mu}{\sigma}$$

A	10	15	20	25	30	35	40	45	50	55
1.0	.0030	.0017	.0011	.0008	.0006	.0005	.0004	.0004	.0003	.0003
1.2	.0020	.0011	.0007	.0005	.0004	.0003	.0003	.0002	.0002	.0002
1.4	.0012	.0007	.0005	.0003	.0003	.0002	.0002	.0002	.0001	.0001
1.6	.0008	.0004	.0003	.0002	.0002	.0001	.0001	.0001	.0001	.0001
1.8	.0005	.0003	.0002	.0001	.0001	.0001	.0001	.0001	.0000	.0000
2.0	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000
2.2	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$G(x) = 1.40$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	60	65	70	75	80	85	90	95	100
1.0	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001	.0001
1.2	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.4	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.6	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 1.53 \quad C = \frac{\theta - \lambda_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	10	15	20	25	30	35	40	45	50	55
1.0	.0023	.0013	.0008	.0006	.0005	.0004	.0003	.0003	.0002	.0002
1.2	.0015	.0008	.0005	.0004	.0003	.0002	.0002	.0002	.0001	.0001
1.4	.0010	.0005	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001
1.6	.0006	.0003	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0000
1.8	.0004	.0002	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000
2.0	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.2	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

B(X) = 1.50

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* \* \*

A	50	65	70	75	80	85	90	95	100
1.0	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.2	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1.4	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 1.62$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	*	10	15	20	25	30	35	40	45	50	55
1.0	*	.0019	.0010	.0006	.0004	.0003	.0003	.0002	.0002	.0001	.0001
1.2	*	.0012	.0006	.0004	.0003	.0002	.0002	.0001	.0001	.0001	.0001
1.4	*	.0008	.0004	.0003	.0002	.0001	.0001	.0001	.0001	.0001	.0001
1.6	*	.0005	.0002	.0002	.0001	.0001	.0001	.0001	.0000	.0000	.0000
1.8	*	.0003	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.0	*	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.2	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$g(x) = 1.60$

		$C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$									
		*****									
A	C	60	65	70	75	80	85	90	95	100	
1.0	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000	
1.2	*	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	
1.4	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
1.6	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
1.8	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.2	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 1.70 \quad C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

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$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$g(x) = 1.7$

A	*	C	*****									
			60	65	70	75	80	85	90	95	100	*****
1.0	*	*	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	
1.2	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
1.4	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
1.6	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
1.8	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.0	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.2	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = -1.83$$

		C = $\frac{\theta - x_0}{\sigma}$ , A = $\frac{x_c - \mu}{\sigma}$									
		*****									
A	*	10	15	20	25	30	35	40	45	50	55
		*****									
1.0	*	.0012	.0006	.0003	.0002	.0002	.0001	.0001	.0001	.0001	.0001
1.2	*	.0008	.0004	.0002	.0001	.0001	.0001	.0001	.0001	.0000	.0000
1.4	*	.0005	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000
1.6	*	.0003	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.8	*	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.2	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 1.90$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	C									
	10	15	20	25	30	35	40	45	50	55
1.0	.0009	.0004	.0002	.0002	.0001	.0001	.0001	.0001	.0000	.0000
1.2	.0006	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000
1.4	.0004	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.6	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.8	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

R(1) = 2.00

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* \* C

A	10	15	20	25	30	35	40	45	50	55
1.0	.0007	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000
1.2	.0005	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000
1.4	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.8	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 2.10$$

$$C = \frac{9-x_0}{\sigma}, \quad A = \frac{x_0-\mu}{\sigma}$$

	10	15	20	25	30	35	40	45	50	55
A	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1.0	.0006	.0003	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000
1.2	.0004	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.4	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.8	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

B(X) = 2.23

		$C = \frac{\theta - X_0}{\sigma}, A = \frac{X_0 - \mu}{\sigma}$									
		10	15	20	25	30	35	40	45	50	55
A	*	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
1.0	*	*****	.0005	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000
1.2	*	*****	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.4	*	*****	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	*	*****	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.8	*	*****	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

236

B(X) = 2.33

		$C = \frac{\theta - X_0}{\sigma}, A = \frac{X_0 - \mu}{\sigma}$									
		10	15	20	25	30	35	40	45	50	55
A	*	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
1.0	*	*****	.0004	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.2	*	*****	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.4	*	*****	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	*	*****	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$$B(X) = 2.49$$

A	C	10	15	20	25	30	35	40	45	50	55
1.0	*	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	*	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.4	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$$B(X) = 2.50$$

A	C	10	15	20	25	30	35	40	45	50	55
1.0	*	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	*	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.4	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.6	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 2.62$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

	10	15	20	25	30	35	40	45	50	55
A	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
C	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
1.0	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

$$B(X) = 2.70$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

	10	15	20	25	30	35	40	45	50	55
A	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
C	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
1.0	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull



$$B(X) = 2.80$$

$$C = \frac{\theta - X_0}{\sigma}, \quad A = \frac{X_0 - \mu}{\sigma}$$

C										
A	10	15	20	25	30	35	40	45	50	55
1.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

$$B(X) = 2.90$$

$$C = \frac{\theta - X_0}{\sigma}, \quad A = \frac{X_0 - \mu}{\sigma}$$

C										
A	10	15	20	25	30	35	40	45	50	55
1.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$B(X) = 3.00$   
 $C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$   
 \* \* \* C  
 \*  
 A \* 10 15 20 25 30 35 40 45 50 55  
 \*\*\*\*\*  
 \*  
 1.0 \* .0001 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000  
 \*  
 1.2 \* .0001 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000

$B(X) = 3.10$   
 $C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$   
 \* \* \* C  
 \*  
 A \* 10 15 20 25 30 35 40 45 50 55  
 \*\*\*\*\*  
 \*  
 1.0 \* .0001 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000

$B(X) = 3.20$   
 $C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$   
 \* \* \* C  
 \*  
 A \* 10 15 20 25 30 35 40 45 50 55  
 \*\*\*\*\*  
 \*  
 1.0 \* .0001 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000

STRESS DISTRIBUTION - Normal  
 STRENGTH DISTRIBUTION - Weibull

$$B(X) = 1.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\*  
\*  
\*

A	10	15	20	25	30	35	40	45	50	55
8	.0115	.0078	.0059	.0047	.0039	.0034	.0030	.0026	.0024	.0022
6	.0160	.0109	.0082	.0066	.0055	.0047	.0042	.0037	.0033	.0030
4	.0218	.0148	.0112	.0090	.0075	.0065	.0057	.0051	.0046	.0041
2	.0290	.0197	.0149	.0120	.0100	.0086	.0076	.0067	.0061	.0055
0	.0375	.0255	.0193	.0156	.0130	.0112	.0098	.0087	.0079	.0072
-2	.0475	.0323	.0245	.0197	.0165	.0142	.0125	.0111	.0100	.0091
-4	.0588	.0401	.0304	.0245	.0205	.0176	.0155	.0138	.0124	.0113
-6	.0713	.0487	.0370	.0298	.0250	.0215	.0189	.0168	.0151	.0138
-8	.0849	.0581	.0442	.0356	.0298	.0257	.0225	.0201	.0181	.0165
-10	.0994	.0681	.0518	.0418	.0351	.0302	.0265	.0236	.0213	.0194
-12	.1301	.0896	.0683	.0552	.0463	.0399	.0350	.0312	.0282	.0256
-14	.1620	.1121	.0857	.0693	.0582	.0502	.0441	.0393	.0355	.0323
-16	.1940	.1348	.1033	.0837	.0704	.0607	.0533	.0476	.0430	.0391
-18	.2252	.1574	.1209	.0981	.0826	.0713	.0627	.0559	.0505	.0460
-20	.2555	.1795	.1382	.1124	.0947	.0818	.0720	.0643	.0581	.0529
-22	.2847	.2010	.1553	.1265	.1067	.0922	.0812	.0725	.0656	.0598
-24	.3127	.2221	.1720	.1403	.1185	.1025	.0903	.0808	.0730	.0666
-26	.3397	.2425	.1884	.1540	.1302	.1127	.0994	.0889	.0804	.0734
-28	.3656	.2625	.2045	.1674	.1417	.1228	.1084	.0969	.0877	.0801
-30	.3904	.2819	.2202	.1806	.1530	.1328	.1172	.1049	.0950	.0857
-32	.4202	.3054	.2395	.1968	.1670	.1451	.1282	.1148	.1040	.0950
-34	.4484	.3282	.2583	.2127	.1808	.1572	.1390	.1246	.1129	.1032
-36	.4753	.3502	.2766	.2283	.1944	.1692	.1497	.1343	.1217	.1113
-38	.5009	.3715	.2944	.2436	.2077	.1809	.1603	.1438	.1305	.1194
-40	.5284	.4120	.3288	.2733	.2336	.2040	.1810	.1627	.1477	.1352
-42	.5514	.4500	.3616	.3018	.2588	.2264	.2012	.1811	.1646	.1508
-44	.5703	.4854	.3927	.3291	.2831	.2482	.2210	.1991	.1811	.1661

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$R(X) = 1 -$	*	$C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$									
A	*	60	65	70	75	80	85	90	95	100	
8	*	.0020	.0018	.0017	.0016	.0015	.0014	.0013	.0013	.0012	
6	*	.0028	.0026	.0024	.0022	.0021	.0020	.0019	.0018	.0017	
4	*	.0038	.0035	.0033	.0031	.0029	.0027	.0025	.0024	.0023	
2	*	.0051	.0047	.0043	.0041	.0038	.0036	.0034	.0032	.0031	
0	*	.0066	.0061	.0056	.0053	.0049	.0047	.0044	.0042	.0040	
-2	*	.0084	.0077	.0072	.0067	.0063	.0059	.0056	.0053	.0050	
-4	*	.0104	.0096	.0089	.0083	.0078	.0074	.0069	.0066	.0063	
-6	*	.0126	.0117	.0109	.0101	.0095	.0090	.0085	.0080	.0075	
-8	*	.0151	.0140	.0130	.0121	.0114	.0107	.0101	.0096	.0091	
-10	*	.0178	.0164	.0153	.0143	.0134	.0126	.0119	.0113	.0107	
-12	*	.0205	.0188	.0175	.0163	.0152	.0141	.0132	.0124	.0117	
-14	*	.0235	.0214	.0199	.0185	.0172	.0159	.0148	.0138	.0129	
-16	*	.0267	.0242	.0224	.0208	.0193	.0178	.0165	.0153	.0142	
-18	*	.0297	.0274	.0255	.0238	.0224	.0211	.0199	.0189	.0179	
-20	*	.0330	.0302	.0281	.0262	.0247	.0232	.0221	.0209	.0198	
-22	*	.0360	.0332	.0309	.0289	.0271	.0255	.0241	.0229	.0218	
-24	*	.0423	.0391	.0364	.0340	.0319	.0301	.0284	.0270	.0256	
-26	*	.0486	.0450	.0419	.0391	.0367	.0346	.0327	.0310	.0295	
-28	*	.0550	.0509	.0473	.0442	.0415	.0391	.0370	.0351	.0334	
-30	*	.0612	.0567	.0527	.0493	.0463	.0437	.0413	.0392	.0372	
-32	*	.0675	.0625	.0581	.0544	.0511	.0481	.0455	.0432	.0411	
-34	*	.0737	.0682	.0635	.0594	.0558	.0526	.0498	.0472	.0449	
-36	*	.0798	.0739	.0688	.0644	.0605	.0571	.0540	.0512	.0487	
-38	*	.0875	.0810	.0755	.0706	.0664	.0626	.0592	.0562	.0535	
-40	*	.0950	.0881	.0821	.0768	.0722	.0681	.0644	.0612	.0582	
-42	*	.1025	.0951	.0886	.0829	.0780	.0736	.0696	.0661	.0629	
-44	*	.1100	.1020	.0951	.0890	.0837	.0796	.0748	.0710	.0676	
-46	*	.1247	.1157	.1079	.1011	.0951	.0898	.0850	.0807	.0768	
-48	*	.1392	.1292	.1206	.1130	.1063	.1004	.0951	.0903	.0860	
-50	*	.1534	.1425	.1330	.1247	.1174	.1109	.1051	.0999	.0951	

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 2.00$$

$$C = \frac{9-x_0}{\sigma}, \quad A = \frac{x_0-\mu}{\sigma}$$

\* C

A	10	15	20	25	30	35	40	45	50	55
* .8	.0011	.0005	.0003	.0002	.0001	.0001	.0001	.0001	.0000	.0000
* .6	.0017	.0008	.0004	.0003	.0002	.0001	.0001	.0001	.0001	.0001
* .4	.0025	.0011	.0006	.0004	.0003	.0002	.0002	.0001	.0001	.0001
* .2	.0035	.0016	.0009	.0006	.0004	.0003	.0002	.0002	.0001	.0001
* .0	.0049	.0022	.0012	.0008	.0006	.0004	.0003	.0002	.0002	.0002
* -.2	.0067	.0030	.0017	.0011	.0008	.0006	.0004	.0003	.0003	.0002
* -.4	.0089	.0040	.0023	.0014	.0010	.0007	.0006	.0004	.0004	.0003
* -.6	.0116	.0052	.0030	.0019	.0012	.0010	.0007	.0006	.0005	.0004
* -.8	.0149	.0067	.0038	.0024	.0017	.0012	.0010	.0008	.0006	.0005
* -1.0	.0188	.0085	.0048	.0031	.0021	.0016	.0012	.0009	.0008	.0006
* -1.4	.0284	.0128	.0073	.0047	.0032	.0024	.0018	.0014	.0012	.0010
* -1.8	.0407	.0185	.0105	.0067	.0047	.0034	.0025	.0021	.0017	.0014
* -2.2	.0557	.0254	.0144	.0093	.0065	.0047	.0036	.0029	.0023	.0019
* -2.6	.0733	.0336	.0191	.0123	.0086	.0063	.0048	.0035	.0031	.0026
* -3.0	.0935	.0431	.0246	.0158	.0110	.0081	.0062	.0049	.0040	.0033
* -3.4	.1159	.0538	.0308	.0198	.0138	.0102	.0078	.0062	.0050	.0041
* -3.8	.1406	.0658	.0377	.0243	.0170	.0125	.0096	.0076	.0062	.0051
* -4.2	.1671	.0789	.0453	.0293	.0205	.0151	.0116	.0092	.0074	.0061
* -4.6	.1954	.0930	.0536	.0347	.0243	.0179	.0137	.0109	.0088	.0073
* -5.0	.2251	.1082	.0626	.0406	.0284	.0210	.0161	.0127	.0103	.0086
* -5.5	.2640	.1286	.0748	.0486	.0341	.0251	.0193	.0153	.0124	.0103
* -6.0	.3043	.1504	.0879	.0573	.0402	.0297	.0228	.0181	.0147	.0121
* -6.5	.3457	.1735	.1020	.0667	.0468	.0346	.0266	.0211	.0171	.0142
* -7.0	.3876	.1977	.1170	.0767	.0539	.0399	.0307	.0244	.0198	.0164
* -7.5	.4313	.2490	.1493	.0985	.0695	.0516	.0398	.0316	.0256	.0212
* -8.0	.4713	.2902	.1845	.1226	.0869	.0646	.0499	.0396	.0322	.0267
* -9.0	.5525	.3032	.1845	.1226	.0869	.0646	.0499	.0396	.0322	.0267
* -10.0	.6285	.3591	.2222	.1488	.1059	.0790	.0611	.0486	.0396	.0323

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 2.33 \frac{x_0 - \mu}{\sigma}$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	* *	* *	60	65	70	75	80	85	90	95	100
0.8	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.6	*	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.4	*	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
0.2	*	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
0.0	*	*	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-0.2	*	*	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-0.4	*	*	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-0.6	*	*	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004
-0.8	*	*	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0005
-1.0	*	*	.0008	.0008	.0008	.0008	.0008	.0008	.0008	.0008	.0008
-1.4	*	*	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012
-1.8	*	*	.0016	.0016	.0016	.0016	.0016	.0016	.0016	.0016	.0016
-2.2	*	*	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022
-2.6	*	*	.0028	.0028	.0028	.0028	.0028	.0028	.0028	.0028	.0028
-3.0	*	*	.0035	.0035	.0035	.0035	.0035	.0035	.0035	.0035	.0035
-3.4	*	*	.0043	.0043	.0043	.0043	.0043	.0043	.0043	.0043	.0043
-3.8	*	*	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.0052
-4.2	*	*	.0061	.0061	.0061	.0061	.0061	.0061	.0061	.0061	.0061
-4.6	*	*	.0072	.0072	.0072	.0072	.0072	.0072	.0072	.0072	.0072
-5.0	*	*	.0086	.0086	.0086	.0086	.0086	.0086	.0086	.0086	.0086
-5.5	*	*	.0102	.0102	.0102	.0102	.0102	.0102	.0102	.0102	.0102
-6.0	*	*	.0119	.0119	.0119	.0119	.0119	.0119	.0119	.0119	.0119
-6.5	*	*	.0138	.0138	.0138	.0138	.0138	.0138	.0138	.0138	.0138
-7.0	*	*	.0179	.0179	.0179	.0179	.0179	.0179	.0179	.0179	.0179
-8.0	*	*	.0225	.0225	.0225	.0225	.0225	.0225	.0225	.0225	.0225
-9.0	*	*	.0277	.0277	.0277	.0277	.0277	.0277	.0277	.0277	.0277
-10.0	*	*	.0277	.0277	.0277	.0277	.0277	.0277	.0277	.0277	.0277

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 3.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* C

A	10	15	20	25	30	35	40	45	50	55
* .8	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .6	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .4	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .2	.0005	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .0	.0008	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* -.2	.0011	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* -.4	.0016	.0005	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000
* -.6	.0022	.0007	.0003	.0001	.0001	.0001	.0000	.0000	.0000	.0000
* -.8	.0030	.0009	.0004	.0002	.0001	.0001	.0000	.0000	.0000	.0000
* -1.0	.0041	.0012	.0005	.0003	.0002	.0001	.0001	.0000	.0000	.0000
* -1.4	.0069	.0021	.0009	.0004	.0003	.0002	.0001	.0001	.0001	.0000
* -1.6	.0111	.0033	.0014	.0007	.0004	.0003	.0002	.0001	.0001	.0001
* -2.2	.0169	.0051	.0022	.0011	.0006	.0004	.0003	.0002	.0001	.0001
* -2.6	.0247	.0075	.0032	.0016	.0009	.0006	.0004	.0003	.0002	.0002
* -3.0	.0349	.0106	.0045	.0023	.0013	.0008	.0006	.0004	.0003	.0002
* -3.4	.0475	.0145	.0062	.0032	.0018	.0012	.0008	.0005	.0004	.0003
* -3.8	.0630	.0193	.0082	.0042	.0024	.0015	.0010	.0007	.0005	.0004
* -4.2	.0815	.0252	.0108	.0055	.0032	.0020	.0014	.0010	.0007	.0005
* -4.6	.1031	.0322	.0138	.0071	.0041	.0026	.0017	.0012	.0009	.0007
* -5.0	.1279	.0404	.0173	.0089	.0052	.0033	.0022	.0015	.0011	.0008
* -5.5	.1634	.0524	.0225	.0116	.0067	.0043	.0029	.0020	.0015	.0011
* -6.0	.2037	.0665	.0287	.0148	.0086	.0054	.0036	.0026	.0019	.0014
* -6.5	.2485	.0828	.0360	.0186	.0108	.0068	.0046	.0032	.0023	.0018
* -7.0	.2973	.1013	.0443	.0230	.0134	.0084	.0057	.0040	.0029	.0022
* -8.0	.4039	.1454	.0645	.0336	.0196	.0124	.0083	.0059	.0043	.0032
* -9.0	.5165	.1985	.0897	.0471	.0276	.0175	.0117	.0083	.0060	.0045
* -10.0	.6269	.2600	.1202	.0636	.0374	.0237	.0160	.0112	.0082	.0062

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 3.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* \* \*

A	60	65	70	75	80	85	90	95	100
* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.8	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-2.2	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-2.6	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.0	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.8	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-4.2	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004
-4.6	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0005
-5.0	.0006	.0006	.0006	.0006	.0006	.0006	.0006	.0006	.0006
-5.5	.0008	.0007	.0007	.0007	.0007	.0007	.0007	.0007	.0007
-6.0	.0011	.0009	.0009	.0009	.0009	.0009	.0009	.0009	.0009
-6.5	.0014	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0011
-7.0	.0017	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013
-8.0	.0025	.0019	.0016	.0013	.0010	.0009	.0007	.0006	.0005
-9.0	.0035	.0027	.0022	.0018	.0015	.0012	.0010	.0009	.0008
-10.0	.0048	.0037	.0030	.0024	.0020	.0017	.0014	.0012	.0010

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull



$$B(X) = 4.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	5	10	15	20	25	30	35	40	45	50	55
0.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.2	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.4	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.6	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.8	.0007	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.0	.0010	.0002	.0000	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.4	.0018	.0004	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.8	.0033	.0006	.0002	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
-2.2	.0055	.0011	.0003	.0003	.0001	.0001	.0000	.0000	.0000	.0000	.0000
-2.6	.0088	.0018	.0006	.0006	.0002	.0001	.0001	.0000	.0000	.0000	.0000
-3.0	.0136	.0027	.0009	.0009	.0004	.0002	.0001	.0001	.0000	.0000	.0000
-3.4	.0201	.0041	.0013	.0013	.0005	.0003	.0001	.0001	.0001	.0000	.0000
-3.8	.0289	.0059	.0019	.0019	.0008	.0004	.0002	.0001	.0001	.0000	.0000
-4.2	.0404	.0082	.0026	.0026	.0011	.0005	.0003	.0002	.0001	.0001	.0000
-4.6	.0551	.0113	.0036	.0036	.0015	.0007	.0004	.0002	.0001	.0001	.0001
-5.0	.0732	.0152	.0048	.0048	.0020	.0010	.0005	.0003	.0002	.0001	.0001
-5.5	.1015	.0214	.0068	.0068	.0028	.0014	.0007	.0004	.0003	.0002	.0001
-6.0	.1366	.0293	.0094	.0094	.0039	.0019	.0010	.0006	.0004	.0002	.0002
-6.5	.1788	.0392	.0126	.0126	.0052	.0025	.0014	.0008	.0005	.0003	.0002
-7.0	.2282	.0515	.0167	.0167	.0069	.0033	.0018	.0011	.0007	.0004	.0003
-8.0	.3466	.0839	.0275	.0275	.0114	.0055	.0030	.0017	.0011	.0007	.0005
-9.0	.4838	.1284	.0429	.0429	.0179	.0087	.0047	.0027	.0017	.0011	.0008
-10.0	.6252	.1862	.0638	.0638	.0267	.0130	.0070	.0041	.0026	.0017	.0012

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 4.00$$

$$C = \frac{\theta - X_0}{\sigma}, \quad A = \frac{X_0 - \mu}{\sigma}$$

\* \* \*

A	60	65	70	75	80	85	90	95	100
* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-2.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-2.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-4.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-4.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-5.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-5.5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-6.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-6.5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-7.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-8.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-9.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-10.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

B(X) = 5.00

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

A	10	15	20	25	30	35	40	45	50	55
0.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.2	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.4	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.6	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-0.8	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.0	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.4	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-1.8	.0010	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-2.2	.0019	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-2.6	.0033	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.0	.0055	.0007	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
-3.4	.0089	.0012	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0137	.0018	.0004	.0001	.0001	.0000	.0000	.0000	.0000	.0000
-4.2	.0206	.0028	.0007	.0002	.0001	.0000	.0000	.0000	.0000	.0000
-4.6	.0300	.0041	.0010	.0003	.0001	.0001	.0000	.0000	.0000	.0000
-5.0	.0426	.0058	.0014	.0005	.0002	.0001	.0000	.0000	.0000	.0000
-5.5	.0639	.0089	.0021	.0007	.0003	.0001	.0001	.0000	.0000	.0000
-6.0	.0924	.0131	.0031	.0010	.0004	.0002	.0001	.0001	.0000	.0000
-6.5	.1295	.0187	.0045	.0015	.0006	.0003	.0001	.0001	.0000	.0000
-7.0	.1761	.0263	.0063	.0021	.0008	.0004	.0002	.0001	.0001	.0000
-8.0	.2988	.0484	.0118	.0039	.0016	.0007	.0004	.0002	.0001	.0001
-9.0	.4546	.0828	.0205	.0068	.0027	.0013	.0006	.0004	.0002	.0001
-10.0	.6235	.1328	.0337	.0112	.0045	.0021	.0011	.0006	.0004	.0002

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 5.00 \quad C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$$F(x) = 0 \quad \text{when } x > 70$$

A		60	65	70
	*****			
.8	*	.0000	.0000	.0000
.6	*	.0000	.0000	.0000
.4	*	.0000	.0000	.0000
.2	*	.0000	.0000	.0000
.0	*	.0000	.0000	.0000
-.2	*	.0000	-.0000	-.0000
-.4	*	-.0000	-.0000	-.0000
-.6	*	-.0000	-.0000	-.0000
-.8	*	-.0000	-.0000	-.0000
-1.0	*	-.0000	-.0000	-.0000
-1.4	*	.0000	.0000	-.0000
-1.8	*	.0000	.0000	.0000
-2.2	*	.0000	.0000	.0000
-2.6	*	.0000	.0000	.0000
-3.0	*	.0000	.0000	.0000
-3.4	*	.0000	.0000	.0000
-3.8	*	.0000	.0000	.0000
-4.2	*	.0000	.0000	.0000
-4.6	*	.0000	.0000	.0000
-5.0	*	.0000	.0000	.0000
-5.5	*	.0000	.0000	.0000
-6.0	*	.0000	.0000	.0000
-6.5	*	.0000	.0000	.0000
-7.0	*	.0000	.0000	.0000
-8.0	*	.0000	.0000	.0000
-9.0	*	.0001	.0001	.0000
-10.0	*	.0001	.0001	.0001

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

B(X) = 6.00

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$F(x) = 0$  when  $c > 50$

\* C

A	5	10	15	20	25	30	35	40	45	50
* .8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.4	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.8	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -2.2	.0007	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -2.6	.0013	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.0	.0023	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.4	.0040	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.8	.0067	.0006	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -4.2	.0108	.0010	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -4.6	.0168	.0015	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -5.0	.0253	.0023	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -5.5	.0408	.0037	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -6.0	.0634	.0059	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -6.5	.0949	.0091	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -7.0	.1371	.0135	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -8.0	.2591	.0280	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -9.0	.4286	.0534	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -10.0	.6215	.0947	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$B(X) = 7.00$

$C = \frac{\theta - x_0}{\sigma}, A = \frac{x_0 - \mu}{\sigma}$

$F(x) = 0$  when  $c > 40$

A	10	15	20	25	30	35	40
* .8	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .6	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .4	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .2	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -1.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -2.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -2.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.4	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -4.2	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -4.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -5.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -5.5	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -6.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -6.5	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -7.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -8.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -9.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* -10.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(X) = 8.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* C F(x) = 0 when c > 35 \* > 35

A	*	10	15	20	25	30	35
*****							
.8	*	.0000	.0000	.0000	.0000	.0000	.0000
.6	*	.0000	.0000	.0000	.0000	.0000	.0000
.4	*	.0000	.0000	.0000	.0000	.0000	.0000
.2	*	.0000	.0000	.0000	.0000	.0000	.0000
.0	*	.0000	.0000	.0000	.0000	.0000	.0000
-.2	*	.0000	.0000	.0000	.0000	.0000	.0000
-.4	*	.0000	.0000	.0000	.0000	.0000	.0000
-.6	*	.0000	.0000	.0000	.0000	.0000	.0000
-.8	*	.0000	.0000	.0000	.0000	.0000	.0000
-1.0	*	.0000	.0000	.0000	.0000	.0000	.0000
-1.4	*	.0000	.0000	.0000	.0000	.0000	.0000
-1.8	*	.0000	.0000	.0000	.0000	.0000	.0000
-2.2	*	.0001	.0000	.0000	.0000	.0000	.0000
-2.6	*	.0002	.0000	.0000	.0000	.0000	.0000
-3.0	*	.0005	.0000	.0000	.0000	.0000	.0000
-3.4	*	.0009	.0000	.0000	.0000	.0000	.0000
-3.8	*	.0018	.0001	.0000	.0000	.0000	.0000
-4.2	*	.0032	.0001	.0000	.0000	.0000	.0000
-4.6	*	.0056	.0002	.0000	.0000	.0000	.0000
-5.0	*	.0095	.0004	.0000	.0000	.0000	.0000
-5.5	*	.0176	.0007	.0001	.0000	.0000	.0000
-6.0	*	.0312	.0013	.0001	.0000	.0000	.0000
-6.5	*	.0528	.0022	.0002	.0000	.0000	.0000
-7.0	*	.0858	.0037	.0004	.0001	.0000	.0000
-8.0	*	.1989	.0097	.0010	.0002	.0000	.0000
-9.0	*	.3851	.0227	.0023	.0004	.0001	.0000
-10.0	*	.6168	.0487	.0051	.0009	.0002	.0001

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

$$B(x) = 9.00$$

$$C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

$$F(x) = 0 \quad \text{when } c > 30$$

A	*	10	15	20	25	30
*****						
.8	*	.0000	.0000	.0000	.0000	.0000
.6	*	.0000	.0000	.0000	.0000	.0000
.4	*	.0000	.0000	.0000	.0000	.0000
.2	*	.0000	.0000	.0000	.0000	.0000
.0	*	.0000	.0000	.0000	.0000	.0000
-.2	*	.0000	.0000	.0000	.0000	.0000
-.4	*	.0000	.0000	.0000	.0000	.0000
-.6	*	.0000	.0000	.0000	.0000	.0000
-.8	*	.0000	.0000	.0000	.0000	.0000
-1.0	*	.0000	.0000	.0000	-.0000	-.0000
-1.4	*	.0000	.0000	.0000	.0000	.0000
-1.8	*	.0000	.0000	.0000	.0000	.0000
-2.2	*	.0000	.0000	.0000	.0000	.0000
-2.6	*	.0001	.0000	.0000	.0000	.0000
-3.0	*	.0002	.0000	.0000	.0000	.0000
-3.4	*	.0005	.0000	.0000	.0000	.0000
-3.8	*	.0009	.0000	.0000	.0000	.0000
-4.2	*	.0018	.0000	.0000	.0000	.0000
-4.6	*	.0034	.0001	.0000	.0000	.0000
-5.0	*	.0060	.0002	.0000	.0000	.0000
-5.5	*	.0119	.0003	.0000	.0000	.0000
-6.0	*	.0223	.0006	.0000	.0000	.0000
-6.5	*	.0402	.0011	.0001	.0000	.0000
-7.0	*	.0690	.0020	.0002	.0000	.0000
-8.0	*	.1762	.0058	.0004	.0001	.0000
-9.0	*	.3671	.0149	.0011	.0002	.0000
-10.0	*	.6142	.0352	.0027	.0004	.0001

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull



$$B(x) = 10.00 \quad C = \frac{\theta - x_0}{\sigma}, \quad A = \frac{x_0 - \mu}{\sigma}$$

\* C F(x) = 0 when c > 25

A		10	15	20	25
*****					
.8	*	.0000	.0000	.0000	.0000
.6	*	.0000	.0000	.0000	.0000
.4	*	.0000	.0000	.0000	.0000
.2	*	.0000	.0000	.0000	.0000
.0	*	.0000	.0000	.0000	.0000
-.2	*	.0000	.0000	.0000	.0000
-.4	*	.0000	.0000	.0000	.0000
-.6	*	.0000	.0000	.0000	.0000
-.8	*	.0000	.0000	.0000	.0000
-1.0	*	.0000	.0000	.0000	.0000
-1.4	*	.0000	.0000	.0000	.0000
-1.8	*	.0000	.0000	.0000	.0000
-2.2	*	.0000	.0000	.0000	.0000
-2.6	*	.0000	.0000	.0000	.0000
-3.0	*	.0001	.0000	.0000	.0000
-3.4	*	.0002	.0000	.0000	.0000
-3.8	*	.0005	.0000	.0000	.0000
-4.2	*	.0011	.0000	.0000	.0000
-4.6	*	.0021	.0000	.0000	.0000
-5.0	*	.0039	.0001	.0000	.0000
-5.5	*	.0081	.0001	.0000	.0000
-6.0	*	.0163	.0003	.0000	.0000
-6.5	*	.0310	.0006	.0000	.0000
-7.0	*	.0561	.0011	.0001	.0000
-8.0	*	.1573	.0035	.0002	.0000
-9.0	*	.3511	.0099	.0006	.0001
-10.0	*	.6114	.0256	.0015	.0002

STRESS DISTRIBUTION - Normal  
STRENGTH DISTRIBUTION - Weibull

A-2.2 STRESS DISTRIBUTION - WEIBULL  
STRENGTH DISTRIBUTION - WEIBULL

(XC - Y0)/THEIA1 = .000  
THEIA2/THEIA1 = 1.000

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

\* \* 61

81/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
.1	.6110									
.2	.5917	.5917	.5917							
.3	.5742	.5742	.5742	.5742	.5742					
.4	.5589	.5589	.5589	.5589	.5589	.5589				
.5	.5456	.5456	.5456	.5456	.5456	.5456	.5456	.5456	.5456	
.6	.5341	.5341	.5341	.5341	.5341	.5341	.5341	.5341	.5341	.5341
.7	.5240	.5240	.5240	.5240	.5240	.5240	.5240	.5240	.5240	.5240
.8	.5150	.5150	.5150	.5150	.5150	.5150	.5150	.5150	.5150	.5150
.9	.5071	.5071	.5071	.5071	.5071	.5071	.5071	.5071	.5071	.5071
1.0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
2.0			.4544	.4544	.4544	.4544	.4544	.4544	.4544	.4544
3.0					.4311	.4311	.4311	.4311	.4311	.4311
4.0						.4173	.4173	.4173	.4173	.4173
5.0									.4083	.4083

STRESS DISTRIBUTION - Wedball  
STRENGTH DISTRIBUTION - Wedball

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = 1.000

THETA 1 = 0 - X  
THETA 2 = 0 - Y

\* \* B1

B1/82	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	.5341								
.7	.5240	.5240	.5240						
.8	.5150	.5150	.5150	.5150	.5150				
.9	.5071	.5071	.5071	.5071	.5071	.5071	.5071		
1.0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
2.0	.4544	.4544	.4544	.4544	.4544	.4544	.4544	.4544	.4544
3.0	.4311	.4311	.4311	.4311	.4311	.4311	.4311	.4311	.4311
4.0	.4173	.4173	.4173	.4173	.4173	.4173	.4173	.4173	.4173
5.0	.4083	.4083	.4083	.4083	.4083	.4083	.4083	.4083	.4083
6.0	.4020	.4020	.4020	.4020	.4020	.4020	.4020	.4020	.4020
7.0		.3974	.3974	.3974	.3974	.3974	.3974	.3974	.3974
8.0			.3939	.3939	.3939	.3939	.3939	.3939	.3939
9.0					.3911	.3911	.3911	.3911	.3911
10.0									.3886

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

$$\begin{aligned} \text{THETA } 1 &= \theta - \Sigma \theta_i \\ \text{THETA } 2 &= \theta - Y_0 \end{aligned}$$
[illegible]

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NEW YORK 17, N. Y.**

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = .000

THETA 1 =  $\theta - x_0$   
THETA 2 =  $\theta - y_0$

\* \* B1

B1/B2 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6 *	.2003								
.7 *	.2013	.1831	.1663						
.8 *	.2027	.1846	.1679	.1525	.1363				
.9 *	.2049	.1870	.1703	.1549	.1407	.1275	.1155		
1.0 *	.2077	.1899	.1734	.1579	.1437	.1305	.1183	.1072	.0970
2.0 *	.2487	.2334	.2187	.2044	.1907	.1776	.1650	.1530	.1416
3.0 *	.2805	.2686	.2568	.2453	.2339	.2227	.2118	.2012	.1908
4.0 *	.3003	.2908	.2813	.2720	.2627	.2536	.2445	.2356	.2267
5.0 *	.3130	.3052	.2974	.2897	.2820	.2744	.2668	.2592	.2516
6.0 *	.3218	.3152	.3086	.3021	.2955	.2890	.2825	.2761	.2697
7.0 *			.3168	.3111	.3055	.2998	.2942	.2885	.2830
8.0 *					.3130	.3080	.3031	.2981	.2931
9.0 *							.3101	.3056	.3012
10.0 *									.3077

STRESS DISTRIBUTION - Wedbells  
STRAIN DISTRIBUTION - Wedbells

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = .667

THETA 1 =  $\theta_x - X_0$   
THETA 2 =  $\theta_y - Y_0$

\* 81

B1/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	* .4681									
* .2	* .4524	* .3893	* .3321							
* .3	* .4395	* .3784	* .3231	* .2738	* .2305					
* .4	* .4291	* .3701	* .3184	* .2685	* .2264	* .1898	* .1584			
* .5	* .4208	* .3638	* .3117	* .2651	* .2239	* .1880	* .1571	* .1308	* .1085	
* .6	* .4143	* .3593	* .3087	* .2632	* .2228	* .1875	* .1570	* .1309	* .1087	* .0900
* .7	* .4093	* .3562	* .3071	* .2626	* .2229	* .1881	* .1578	* .1318	* .1097	* .0939
* .8	* .4054	* .3542	* .3065	* .2630	* .2240	* .1896	* .1595	* .1335	* .1113	* .0925
* .9	* .4024	* .3530	* .3068	* .2643	* .2260	* .1919	* .1619	* .1359	* .1136	* .0945
* 1.0	* .4000	* .3525	* .3077	* .2663	* .2286	* .1948	* .1649	* .1389	* .1164	* .0971
* 2.0			* .3261	* .2955	* .2659	* .2377	* .2110	* .1860	* .1629	* .1417
* 3.0					* .2938	* .2719	* .2506	* .2299	* .2100	* .1909
* 4.0							* .2763	* .2595	* .2430	* .2269
* 5.0									* .2655	* .2519

STRESS DISTRIBUTION - Weibull  
STRESS DISTRIBUTION - Weibull

(X2 - VO)/THETA1 = .000  
THETA2/THETA1 = .667

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* 81										
81/82	*									
.6	*	.0744								
.7	*	.0752	.0021	.0511						
.8	*	.0766	.0633	.0522	.0429	.0353				
.9	*	.0784	.0649	.0536	.0441	.0363	.0298	.0245		
1.0	*	.0807	.0669	.0553	.0456	.0376	.0309	.0254	.0208	.0171
2.0	*	.1225	.1052	.0899	.0764	.0645	.0543	.0455	.0380	.0316
3.0	*	.1727	.1554	.1392	.1240	.1100	.0970	.0851	.0742	.0645
4.0	*	.2112	.1959	.1812	.1671	.1535	.1405	.1281	.1164	.1054
5.0	*	.2385	.2254	.2125	.1999	.1877	.1758	.1642	.1531	.1423
6.0	*	.2582	.2468	.2356	.2245	.2136	.2030	.1925	.1823	.1723
7.0	*			.2530	.2432	.2335	.2240	.2146	.2054	.1963
8.0	*					.2491	.2406	.2321	.2237	.2154
9.0	*							.2461	.2385	.2309
10.0	*									.2437

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/THETA1 = -.000  
 THETA2/THETA1 = .571

THETA 1 =  $\theta - X_0$   
 THETA 2 =  $\theta - Y_0$

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$\text{THETA 1} = \theta - x_0$   
 $\text{THETA 2} = \theta - y_0$

$(X0 - Y0)/\text{THETA1} = .000$   
 $\text{THETA2}/\text{THETA1} = .571$

* B1	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
61/B2 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	.3304								
.7	.0309	.0235	.0178						
.8	.0316	.0240	.0182	.0138	.0105				
.9	.0325	.0247	.0188	.0143	.0108	.0082	.0063		
1.0	.0336	.0256	.0195	.0148	.0113	.0085	.0065	.0049	.0037
2.0	.0586	.0460	.0358	.0277	.0213	.0164	.0125	.0096	.0073
3.0	.1025	.0857	.0710	.0582	.0473	.0381	.0304	.0241	.0189
4.0	.1461	.1286	.1128	.0980	.0846	.0724	.0615	.0518	.0433
5.0	.1809	.1649	.1496	.1350	.1212	.1083	.0962	.0850	.0747
6.0	.2076	.1931	.1791	.1655	.1524	.1399	.1279	.1165	.1057
7.0			.2024	.1900	.1779	.1661	.1547	.1438	.1332
8.0					.1985	.1877	.1770	.1667	.1566
9.0						.1956	.1859	.1765	
10.0									.1932

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - YD)/THEIA1 = .039  
THEIA2/THEIA1 = .500

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

\* B1  
\*

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
1	.3775									
2	.3645	.2752	.2040							
3	.3545	.2675	.1988	.1455	.1055					
4	.3469	.2628	.1954	.1433	.1040	.0749	.0537			
5	.3414	.2595	.1935	.1422	.1034	.0746	.0535	.0382	.0272	
6	.3375	.2577	.1929	.1422	.1036	.0749	.0538	.0384	.0274	.0195
7	.3351	.2572	.1934	.1431	.1045	.0757	.0544	.0390	.0278	.0198
8	.3337	.2578	.1948	.1448	.1062	.0771	.0555	.0398	.0284	.0202
9	.3332	.2592	.1971	.1472	.1084	.0789	.0570	.0409	.0293	.0209
1.0	.3333	.2612	.2000	.1502	.1111	.0812	.0588	.0423	.0303	.0216
2.0		.2421	.1971	.1573	.1231	.0947	.0716	.0534	.0393	
3.0				.2050	.1733	.1443	.1185	.0958	.0763	
4.0						.1859	.1518	.1393	.1187	
5.0								.1747	.1552	

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

(X2 - Y2)/IMEIA1 = .000  
 IMEIA2/IMEIA1 = .500

* * H1		*****									
B1/B2	*	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	*****
.6	*	.0138									
.7	*	.0140	.0100	.0071							
.8	*	.0144	.0102	.0073	.0052	.0037					
.9	*	.0148	.0105	.0075	.0053	.0038	.0027	.0019			
1.0	*	.0154	.0109	.0078	.0055	.0039	.0028	.0020	.0014	.0010	
2.0	*	.0267	.0208	.0150	.0107	.0076	.0054	.0039	.0028	.0020	
3.0	*	.0598	.0463	.0353	.0266	.0198	.0146	.0107	.0077	.0056	
4.0	*	.1000	.0833	.0685	.0557	.0448	.0355	.0278	.0216	.0165	
5.0	*	.1369	.1199	.1041	.0890	.0765	.0647	.0542	.0450	.0370	
6.0	*	.1673	.1511	.1357	.1212	.1076	.0949	.0832	.0724	.0626	
7.0	*			.1622	.1483	.1351	.1225	.1105	.0993	.0887	
8.0	*					.1584	.1463	.1347	.1235	.1129	
9.0	*							.1550	.1448	.1345	
10.0	*									.1533	

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = .COC  
THEIA2/THEIA1 = .444

THETA 1 =  $\theta - x_0$   
THETA 2 =  $\theta - y_0$

\* B1

B1/62	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	* .3439									
* .2	* .3221	.2367	.1651							
* .3	* .3231	.2305	.1610	.1108	.0755					
* .4	* .3164	.2264	.1584	.1092	.0744	.0504	.0339			
* .5	* .3117	.2239	.1571	.1085	.0741	.0502	.0338	.0227	.0152	
* .6	* .3087	.2228	.1570	.1087	.0744	.0505	.0340	.0229	.0153	.0103
* .7	* .3071	.2229	.1578	.1097	.0752	.0511	.0345	.0232	.0156	.0104
* .8	* .3065	.2240	.1595	.1113	.0766	.0522	.0353	.0237	.0159	.0107
* .9	* .3068	.2260	.1619	.1136	.0784	.0536	.0363	.0245	.0164	.0110
1.0	* .3077	.2286	.1649	.1164	.0807	.0553	.0376	.0254	.0171	.0114
2.0			.2110	.1629	.1225	.0899	.0645	.0455	.0316	.0217
3.0					.1727	.1392	.1100	.0851	.0645	.0479
4.0							.1535	.1281	.1054	.0853
5.0								.1423		.1220

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = .444

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\* \* b1

B1/B2	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* .6	.0070								
* .7	.0070	.0047	.0031						
* .8	.0072	.0048	.0032	.0021	.0014				
* .9	.0074	.0050	.0033	.0022	.0015	.0010	.0007		
* 1.0	.0077	.0052	.0034	.0023	.0015	.0010	.0007	.0005	.0003
* 2.0	.0148	.0100	.0067	.0045	.0030	.0020	.0014	.0009	.0006
* 3.0	.0349	.0250	.0176	.0123	.0085	.0058	.0039	.0027	.0018
* 4.0	.0680	.0533	.0411	.0311	.0232	.0170	.0123	.0088	.0062
* 5.0	.1035	.0868	.0719	.0588	.0475	.0378	.0297	.0230	.0174
* 6.0	.1351	.1183	.1027	.0884	.0754	.0637	.0532	.0441	.0361
* 7.0			.1302	.1158	.1024	.0899	.0783	.0678	.0582
* 8.0					.1266	.1141	.1022	.0912	.0808
* 9.0							.1239	.1128	.1023
* 10.0									.1218

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - F bull

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = .400

THETA 1 =  $\theta_x - X_0$   
THETA 2 =  $\theta_y - Y_0$

\* B1

81/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.3150									
.2	.3049	.2061	.1361							
.3	.2967	.2009	.1328	.0864	.0557					
.4	.2908	.1974	.1308	.0852	.0549	.0352	.0224			
.5	.2868	.1955	.1299	.0848	.0547	.0351	.0223	.0142	.0091	
.6	.2844	.1948	.1295	.0851	.0550	.0353	.0225	.0143	.0091	.0059
.7	.2834	.1953	.1309	.0859	.0557	.0358	.0229	.0145	.0093	.0059
.8	.2834	.1968	.1326	.0874	.0568	.0366	.0234	.0149	.0095	.0061
.9	.2842	.1990	.1349	.0894	.0583	.0376	.0241	.0154	.0098	.0062
1.0	.2857	.2019	.1379	.0919	.0602	.0389	.0250	.0159	.0102	.0065
2.0			.1851	.1358	.0964	.0666	.0449	.0297	.0193	.0125
3.0					.1462	.1124	.0842	.0614	.0437	.0303
4.0							.1272	.1018	.0799	.0614
5.0									.1163	.0961

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = .000  
THEIA2/THEIA1 = .400

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* 81										
01/02	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
* .6	* .0037									
* .7	* .0038	* .0024	* .0015							
* .8	* .0039	* .0024	* .0015	* .0010	* .0006					
* .9	* .0040	* .0025	* .0016	* .0010	* .0006	* .0004	* .0003			
* 1.0	* .0041	* .0026	* .0016	* .0010	* .0007	* .0004	* .0003	* .0002	* .0001	
* 2.0	* .0080	* .0051	* .0033	* .0021	* .0013	* .0008	* .0005	* .0003	* .0002	
* 3.0	* .0206	* .0138	* .0091	* .0059	* .0038	* .0024	* .0016	* .0010	* .0006	
* 4.0	* .0462	* .0340	* .0245	* .0173	* .0120	* .0082	* .0055	* .0036	* .0024	
* 5.0	* .0782	* .0627	* .0454	* .0383	* .0291	* .0218	* .0160	* .0115	* .0082	
* 6.0	* .1094	* .0927	* .0776	* .0642	* .0525	* .0423	* .0336	* .0264	* .0204	
* 7.0			* .1047	* .0905	* .0775	* .0657	* .0552	* .0459	* .0377	
* 8.0					* .1013	* .0889	* .0775	* .0670	* .0575	
* 9.0							* .0988	* .0878	* .0776	
* 10.0									* .0968	

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/TNETA1 = .000  
TNETA2/TNETA1 = .364

THETA 1 = 0 - X<sub>0</sub>  
THETA 2 = 0 - Y<sub>0</sub>

\* 81

BL/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.2518									
.2	.2817	.1815	.1140							
.3	.2742	.1769	.1112	.0688	.0421					
.4	.2689	.1740	.1090	.0679	.0416	.0253	.0152			
.5	.2655	.1724	.1089	.0676	.0415	.0253	.0153	.0093	.0057	
.6	.2636	.1721	.1091	.0679	.0417	.0254	.0154	.0094	.0058	.0034
.7	.2630	.1728	.1101	.0687	.0423	.0258	.0157	.0095	.0058	.0035
.8	.2634	.1744	.1117	.0700	.0432	.0264	.0161	.0098	.0063	.0036
.9	.2647	.1768	.1140	.0717	.0444	.0272	.0166	.0101	.0061	.0037
1.0	.2667	.1798	.1168	.0738	.0459	.0282	.0172	.0105	.0064	.0039
2.0			.1633	.1141	.0767	.0500	.0318	.0199	.0123	.0075
3.0					.1245	.0913	.0648	.0446	.0299	.0195
4.0							.1058	.0812	.0607	.0442
5.0									.0953	.0758

321

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .000  
THETA2/THETA1 = .364

THETA 1 =  $\theta_x - \theta_0$   
THETA 2 =  $\theta_y - \theta_0$

* * B1 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
B1/B2 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
* .6	* .0021								
* .7	* .0021	* .0013	* .0008						
* .8	* .0022	* .0013	* .0008	* .0005	* .0003				
* .9	* .0022	* .0013	* .0008	* .0005	* .0003	* .0002	* .0001		
1.0	* .0023	* .0014	* .0008	* .0005	* .0003	* .0002	* .0001	* .0001	* .0000
2.0	* .0046	* .0028	* .0017	* .0010	* .0006	* .0004	* .0002	* .0001	* .0001
3.0	* .0124	* .0078	* .0046	* .0030	* .0018	* .0011	* .0007	* .0004	* .0003
4.0	* .0314	* .0217	* .0147	* .0097	* .0063	* .0040	* .0025	* .0016	* .0010
5.0	* .0592	* .0452	* .0339	* .0248	* .0178	* .0125	* .0085	* .0057	* .0038
6.0	* .0888	* .0727	* .0587	* .0466	* .0364	* .0279	* .0211	* .0156	* .0113
7.0	*	*	* .0844	* .0707	* .0586	* .0479	* .0387	* .0308	* .0242
8.0	*	*	*	*	* .0812	* .0694	* .0587	* .0491	* .0407
9.0	*	*	*	*	*	*	* .0789	* .0684	* .0589
10.0	*	*	*	*	*	*	*	*	* .0771

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

IXO - VOI/THETA1 = .000  
 THETA2/THETA1 = .333

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

01/02	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.2712									
.2	.2618	.1613	.0967							
.3	.2549	.1573	.0944	.0556	.0326					
.4	.2501	.1547	.0931	.0556	.0322	.0187	.0138			
.5	.2471	.1535	.0926	.0549	.0321	.0187	.0108	.0365	.0336	
.6	.2450	.1534	.0928	.0551	.0323	.0186	.0119	.0365	.0337	.0321
.7	.2434	.1542	.0938	.0550	.0328	.0191	.0111	.0365	.0338	.0322
.8	.2401	.1559	.0953	.0569	.0335	.0196	.0114	.0367	.0339	.0322
.9	.2477	.1583	.0974	.0584	.0345	.0202	.0116	.0368	.0340	.0323
1.0	.2500	.1614	.1000	.0603	.0357	.0209	.0122	.0371	.0341	.0324
2.0			.1450	.0966	.0618	.0382	.0231	.0137	.0080	.0047
3.0					.1365	.0746	.0503	.0328	.0207	.0127
4.0							.0884	.0650	.0463	.0320
5.0									.0784	.0630

2-138 DISTRIBUTION - Redball  
 STRENGTH DISTRIBUTION - Redball

$(X_0 - Y_0)/\theta_1 = .000$   
 $\theta_2/\theta_1 = .233$

$\theta_1 = \theta - X_0$   
 $\theta_2 = \theta - Y_0$

* B1		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
B1/B2	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0012								
.7	*	.0012	.0007	.0004						
.8	*	.0013	.0007	.0004	.0002	.0001				
.9	*	.0013	.0008	.0004	.0003	.0001	.0001	.0001		
1.0	*	.0014	.0008	.0005	.0003	.0002	.0001	.0001	.0000	.0000
2.0	*	.0027	.0016	.0009	.0005	.0003	.0002	.0001	.0001	.0001
3.0	*	.0077	.0046	.0027	.0016	.0009	.0005	.0003	.0002	.0001
4.0	*	.0215	.0140	.0089	.0055	.0033	.0020	.0012	.0007	.0004
5.0	*	.0448	.0327	.0232	.0161	.0108	.0071	.0044	.0029	.0018
6.0	*	.0722	.0571	.0443	.0338	.0252	.0164	.0131	.0092	.0062
7.0	*			.0681	.0553	.0443	.0349	.0276	.0206	.0154
8.0	*					.0652	.0541	.0444	.0367	.0287
9.0	*							.0631	.0533	.0446
10.0	*									.0614

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(MO - VOF)/TOTAL = .250  
 (MO - VOF)/TOTAL = 1.000

TAREA 1 - 0 - X  
 TAREA 2 - 0 - Y

01/02	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
01	.5007									
02	.4757	.4321	.3853							
03	.4535	.4151	.3754	.3373	.3015					
04	.4303	.4021	.3652	.3292	.2952	.2635	.2343			
05	.4220	.3922	.3581	.3241	.2916	.2611	.2328	.2068	.1832	
06	.4114	.3800	.3454	.3114	.2801	.2607	.2332	.2078	.1846	.1634
07	.4003	.3679	.3304	.3000	.2903	.2619	.2351	.2103	.1873	.1654
08	.3974	.3602	.3208	.3000	.2915	.2641	.2382	.2138	.1912	.1704
09	.3924	.3536	.3140	.3000	.2936	.2672	.2420	.2181	.1959	.1752
1.0	.3894	.3478	.3048	.3000	.2961	.2707	.2463	.2230	.2011	.1807
2.0		.3539	.3062	.3000	.3217	.3049	.2880	.2713	.2547	.2385
3.0					.3357	.3238	.3117	.2996	.2874	.2753
4.0							.3250	.3156	.3062	.2967
5.0								.3180	.3103	

STRESS DISTRIBUTION - Wellbore  
 STRESS DISTRIBUTION - Wellbore

(X0 - Y0)/THETA1 = .250  
THETA2/THETA1 = 1.000

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
B1/B2 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6 *	.1443								
.7 *	.1473	.1301	.1146						
.8 *	.1514	.1341	.1184	.1044	.0918				
.9 *	.1562	.1388	.1231	.1088	.0959	.0844	.0742		
1.0 *	.1617	.1443	.1283	.1136	.1007	.0889	.0783	.0688	.0604
2.0 *	.2227	.2073	.1925	.1782	.1644	.1513	.1389	.1271	.1159
3.0 *	.2632	.2513	.2396	.2280	.2167	.2055	.1946	.1840	.1736
4.0 *	.2872	.2778	.2685	.2592	.2500	.2409	.2318	.2229	.2141
5.0 *	.3026	.2949	.2872	.2795	.2719	.2643	.2567	.2493	.2418
6.0 *	.3131	.3066	.3001	.2936	.2871	.2807	.2742	.2678	.2614
7.0 *			.3095	.3039	.2983	.2927	.2871	.2815	.2759
8.0 *					.3067	.3018	.2969	.2919	.2870
9.0 *							.3046	.3002	.2958
10.0 *									.3028

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

$\theta = \theta - \theta_0$   
 $\theta = \theta - \theta_0$

$(X_0 - Y_0)/\theta = .253$   
 $\theta = .600$

B1/B2	*****										*****									
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	*****									
.1	.3570										*****									
.2	.3652	.2468	.2333								*****									
.3	.3572	.2875	.2258	.1747	.1337						*****									
.4	.3440	.2802	.2217	.1724	.1325	.1009	.0764				*****									
.5	.3351	.2761	.2202	.1724	.1332	.1018	.0773	.0583	.0438		*****									
.6	.3295	.2744	.2209	.1742	.1353	.1040	.0792	.0600	.0452	.0339	*****									
.7	.3203	.2746	.2231	.1773	.1367	.1072	.0821	.0624	.0471	.0355	*****									
.8	.3241	.2761	.2265	.1816	.1432	.1114	.0856	.0655	.0496	.0375	*****									
.9	.3247	.2785	.2300	.1867	.1484	.1164	.0902	.0692	.0527	.0399	*****									
1.0	.3202	.2614	.2353	.1923	.1543	.1220	.0952	.0735	.0563	.0428	*****									
2.0			.2803	.2467	.2178	.1883	.1609	.1358	.1132	.0933	*****									
3.0					.2596	.2365	.2140	.1923	.1716	.1520	*****									
4.0							.2479	.2300	.2126	.1956	*****									
5.0									.2405	.2200	*****									

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - YC)/THETA1 = .250  
 THETA2/THETA1 = .800

THETA 1 =  $\frac{9 - X}{y}$   
 THETA 2 =  $\frac{9 - Y}{y}$

* B1	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
81/82 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
* .6	.0254								
* .7	.0266	.0199	.0148						
* .8	.0282	.0211	.0158	.0118	.0088				
* .9	.0301	.0226	.0169	.0127	.0094	.0070	.0053		
* 1.0	.0324	.0244	.0183	.0137	.0103	.0077	.0057	.0043	.0032
* 2.0	.0760	.0612	.0488	.0385	.0301	.0234	.0180	.0138	.0105
* 3.0	.1336	.1165	.1008	.0865	.0736	.0620	.0519	.0430	.0354
* 4.0	.1791	.1633	.1482	.1338	.1202	.1074	.0955	.0844	.0742
* 5.0	.2118	.1979	.1843	.1712	.1585	.1463	.1345	.1233	.1126
* 6.0	.2355	.2233	.2113	.1996	.1881	.1769	.1660	.1554	.1451
* 7.0			.2318	.2213	.2110	.2009	.1909	.1811	.1716
* 8.0					.2290	.2198	.2108	.2018	.1931
* 9.0							.2269	.2187	.2106
* 10.0									.2251

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



IXC - Y01/THETA1 = .250  
 THETA2/THETA1 = .667

THETA 1 =  $\theta - \frac{x}{y}$   
 THETA 2 =  $\theta - \frac{y}{x}$

		*****										*****									
		1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5										
81/82	*	*****																			
.1	*	.3172																			
.2	*	.2972	.2077	.1409																	
.3	*	.2825	.2003	.1370	.0918	.0607															
.4	*	.2732	.1966	.1357	.0915	.0603	.0400	.0261													
.5	*	.2683	.1958	.1360	.0928	.0619	.0409	.0268	.0175	.0114											
.6	*	.2665	.1972	.1391	.0953	.0640	.0425	.0279	.0183	.0119	.0078										
.7	*	.2669	.2003	.1430	.0989	.0670	.0447	.0295	.0193	.0126	.0092										
.8	*	.2688	.2045	.1479	.1034	.0706	.0474	.0315	.0207	.0136	.0099										
.9	*	.2710	.2095	.1536	.1088	.0750	.0508	.0339	.0224	.0147	.0096										
1.0	*	.2749	.2150	.1599	.1147	.0800	.0546	.0367	.0244	.0161	.0106										
2.0	*			.2241	.1833	.1462	.1137	.0867	.0640	.0464	.0330										
3.0	*					.2019	.1726	.1454	.1205	.0984	.0790										
4.0	*							.1899	.1673	.1461	.1262										
5.0	*									.1825	.1642										

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - Y0)/THETA1 = .250  
THETA2/THETA1 = .667

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

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(X0 - Y0)/THEIA1 = .250  
THEIA2/THEIA1 = .571

THETA 1 =  $\theta_x - X_0$   
THETA 2 =  $\theta_y - Y_0$

		*****										*****										*****									
		*****										*****										*****									
B1/32	*	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5																				
.1	*	.2528																													
.2	*	.2353	.1448	.0859																											
.3	*	.2298	.1402	.0839	.0491	.0284																									
.4	*	.2179	.1388	.0840	.0495	.0287	.0165	.0094																							
.5	*	.2162	.1400	.0857	.0509	.0296	.0171	.0098	.0056	.0032																					
.6	*	.2173	.1430	.0887	.0531	.0312	.0181	.0104	.0060	.0034	.0019																				
.7	*	.2203	.1475	.0928	.0562	.0332	.0193	.0112	.0064	.0037	.0021																				
.8	*	.2245	.1530	.0977	.0599	.0357	.0209	.0121	.0070	.0040	.0023																				
.9	*	.2294	.1592	.1035	.0643	.0388	.0229	.0133	.0077	.0045	.0026																				
1.0	*	.2343	.1660	.1098	.0693	.0423	.0252	.0148	.0086	.0050	.0029																				
2.0	*			.1607	.1258	.0981	.0682	.0457	.0297	.0187	.0115																				
3.0	*					.1580	.1261	.0982	.0745	.0550	.0395																				
4.0	*							.1460	.1210	.0997	.0803																				
5.0	*										.1368	.1172																			

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X3 - Y31/INITIAL = .250  
INITIAL2/INITIAL = .571

THETA 1 = 9 - X<sup>0</sup>  
THETA 2 = 9 - Y<sup>0</sup>

\* \* 31

BI/82	* 8.0	* 7.5	* 7.0	* 6.5	* 6.0	* 5.5	* 5.0	* 4.5	* 4.0	* 3.5	* 3.0	* 2.5	* 2.0	* 1.5	* 1.0
.6	.0011														
.7	.0012	.0007	.0004												
.8	.0013	.0007	.0004	.0002	.0001										
.9	.0015	.0008	.0005	.0003	.0002	.0001									
1.0	.0016	.0009	.0005	.0003	.0002	.0001	.0001								
2.0	.0070	.0042	.0025	.0015	.0009	.0005	.0003	.0002	.0001						
3.0	.0276	.0168	.0124	.0081	.0051	.0032	.0020	.0012	.0007	.0004					
4.0	.0635	.0492	.0374	.0278	.0203	.0145	.0091	.0064	.0046						
5.0	.1011	.0848	.0702	.0574	.0462	.0367	.0286	.0220	.0166	.0115					
6.0	.1340	.1176	.1024	.0883	.0755	.0644	.0536	.0444	.0364	.0297	.0243	.0193	.0154	.0125	.0100
7.0			.1397	.1166	.1034	.0911	.0797	.0693	.0597	.0503	.0413	.0333	.0261	.0195	.0148
8.0					.1282	.1159	.1043	.0933	.0831	.0736	.0646	.0561	.0481	.0406	.0336
9.0							.1263	.1154	.1050	.0951	.0857	.0768	.0684	.0605	.0531
10.0															

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .250  
THETA2/THETA1 = .500

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

\* \* 81

BI/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.2007									
.2	.1855	.1008	.0525							
.3	.1770	.0981	.0516	.0265	.0134					
.4	.1741	.0983	.0523	.0271	.0138	.0070	.0036			
.5	.1749	.1005	.0542	.0283	.0145	.0074	.0038	.0019	.0009	
.6	.1782	.1044	.0571	.0301	.0155	.0079	.0040	.0020	.0010	.0005
.7	.1831	.1095	.0609	.0324	.0169	.0087	.0044	.0022	.0011	.0006
.8	.1889	.1155	.0654	.0353	.0185	.0096	.0049	.0025	.0013	.0006
.9	.1954	.1222	.0705	.0387	.0205	.0107	.0055	.0028	.0014	.0007
1.0	.2022	.1294	.0763	.0425	.0229	.0120	.0062	.0032	.0016	.0008
2.0			.1469	.1013	.0661	.0411	.0244	.0139	.0077	.0042
3.0					.1243	.0924	.0662	.0457	.0303	.0194
4.0							.1128	.0885	.0678	.0505
5.0									.1059	.0865

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(K0 - Y0)/THETA1 = .250  
 THETA2/THETA1 = .500

THETA 1 =  $\theta_x - x_o$   
 THETA 2 =  $\theta_y - y_o$

		*****										*****									
BI/B2	*	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	*****										
.6	*	.0093									*****										
.7	*	.0003	.0001	.0001							*****										
.8	*	.0003	.0002	.0001	.0001	.0000					*****										
.9	*	.0004	.0002	.0001	.0001	.0000	.0000	.0000			*****										
1.0	*	.0004	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	*****										
2.0	*	.0022	.0012	.0006	.0003	.0002	.0001	.0001	.0000	.0000	*****										
3.0	*	.0120	.0071	.0041	.0023	.0013	.0007	.0004	.0002	.0001	*****										
4.0	*	.0367	.0258	.0177	.0117	.0076	.0047	.0029	.0017	.0010	*****										
5.0	*	.0693	.0546	.0421	.0319	.0236	.0170	.0120	.0083	.0056	*****										
6.0	*	.1015	.0853	.0707	.0579	.0467	.0371	.0290	.0223	.0168	*****										
7.0	*			.0984	.0845	.0719	.0605	.0504	.0415	.0338	*****										
8.0	*					.0961	.0840	.0728	.0627	.0535	*****										
9.0	*							.0944	.0836	.0736	*****										
10.0	*									.0930	*****										

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .250  
THETA2/THETA1 = .444

BI/b2	* * BI									
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.1517									
.2	.1448	.0695	.0318							
.3	.1394	.0683	.0316	.0143	.0065					
.4	.1390	.0695	.0326	.0148	.0067	.0030	.0013			
.5	.1419	.0724	.0344	.0158	.0072	.0032	.0014	.0006	.0003	
.6	.1468	.0766	.0370	.0172	.0078	.0035	.0016	.0007	.0003	.0001
.7	.1530	.0818	.0403	.0190	.0087	.0040	.0018	.0008	.0004	.0002
.8	.1601	.0878	.0441	.0211	.0098	.0045	.0020	.0009	.0004	.0002
.9	.1676	.0945	.0486	.0236	.0111	.0051	.0023	.0011	.0005	.0002
1.0	.1753	.1016	.0536	.0265	.0126	.0059	.0027	.0012	.0006	.0003
2.0			.1202	.0761	.0449	.0250	.0132	.0067	.0033	.0016
3.0					.0984	.0679	.0447	.0280	.0167	.0095
4.0							.0874	.0645	.0460	.0316
5.0									.0811	.0628

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull





THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

IXO - YOI/THETA1 = .250  
 ITHETA2/THETA1 = .400

		1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* B1	*	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
B1/82	*	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
.1	*	.1217									
.2	*	.1115	.0470	.0189							
.3	*	.1069	.0470	.0191	.0076	.0030					
.4	*	.1108	.0488	.0201	.0081	.0032	.0013	.0005			
.5	*	.1153	.0521	.0218	.0088	.0035	.0014	.0006	.0002	.0001	
.6	*	.1214	.0563	.0241	.0099	.0040	.0016	.0006	.0002	.0001	.0000
.7	*	.1285	.0614	.0268	.0112	.0045	.0018	.0007	.0003	.0001	.0001
.8	*	.1364	.0672	.0300	.0127	.0052	.0021	.0008	.0003	.0001	.0001
.9	*	.1445	.0736	.0338	.0146	.0061	.0025	.0010	.0004	.0002	.0001
1.0	*	.1529	.0804	.0380	.0167	.0071	.0029	.0012	.0005	.0002	.0001
2.0	*		.0991	.0575	.0308	.0308	.0154	.0073	.0033	.0015	.0006
3.0	*				.0783		.0502	.0304	.0172	.0092	.0047
4.0	*							.0431	.0471	.0312	.0197
5.0	*									.0623	.0456

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .25  
THETA2/THETA1 = .40

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

* #	* H1	* #	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
81/82	*	*	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	*	.0000										
.7	*	*	.0000	.0000									
.8	*	*	.0000	.0000	.0000								
.9	*	*	.0000	.0000	.0000	.0000							
1.0	*	*	.0000	.0000	.0000	.0000	.0000						
2.0	*	*	.0003	.0001	.0000	.0000	.0000	.0000					
3.0	*	*	.0023	.0011	.0005	.0002	.0001	.0000	.0000				
4.0	*	*	.0119	.0069	.0038	.0020	.0010	.0005	.0003	.0001			
5.0	*	*	.0323	.0222	.0147	.0094	.0057	.0034	.0019	.0011	.0006		
6.0	*	*	.0586	.0448	.0334	.0243	.0172	.0119	.0079	.0051	.0032		
7.0	*	*			.0501	.0443	.0344	.0262	.0195	.0142	.0101		
8.0	*	*					.0542	.0440	.0352	.0277	.0215		
9.0	*	*							.0529	.0438	.0359		
10.0	*	*									.0518		

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/IHEIA1 = .250  
 YHEIA2/IHEIA1 = .364

THETA 1 =  $\theta - X_0$   
 THETA 2 =  $\theta - Y_0$

\* 01

BI/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0912									
.2	.0841	.0310	.0108							
.3	.0842	.0317	.0113	.0040	.0013					
.4	.0874	.0340	.0123	.0043	.0015	.0005	.0002			
.5	.0936	.0373	.0137	.0049	.0017	.0006	.0002	.0001	.0000	
.6	.1006	.0414	.0156	.0056	.0020	.0007	.0002	.0001	.0000	.0000
.7	.1084	.0462	.0179	.0066	.0024	.0008	.0003	.0001	.0000	.0000
.8	.1167	.0517	.0205	.0077	.0028	.0010	.0004	.0001	.0001	.0000
.9	.1253	.0576	.0236	.0091	.0034	.0012	.0004	.0002	.0001	.0000
1.0	.1341	.0641	.0271	.0107	.0040	.0015	.0005	.0002	.0001	.0000
2.0			.0821	.0439	.0214	.0096	.0041	.0017	.0007	.0003
3.0					.0626	.0373	.0206	.0107	.0052	.0024
4.0							.0532	.0345	.0212	.0123
5.0									.0479	.0332

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X2 - Y2)/THETA1 = .253  
 THETA2/THETA1 = .364

THETA 1 =  $\theta_x - X_0$   
 THETA 2 =  $\theta_y - Y_0$

\*  
 \* 81  
 \*

B1/B2 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	.0000								
.7	.0000	.0000							
.8	.0000	.0000	.0000	.0000	.0000				
.9	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0011	.0005	.0002	.0001	.0000	.0000	.0000	.0000	.0000
4.0	.0068	.0035	.0017	.0008	.0004	.0002	.0001	.0000	.0000
5.0	.0221	.0141	.0086	.0050	.0028	.0015	.0008	.0004	.0002
6.0	.0440	.0325	.0229	.0157	.0104	.0066	.0041	.0024	.0014
7.0			.0424	.0321	.0237	.0171	.0120	.0082	.0054
8.0					.0408	.0319	.0244	.0183	.0135
9.0							.0397	.0317	.0250
10.0									.0388

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

(X0 - Y0)/THETA1 = .250  
 THETA2/THETA1 = .333

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	.0655									
* .2	.0618	.0196	.0059							
* .3	.0642	.0209	.0064	.0019	.0006					
* .4	.0694	.0234	.0073	.0022	.0007	.0002	.0001			
* .5	.0760	.0266	.0085	.0027	.0008	.0002	.0001	.0000		
* .6	.0835	.0304	.0101	.0032	.0010	.0003	.0001	.0000	.0000	
* .7	.0917	.0349	.0119	.0039	.0012	.0004	.0001	.0000	.0000	.0000
* .8	.1002	.0399	.0141	.0047	.0015	.0005	.0002	.0001	.0000	.0000
* .9	.1091	.0454	.0166	.0057	.0019	.0006	.0002	.0001	.0000	.0000
* 1.0	.1181	.0513	.0195	.0068	.0023	.0008	.0002	.0001	.0000	.0000
* 2.0			.0685	.0337	.0150	.0061	.0024	.0009	.0003	.0001
* 3.0					.0503	.0278	.0142	.0067	.0029	.0012
* 4.0						.0418	.0254	.0145	.0077	.0030
* 5.0								.0370	.0242	

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - VC)/TPEIAL = .250  
 THEA2/TPEIAL = .333

THETA 1 =  $\theta_x - x_o$   
 THETA 2 =  $\theta_y - y_o$

BI/82	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* .6	.0000								
* .7	.0000	.0000	.0000						
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0005	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0039	.0018	.0008	.0004	.0002	.0001	.0000	.0000	.0000
* 5.0	.0151	.0089	.0050	.0027	.0013	.0007	.0003	.0001	.0001
* 6.0	.0341	.0236	.0157	.0101	.0062	.0036	.0021	.0011	.0006
* 7.0			.0322	.0233	.0163	.0111	.0073	.0047	.0029
* 8.0					.0308	.0231	.0169	.0121	.0084
* 9.0							.0298	.0230	.0174
* 10.0									.0290

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

$$\begin{aligned}\text{THETA } 1 &= \theta - x_0 \\ \text{THETA } 2 &= \theta - y_0\end{aligned}$$

**STRESS DISTRIBUTION - Weibull**  
**STRENGTH DISTRIBUTION - Weibull**

(X0 - V0)/THETA1 = .500  
THETA2/THETA1 = 1.000

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\* \* 81

81/82	*	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0101								
.7	*	.0177	.0127	.0051						
.8	*	.0197	.0142	.0101	.0073	.0052				
.9	*	.0222	.0160	.0115	.0082	.0059	.0042	.0030		
1.0	*	.0251	.0182	.0132	.0095	.0068	.0049	.0035	.0025	.0018
2.0	*	.0796	.0644	.0514	.0400	.0316	.0244	.0186	.0141	.0105
3.0	*	.1431	.1264	.1108	.0964	.0833	.0713	.0605	.0510	.0425
4.0	*	.1893	.1743	.1599	.1461	.1329	.1203	.1084	.0972	.0868
5.0	*	.2211	.2082	.1956	.1832	.1712	.1595	.1482	.1373	.1269
6.0	*	.2438	.2326	.2215	.2106	.1999	.1894	.1791	.1691	.1593
7.0	*			.2410	.2314	.2218	.2124	.2031	.1940	.1850
8.0	*					.2383	.2304	.2220	.2137	.2056
9.0	*							.2372	.2296	.2222
10.0	*									.2358

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull



(XC - Y0)/THEIA1 = .500  
THEIA2/THEIA1 = .800

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

		*****										*****									
		1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5										
BI/82	*	*****										*****									
.1	*	.2269																			
.2	*	.2360	.1255	.0728																	
.3	*	.1981	.1237	.0729	.0417	.0234															
.4	*	.1980	.1265	.0759	.0439	.0249	.0140	.0078													
.5	*	.2019	.1322	.0809	.0476	.0273	.0154	.0086	.0048	.0027											
.6	*	.2080	.1396	.0874	.0523	.0304	.0173	.0098	.0055	.0031	.0017										
.7	*	.2152	.1482	.0951	.0581	.0343	.0198	.0113	.0064	.0036	.0020										
.8	*	.2228	.1573	.1035	.0647	.0389	.0228	.0131	.0075	.0042	.0024										
.9	*	.2304	.1667	.1126	.0721	.0443	.0264	.0154	.0088	.0050	.0028										
1.0	*	.2379	.1761	.1220	.0801	.0503	.0306	.0181	.0105	.0060	.0034										
2.0	*			.2051	.1644	.1278	.0962	.0702	.0495	.0339	.0225										
3.0	*					.1899	.1608	.1340	.1096	.0880	.0693										
4.0	*							.1814	.1590	.1380	.1185										
5.0	*									.1760	.1579										

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(XC - YC)/IPETAL = .011  
 THETA2/IPETAL = .007

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

* * b1 *	* * b1 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
b1/82 *	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0009								
.7	*	.0011	.0006	.0003						
.8	*	.0013	.0007	.0004	.0002	.0001				
.9	*	.0015	.0009	.0005	.0003	.0002	.0001	.0001		
1.0	*	.0019	.0011	.0006	.0003	.0002	.0001	.0001	.0001	.0000
2.0	*	.0145	.0092	.0057	.0034	.0021	.0012	.0007	.0004	.0003
3.0	*	.0534	.0403	.0298	.0215	.0152	.0105	.0071	.0047	.0030
4.0	*	.1005	.0843	.0698	.0570	.0459	.0364	.0285	.0219	.0165
5.0	*	.1407	.1245	.1093	.0952	.0825	.0705	.0598	.0503	.0418
6.0	*	.1723	.1571	.1426	.1288	.1157	.1034	.0918	.0811	.0711
7.0	*			.1650	.1565	.1449	.1320	.1205	.1096	.0992
8.0	*					.1675	.1562	.1451	.1345	.1243
9.0	*							.1660	.1558	.1460
10.0	*									.1647

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

THEIA2/THETA1 = .667  
(XO - YO)/THETA1 = .500

* * *
* * B1
* * *
B1/B2
*****
1.0 *****
1.5 *****
2.0 *****
2.5 *****
3.0 *****
3.5 *****
4.0 *****
4.5 *****
5.0 *****
5.5 *****
*****

347



(X0 - Y0)/THETA1 = .500  
THETA2/THETA1 = .571

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\* B1  
\*

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0523									
.2	.0576	.0179	.0052							
.3	.0676	.0226	.0069	.0020	.0006					
.4	.0791	.0285	.0092	.0028	.0008	.0003	.0001			
.5	.0912	.0354	.0122	.0039	.0012	.0004	.0001	.0000		
.6	.1036	.0431	.0156	.0053	.0017	.0005	.0002	.0001	.0000	.0000
.7	.1160	.0516	.0200	.0071	.0024	.0008	.0003	.0001	.0000	.0000
.8	.1282	.0607	.0250	.0093	.0033	.0011	.0004	.0001	.0000	.0000
.9	.1401	.0703	.0306	.0120	.0044	.0016	.0005	.0002	.0001	.0000
1.0	.1516	.0802	.0369	.0152	.0058	.0021	.0007	.0003	.0001	.0000
2.0			.1167	.0733	.0424	.0226	.0112	.0052	.0023	.0010
3.0					.1020	.0716	.0478	.0303	.0181	.0103
4.0							.0943	.0711	.0519	.0367
5.0									.0896	.0716

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

$$\begin{aligned} (K_2 - Y_0)/\text{THEIA} &= .590 \\ \text{THEIAZ}/\text{THEIA} &= .571 \end{aligned}$$
$$\begin{aligned} \text{THETA 1} &= \theta - x^0 \\ \text{THETA 2} &= \theta - y^0 \end{aligned}$$

19  
\*

[illegible]

(X0 - Y0)/THETA1 = .500  
THETA2/THETA1 = .500

THETA 1 =  $\theta_x - X_0$   
THETA 2 =  $\theta_y - Y_0$

\* \* d1

BI/B2	*	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	*	.0112									
.2	*	.0229	.0038	.0005							
.3	*	.0349	.0073	.0013	.0002	.0000					
.4	*	.0473	.0116	.0024	.0005	.0001	.0000	.0000			
.5	*	.0600	.0169	.0040	.0009	.0002	.0000	.0000	.0000		
.6	*	.0727	.0229	.0061	.0014	.0003	.0001	.0000	.0000	.0000	
.7	*	.0855	.0298	.0087	.0022	.0005	.0001	.0000	.0000	.0000	
.8	*	.0982	.0374	.0119	.0033	.0009	.0002	.0001	.0000	.0000	
.9	*	.1106	.0456	.0157	.0047	.0013	.0003	.0001	.0000	.0000	
1.0	*	.1226	.0544	.0201	.0065	.0019	.0005	.0001	.0000	.0000	
2.0	*			.0891	.0492	.0243	.0108	.0044	.0017	.0006	.0002
3.0	*					.0754	.0478	.0283	.0155	.0079	.0037
4.0	*							.0684	.0475	.0315	.0199
5.0	*									.0643	.0475

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

$(X_0 - Y_0)/T_{trial} = .511$   
 $THETA2/T_{trial} = .511$

$THETA 1 = \theta - X_0$   
 $THETA 2 = \theta - Y_0$

\* \* 31  
 \*

B1/B2	* 6.0	* 6.5	* 7.0	* 7.5	* 8.0	* 8.5	* 9.0	* 9.5	* 10.0
.6	.0000								
.7	.0000	.0000	.0000						
.8	.0000	.0000	.0000	.0000					
.9	.0000	.0000	.0000	.0000	.0000				
1.0	.0000	.0000	.0000	.0000	.0000	.0000			.0000
2.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0017	.0007	.0003	.0001	.0000	.0000	.0000	.0000	.0000
4.0	.0119	.0067	.0036	.0018	.0009	.0004	.0002	.0001	.0000
5.0	.0340	.0235	.0157	.0100	.0062	.0036	.0020	.0011	.0006
6.0	.0616	.0476	.0359	.0265	.0190	.0132	.0090	.0059	.0037
7.0			.0597	.0477	.0375	.0289	.0218	.0162	.0117
8.0					.0583	.0478	.0387	.0309	.0243
9.0							.0572	.0479	.0397
10.0									.0564

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/THEIA1 = .592  
THEIA2/THEIA1 = .444

THEIA 1 = 0 - X0  
THEIA 2 = 0 - Y0

\* B1

61/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0005									
.2	.0066	.0033	.0000							
.3	.0162	.0017	.0001	.0003	.0000					
.4	.0271	.0041	.0005	.0000	.0000	.0000				
.5	.0387	.0075	.0011	.0001	.0000	.0000	.0000	.0000		
.6	.0508	.0116	.0021	.0003	.0000	.0000	.0000	.0000	.0000	.0000
.7	.0631	.0169	.0036	.0006	.0001	.0000	.0000	.0000	.0000	.0000
.8	.0755	.0229	.0055	.0011	.0002	.0000	.0000	.0000	.0000	.0000
.9	.0878	.0296	.0079	.0018	.0004	.0001	.0000	.0000	.0000	.0000
1.0	.0999	.0370	.0109	.0027	.0006	.0001	.0000	.0000	.0000	.0000
2.0			.0685	.0332	.0140	.0052	.0017	.0005	.0002	.0000
3.0					.0561	.0320	.0167	.0079	.0034	.0014
4.0							.0498	.0317	.0190	.0107
5.0									.0462	.0317

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(K0 - Y0)/INITIAL = .226  
 THETA2/INITIAL = .444

THETA 1 =  $\theta - X_0$   
 THETA 2 =  $\theta - Y_0$

\* d1

BL/82	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	.0000								
.7	.0000	.0000	.0000						
.8	.0000	.0000	.0000	.0000	.0000				
.9	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000		.0000
2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0005	.0012	.0001	.0000	.0000	.0000	.0000	.0000	.0000
4.0	.0056	.0027	.0012	.0005	.0002	.0001	.0000	.0000	.0000
5.0	.0209	.0131	.0078	.0044	.0024	.0012	.0006	.0003	.0001
6.0	.0439	.0319	.0224	.0152	.0100	.0063	.0038	.0022	.0012
7.0			.0423	.0320	.0230	.0170	.0119	.0081	.0054
8.0					.0411	.0321	.0247	.0186	.0137
9.0							.0402	.0323	.0255
10.0									.0395

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/TETAL = .500  
 TETAL2/TETAL = .400

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
91/82 *	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* .1	.0000									
* .2	.0012	.0000	.0000							
* .3	.0065	.0003	.0000	.0000						
* .4	.0148	.0012	.0001	.0000	.0000	.0000				
* .5	.0245	.0031	.0003	.0000	.0000	.0000	.0000			
* .6	.0352	.0058	.0007	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* .7	.0465	.0094	.0014	.0002	.0000	.0000	.0000	.0000	.0000	.0000
* .8	.0582	.0139	.0024	.0003	.0000	.0000	.0000	.0000	.0000	.0000
* .9	.0700	.0192	.0039	.0006	.0001	.0000	.0000	.0000	.0000	.0000
* 1.0	.0819	.0252	.0059	.0011	.0002	.0000	.0000	.0000	.0000	.0000
* 2.0			.0530	.0226	.0081	.0025	.0007	.0002	.0000	.0000
* 3.0					.0419	.0215	.0099	.0040	.0015	.0005
* 4.0							.0364	.0212	.0115	.0057
* 5.0									.0333	.0212

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

$(X_0 - Y_0)/\theta = .50$   
 $\theta = .40$

$\theta = X_0 - Y_0$   
 $\theta = X_0 - Y_0$

* 61	* 62	* 63	* 64	* 65	* 66	* 67	* 68	* 69	* 70	* 71	* 72	* 73	* 74	* 75	* 76	* 77	* 78	* 79	* 80
0.6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
0.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
9.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta_x - X_o$   
 THETA 2 =  $\theta_y - Y_o$

EXO - Y01/THETA1 = .500  
 THETA2/THETA1 = .204

		*****										*****									
		1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	*****									
BL/B2	*	*****																			
.1	*	.0000																			
.2	*	.0001	.0000	.0000																	
.3	*	.0023	.0000	.0000	.0000	.0000															
.4	*	.0076	.0003	.0000	.0000	.0000	.0000	.0000													
.5	*	.0152	.0011	.0000	.0000	.0000	.0000	.0000	.0000	.0000											
.6	*	.0242	.0027	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000										
.7	*	.0343	.0051	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000									
.8	*	.0450	.0083	.0010	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000								
.9	*	.0561	.0124	.0019	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000							
1.0	*	.0674	.0172	.0031	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000						
2.0	*			.0413	.0154	.0047	.0012	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000					
3.0	*					.0315	.0145	.0059	.0021	.0006	.0002										
4.0	*							.0268	.0142	.0069	.0030										
5.0	*									.0241	.0142										

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = .573  
THEIA2/THEIA1 = .364

THEIA 1 =  $\frac{\theta - X_0}{X}$   
THEIA 2 =  $\frac{\theta - Y_0}{Y}$

		*****									
		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	*****
BL/B2	*	*****									
.6	*	.0000									
.7	*	.0000	.0000	.0000							
.8	*	.0000	.0000	.0000	.0000	.0000					
.9	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
3.0	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
4.0	*	.0012	.0004	.0001	.0000	.0000	.0000	.0000	.0000	.0000	
5.0	*	.0078	.0040	.0019	.0008	.0003	.0001	.0000	.0000	.0000	
6.0	*	.0225	.0143	.0086	.0049	.0027	.0014	.0006	.0003	.0001	
7.0	*			.0213	.0144	.0093	.0058	.0034	.0020	.0011	
8.0	*					.0205	.0145	.0099	.0066	.0042	
9.0	*							.0199	.0146	.0104	
10.0	*									.0195	

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta_x - X_0$   
 THETA 2 =  $\theta_y - Y_0$

(X0 - Y0)/THEIA1 = .500  
 THEIA2/THEIA1 = .333

\* \* 81

61/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
1	.0000									
2	.0000	.0000	.0000							
3	.0007	.0000	.0000	.0000	.0000					
4	.0037	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
5	.0092	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.0166	.0012	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7	.0253	.0027	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	.0349	.0050	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000
9	.0451	.0060	.0009	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0558	.0118	.0016	.0002	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0324	.0106	.0028	.0006	.0001	.0000	.0000	.0000
3.0					.0238	.0098	.0035	.0011	.0003	.0001
4.0							.0197	.0096	.0042	.0016
5.0									.0175	.0095

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - YD)/TPEIAT = .577  
TPEIAT2/TPEIAT = .333

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* * dl *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
61/82 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
* .6	.0000								
* .7	.0000	.0000	.0000						
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000			
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0000	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* 5.0	.0048	.0022	.0009	.0004	.0001	.0000	.0000	.0000	.0000
* 6.0	.0161	.0096	.0053	.0028	.0014	.0006	.0003	.0001	.0000
* 7.0			.0152	.0096	.0058	.0034	.0018	.0009	.0005
* 8.0					.0146	.0097	.0063	.0039	.0023
* 9.0							.0141	.0096	.0066
* 10.0									.0137

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/THETA1 = .750  
THETA2/THETA1 = 1.000

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* B1		*****									
91/B2	*	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
*****											
.1	*	.1809									
.2	*	.1668	.0950								
.3	*	.1683	.0992	.0546	.0289	.0150					
.4	*	.1758	.1075	.0610	.0330	.0174	.0090	.0046			
.5	*	.1857	.1180	.0693	.0386	.0208	.0110	.0057	.0029	.0015	
.6	*	.1964	.1296	.0789	.0454	.0251	.0135	.0071	.0037	.0019	.0010
.7	*	.2072	.1417	.0896	.0533	.0303	.0167	.0090	.0048	.0025	.0013
.8	*	.2175	.1539	.1009	.0622	.0365	.0206	.0113	.0061	.0033	.0017
.9	*	.2272	.1657	.1125	.0716	.0435	.0253	.0143	.0079	.0042	.0023
1.0	*	.2362	.1771	.1241	.0820	.0514	.0308	.0178	.0100	.0055	.0030
2.0	*			.2146	.1765	.1414	.1100	.0831	.0610	.0433	.0298
3.0	*					.2036	.1765	.1509	.1272	.1056	.0862
4.0	*							.1971	.1763	.1563	.1375
5.0	*									.1929	.1760

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X2 - Y2)/THETA1 = .750  
THETA2/THETA1 = 1.000

THETA 1 =  $\theta - x_0$   
THETA 2 =  $\theta - y_0$

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(X0 - Y0)/THETA1 = .750  
THETA2/THETA1 = .600

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

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$$\frac{(X_0 - Y_0)/I_{HETAL}}{THEI62/I_{HETAL}} = \frac{.752}{.855}$$

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[illegible]

**STRESS DISTRIBUTION - Weibull**  
**STRENGTH DISTRIBUTION - Weibull**

(X0 - Y0)/THETA1 = .752  
 THETA2/THETA1 = .697

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

BL/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .0007										
* .0098	.0006									
* .0250	.0031	.0003	.0000							
* .0339	.0074	.0011	.0001	.0000						
* .0548	.0133	.0025	.0004	.0001	.0000					
* .0707	.0207	.0047	.0009	.0001	.0000	.0000				
* .0863	.0292	.0078	.0017	.0003	.0001	.0000	.0000			.0000
* .1015	.0388	.0118	.0030	.0007	.0001	.0000	.0000	.0000		.0000
* .1161	.0491	.0167	.0048	.0012	.0003	.0001	.0000	.0000	.0000	.0000
* .1299	.0600	.0226	.0071	.0020	.0005	.0001	.0000	.0000	.0000	.0000
* .1059		.0636		.0346	.0169	.0074	.0030	.0011	.0004	.0000
* .0954					.0656	.0427	.0262	.0150	.0080	.0035
* .0897							.0897	.0670	.0483	.0335
* .0962										.0679

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - Y3)/THETA1 = .750  
THETA2/THETA1 = .007

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

\* 31 \*

BL/82 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	.0000								
.7	.0000	.0000	.0000						
.8	.0000	.0000	.0000	.0000	.0000				
.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0040	.0016	.0008	.0003	.0001	.0001	.0000	.0000	.0000
4.0	.0224	.0143	.0087	.0050	.0028	.0014	.0007	.0003	.0002
5.0	.0522	.0392	.0286	.0203	.0140	.0093	.0060	.0037	.0022
6.0	.0838	.0685	.0550	.0435	.0337	.0256	.0190	.0138	.0098
7.0			.0821	.0690	.0572	.0468	.0378	.0301	.0236
8.0					.0809	.0693	.0589	.0495	.0412
9.0							.0799	.0696	.0602
10.0									.0791

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = .750  
THEIA2/THEIA1 = .571

THEIA 1 =  $\theta - X_0$   
THEIA 2 =  $\theta - Y_0$

\* \* B1

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0000									
.2	.0000	.0000	.0000							
.3	.0059	.0002	.0000	.0000	.0000					
.4	.0155	.0011	.0000	.0000	.0000	.0000				
.5	.0277	.0034	.0002	.0000	.0000	.0000	.0000	.0000		
.6	.0412	.0071	.0008	.0001	.0000	.0000	.0000	.0000	.0000	.0000
.7	.0554	.0122	.0016	.0002	.0000	.0000	.0000	.0000	.0000	.0000
.8	.0698	.0186	.0035	.0005	.0001	.0000	.0000	.0000	.0000	.0000
.9	.0840	.0262	.0059	.0010	.0001	.0000	.0000	.0000	.0000	.0000
1.0	.0979	.0347	.0091	.0019	.0003	.0000	.0000	.0000	.0000	.0000
2.0			.0753	.0381	.0165	.0063	.0020	.0006	.0002	.0000
3.0					.0659	.0398	.0221	.0112	.0052	.0021
4.0							.0609	.0410	.0262	.0158
5.0									.0579	.0419

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/INITIAL = .750  
 THEIA2/INITIAL = .571

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

BL/B2	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* .6	.0000								
* .7	.0000	.0000	.0000						
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000			
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 5.0	.0292	.0196	.0126	.0076	.0045	.0025	.0013	.0007	.0003
* 6.0	.0559	.0425	.0315	.0227	.0159	.0108	.0071	.0044	.0027
* 7.0		.0545	.0430	.0330	.0252	.0187	.0136	.0096	
* 8.0			.0534	.0434	.0347	.0273	.0211		
* 9.0					.0526	.0437	.0358		
* 10.0								.0520	

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/TMEI1 = .750  
TMEI2/TMEI1 = .500

THEIA 1 =  $\theta_x - x_0$   
THEIA 2 =  $\theta_y - y_0$

\* 81

81/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	* .0000									
* .2	* .0000	* .0000	* .0000							
* .3	* .0010	* .0000	* .0000	* .0000						
* .4	* .0054	* .0021	* .0003	* .0003	* .0000					
* .5	* .0133	* .0007	* .0020	* .0000	* .0000	* .0000				
* .6	* .0236	* .0022	* .0001	* .0000	* .0000	* .0000	* .0000			
* .7	* .0354	* .0048	* .0004	* .0000	* .0000	* .0000	* .0000	* .0000		
* .8	* .0481	* .0087	* .0009	* .0001	* .0000	* .0000	* .0000	* .0000	* .0000	
* .9	* .0612	* .0138	* .0020	* .0002	* .0000	* .0000	* .0000	* .0000	* .0000	
* 1.0	* .0744	* .0199	* .0030	* .0004	* .0000	* .0000	* .0000	* .0000	* .0000	
* 2.0			* .0540	* .0228	* .0079	* .0023	* .0005	* .0001	* .0000	* .0000
* 3.0					* .0457	* .0242	* .0114	* .0047	* .0017	* .0006
* 4.0							* .0415	* .0251	* .0141	* .0073
* 5.0									* .0390	* .0258

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X3 - Y3)/THEIA1 = .750  
THEIA2/THEIA1 = .500

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

#	# d1	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
BL/82 *	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0000								
.7	*	.0000	.0000							
.8	*	.0000	.0000	.0000	.0000					
.9	*	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	*	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	*	.0044	.0015	.0006	.0002	.0001	.0000	.0000	.0000	.0000
5.0	*	.0162	.0096	.0054	.0028	.0014	.0006	.0003	.0001	.0000
6.0	*	.0373	.0263	.0179	.0117	.0073	.0044	.0025	.0013	.0007
7.0	*			.0362	.0267	.0192	.0134	.0091	.0059	.0037
8.0	*					.0354	.0271	.0203	.0149	.0106
9.0	*							.0347	.0273	.0212
10.0	*									.0342

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - V01)/THETA1 = .750  
 THETA2/THETA1 = .444

THETA 1 =  $\theta_x - X_0$   
 THETA 2 =  $\theta_y - Y_0$

* B1	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
B1/B2 *	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	.0000									
* .2	.0000	.0000	.0000							
* .3	.0001	.0000	.0000	.0000	.0000					
* .4	.0017	.0000	.0000	.0000	.0000	.0000				
* .5	.0081	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
* .6	.0132	.0006	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .7	.0225	.0018	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .8	.0332	.0039	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .9	.0448	.0071	.0006	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 1.0	.0569	.0114	.0013	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0			.0390	.0137	.0038	.0008	.0001	.0000	.0000	.0000
* 3.0					.0319	.0147	.0058	.0020	.0006	.0001
* 4.0							.0284	.0153	.0075	.0033
* 5.0									.0263	.0152

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - Y0)/IPEIA1 = .752  
IPEIA2/IHEIA1 = .444

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* * B1		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* * B1		*****								
BL/B2	*	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.7	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.8	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	*	.0013	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000
5.0	*	.0069	.0047	.0023	.0010	.0004	.0001	.0001	.0000	.0000
6.0	*	.0250	.0163	.0101	.0059	.0033	.0017	.0008	.0004	.0002
7.0	*		.0241	.0156	.0106	.0110	.0070	.0043	.0025	.0014
8.0	*					.0234	.0169	.0118	.0080	.0053
9.0	*						.0229	.0171	.0125	
10.0	*									.0225

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .750  
THETA2/THETA1 = .400

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\* 81  
\* 81

B1/R2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
1	.0000									
2	.0000	.0000	.0000							
3	.0000	.0000	.0000	.0000						
4	.0004	.0000	.0000	.0000	.0000					
5	.0026	.0000	.0000	.0000	.0000	.0000	.0000			
6	.0073	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
7	.0142	.0006	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	.0229	.0017	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
9	.0329	.0036	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0438	.0065	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0284	.0083	.0019	.0003	.0000	.0000	.0000	.0000
3.0					.0224	.0089	.0030	.0008	.0002	.0000
4.0							.0195	.0094	.0040	.0015
5.0									.0178	.0097

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = .750  
THETA2/THETA1 = .400

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\*  
\* B1  
\*

B1/B2 *	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* .6	.0000								
* .7	.0000	.0000							
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000			
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 5.0	.0049	.0022	.0009	.0003	.0001	.0000	.0000	.0000	.0000
* 6.0	.0168	.0100	.0056	.0030	.0014	.0007	.0003	.0001	.0000
* 7.0			.0161	.0103	.0063	.0037	.0020	.0010	.0005
* 8.0					.0155	.0105	.0068	.0043	.0026
* 9.0							.0152	.0107	.0073
* 10.0									.0149

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(XC - YC)/THETA1 = .750  
THETA2/THETA1 = .364

THETA 1 =  $\theta_x - Z_0$   
THETA 2 =  $\theta_y - Y_0$

\* B1

BL/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	.0000									
* .2	.0000	.0000	.0000							
* .3	.0000	.0000	.0000	.0000						
* .4	.0001	.0000	.0000	.0000	.0000	.0000				
* .5	.0011	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
* .6	.0039	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .7	.0089	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .8	.0158	.0007	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .9	.0243	.0018	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0234	.0037	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0208	.0050	.0009	.0001	.0000	.0000	.0000	.0000
3.0					.0156	.0055	.0015	.0003	.0001	.0000
4.0						.0135	.0057	.0021	.0007	
5.0								.0121	.0060	

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

375

375

(XC - Y0)/IHEIAL = .750  
 IHEIA2/IHEIAL = .364

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

* B1	6.3	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
61/82 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
* .6	.0000								
* .7	.0000	.0000	.0000						
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000			
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 5.0	.0027	.0011	.0004	.0001	.0000	.0000	.0000	.0000	.0000
* 6.0	.0113	.0062	.0032	.0015	.0006	.0002	.0001	.0000	.0000
* 7.0			.0107	.0064	.0036	.0019	.0009	.0004	.0002
* 8.0					.0103	.0065	.0039	.0023	.0012
* 9.0							.0100	.0066	.0042
* 10.0									.0098

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/THEIA1 = .750  
THEIA2/THEIA1 = .333

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

\* B1

EL/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* .1	* .0000									
* .2	* .0000	.0000	.0000							
* .3	* .0000	.0000	.0000	.0000	.0000					
* .4	* .0000	.0000	.0000	.0000	.0000	.0000	.0000			
* .5	* .0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
* .6	* .0020	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .7	* .0055	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .8	* .0109	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* .9	* .0180	.0009	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 1.0	* .0263	.0021	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0			.0153	.0031	.0004	.0000	.0000	.0000	.0000	.0000
* 3.0					.0112	.0033	.0008	.0001	.0000	.0000
* 4.0							.0093	.0035	.0011	.0003
* 5.0										.0003

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = .750  
THEIA2/THEIA1 = .333

THETA 1 =  $\theta_x - X_0$   
THETA 2 =  $\theta_y - Y_0$

* 81	* 82	* 6.0	* 6.5	* 7.0	* 7.5	* 8.0	* 8.5	* 9.0	* 9.5	* 10.0
*	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0000								
.7	*	.0000	.0000							
.8	*	.0000	.0000	.0000	.0000	.0000				
.9	*	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
3.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5.0	*	.0015	.0005	.0002	.0000	.0000	.0000	.0000	.0000	.0000
6.0	*	.0075	.0038	.0018	.0007	.0003	.0001	.0000	.0000	.0000
7.0	*			.0072	.0039	.0020	.0010	.0004	.0001	.0001
8.0	*				.0069	.0040		.0023	.0012	.0006
9.0	*							.0066	.0041	.0025
10.0	*									.0065

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta_x - X_o$   
 THETA 2 =  $\theta_y - Y_o$

(XO - YO)/THETA1 = 1.000  
 THETA2/THETA1 = 1.000

* 01	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* 01	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 02	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 03	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 04	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 05	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 06	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 07	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 08	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 09	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 10	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 20	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 30	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 40	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
* 50	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(XC - Y3)/THEIA1 = 1.000  
THEIA2/THEIA1 = 1.000

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* B1		*****									
B1/B2	*	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	*****
.6	*	.0000									
.7	*	.0000	.0000								
.8	*	.0000	.0000	.0000	.0000	.0000					
.9	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
2.0	*	.0034	.0016	.0007	.0003	.0001	.0001	.0000	.0000	.0000	
3.0	*	.0298	.0202	.0132	.0083	.0050	.0029	.0016	.0008	.0004	
4.0	*	.0732	.0584	.0457	.0350	.0263	.0192	.0138	.0096	.0065	
5.0	*	.1142	.0982	.0835	.0702	.0583	.0478	.0397	.0309	.0243	
6.0	*	.1479	.1323	.1176	.1038	.0909	.0791	.0682	.0583	.0494	
7.0	*			.1462	.1328	.1200	.1080	.0966	.0859	.0760	
8.0	*					.1449	.1331	.1219	.1112	.1010	
9.0	*							.1439	.1334	.1234	
10.0	*									.1430	

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = 1.000  
YTHETA2/THETA1 = .800

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0000									
.2	.0024	.0000								
.3	.0126	.0007	.0000	.0000	.0000					
.4	.0775	.0034	.0002	.0000	.0020	.0000	.0000			
.5	.0444	.0082	.0010	.0001	.0000	.0000	.0000	.0000	.0000	
.6	.0619	.0152	.0026	.0003	.0000	.0000	.0000	.0000	.0000	.0000
.7	.0793	.0240	.0052	.0009	.0001	.0000	.0000	.0000	.0000	.0000
.8	.0962	.0342	.0091	.0019	.0003	.0000	.0000	.0000	.0000	.0000
.9	.1123	.0455	.0141	.0034	.0007	.0001	.0000	.0000	.0000	.0000
1.0	.1273	.0574	.0204	.0058	.0013	.0003	.0000	.0000	.0000	.0000
2.0			.1098	.0672	.0372	.0185	.0082	.0032	.0011	.0004
3.0					.1017	.0716	.0479	.0302	.0179	.0099
4.0							.0971	.0741	.0548	.0391
5.0									.0942	.0756

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(XC - Y0)/THEIA1 = 1.000  
THEIA2/THEIA1 = .800

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

* 61	* 62	* 63	* 64	* 65	* 66	* 67	* 68	* 69	* 70	* 71	* 72	* 73	* 74	* 75	* 76	* 77	* 78	* 79	* 80
61/82	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
.6	*	.0000																	
.7	*	.0000	.0000	.0000															
.8	*	.0000	.0000	.0000	.0000														
.9	*	.0000	.0000	.0000	.0000	.0000													
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000												
2.0	*	.0001	.0000	.0000	.0000	.0000	.0000	.0000											
3.0	*	.0051	.0024	.0011	.0004	.0002	.0001	.0000	.0000										
4.0	*	.0270	.0178	.0113	.0068	.0039	.0021	.0011	.0005	.0002									
5.0	*	.0594	.0457	.0343	.0251	.0174	.0123	.0083	.0053	.0033									
6.0	*	.0922	.0766	.0628	.0506	.0402	.0313	.0239	.0179	.0132									
7.0	*		.0908	.0774	.0652	.0544	.0448	.0365	.0293										
8.0	*				.0697	.0579	.0488	.0387	.0293	.0200									
9.0	*																		
10.0	*																		

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THEIA1 = 1.000  
THEIA2/THEIA1 = .667

THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

\* \* B1

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* 1	.0000									
* 2	.0000	.0000	.0000							
* 3	.0013	.0000	.0000	.0000	.0000					
* 4	.0071	.0002	.0000	.0000	.0000	.0000				
* 5	.0171	.0010	.0000	.0000	.0000	.0000	.0000			
* 6	.0295	.0033	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 7	.0442	.0072	.0006	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 8	.0593	.0127	.0016	.0001	.0000	.0000	.0000	.0000	.0000	.0000
* 9	.0744	.0198	.0034	.0004	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0893	.0281	.0060	.0009	.0001	.0000	.0000	.0000	.0000	.0000
2.0			.0728	.0360	.0151	.0053	.0016	.0004	.0001	.0000
3.0					.0656	.0396	.0219	.0110	.0050	.0020
4.0							.0616	.0417	.0268	.0162
5.0									.0592	.0431

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull





(XC - Y3)/THETA1 = 1.000  
 THETA2/THETA1 = .571

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

* B1	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
B1/B2 *	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
.1	.0000									
.2	.0000	.0000								
.3	.0001	.0000	.0000	.0000						
.4	.0014	.0000	.0000	.0000	.0000					
.5	.0060	.0001	.0000	.0000	.0000	.0000				
.6	.0139	.0006	.0000	.0000	.0000	.0000	.0000			.0000
.7	.0243	.0019	.0001	.0000	.0000	.0000	.0000	.0000		.0000
.8	.0365	.0044	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	.0497	.0083	.0007	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0632	.0136	.0016	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0480	.0192	.0060	.0014	.0003	.0000	.0000	.0000
3.0					.0425	.0218	.0098	.0038	.0013	.0004
4.0							.0393	.0234	.0129	.0065
5.0									.0374	.0245

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

(X0 - Y0)/THETA1 = 1.000  
 THETA2/THETA1 = .571

BI/B2	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
* .6	.0000								
* .7	.0000	.0000	.0000						
* .8	.0000	.0000	.0000	.0000	.0000				
* .9	.0000	.0000	.0000	.0000	.0000	.0000			
* 1.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 2.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 3.0	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
* 4.0	.0029	.0012	.0004	.0001	.0000	.0000	.0000	.0000	.0000
* 5.0	.0152	.0089	.0049	.0025	.0012	.0005	.0002	.0001	.0000
* 6.0	.0361	.0253	.0170	.0110	.0068	.0040	.0022	.0012	.0006
* 7.0			.0352	.0259	.0185	.0128	.0086	.0056	.0035
* 8.0					.0345	.0263	.0197	.0144	.0102
* 9.0							.0340	.0267	.0206
* 10.0									.0336

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = 1.500  
 THETA2/THETA1 = .500

THETA 1 =  $\theta - X_0$   
 THETA 2 =  $\theta - Y_0$

81/82	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
* * *	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****
.1	.0000									
.2	.0000	.0000	.0000							
.3	.0000	.0000	.0000	.0000	.0000					
.4	.0002	.0000	.0000	.0000	.0000	.0000				
.5	.0019	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
.6	.0062	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
.7	.0132	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.8	.0225	.0014	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	.0333	.0034	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0451	.0065	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0328	.0102	.0023	.0004	.0000	.0000	.0000	.0000
3.0					.0277	.0120	.0043	.0013	.0003	.0001
4.0							.0252	.0131	.0061	.0029
5.0									.0236	.0138

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = 1.000  
 THETA2/THETA1 = .500

THETA 1 =  $\theta_x - X_0$   
 THETA 2 =  $\theta_y - Y_0$

* * 81 *		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
81/82 *	*	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6	*	.0000								
.7	*	.0000	.0000							
.8	*	.0000	.0000	.0000	.0000	.0000				
.9	*	.0000	.0000	.0000	.0000	.0000	.0000			
1.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000		.0000
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	*	.0009	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
5.0	*	.0075	.0038	.0017	.0007	.0003	.0001	.0000	.0000	.0000
6.0	*	.0226	.0144	.0087	.0050	.0027	.0013	.0006	.0003	.0001
7.0	*			.0220	.0149	.0097	.0061	.0036	.0020	.0011
8.0	*					.0214	.0152	.0105	.0070	.0045
9.0	*							.0211	.0155	.0112
10.0	*									.0208

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = 1.000  
 THETA2/THETA1 = .444

THETA 1 =  $\theta - X_0$   
 THETA 2 =  $\theta - Y_0$

B1/B2	*****										
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	*****
* * * B1											
* .1	.0000										
* .2	.0000	.0000	.0000								
* .3	.0000	.0000	.0000	.0000							
* .4	.0000	.0000	.0000	.0000	.0000						
* .5	.0005	.0000	.0000	.0000	.0000	.0000					
* .6	.0026	.0000	.0000	.0000	.0000	.0000	.0000				
* .7	.0071	.0001	.0000	.0000	.0000	.0000	.0000	.0000			
* .8	.0138	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
* .9	.0224	.0013	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
* 1.0	.0324	.0030	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
* 2.0			.0222	.0054	.0009	.0001	.0000	.0000	.0000	.0000	
* 3.0					.0182	.0066	.0019	.0004	.0001	.0000	
* 4.0							.0152	.0073	.0029	.0010	
* 5.0									.0150	.0078	

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



(X0 - Y0)/THETA1 = 1.000  
 THETA2/THETA1 = .400

THETA 1 =  $\theta - x_0$   
 THETA 2 =  $\theta - y_0$

B1/B2	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0000									
.2	.0000	.0000	.0000							
.3	.0000	.0000	.0000	.0000	.0000					
.4	.0000	.0000	.0000	.0000	.0000	.0000				
.5	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
.6	.0011	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.7	.0037	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.8	.0055	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	.0152	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0235	.0014	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0152	.0025	.0003	.0000	.0000	.0000	.0000	.0000
3.0					.0120	.0036	.0008	.0001	.0000	.0000
4.0							.0104	.0041	.0013	.0004
5.0									.0095	.0044

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

(X0 - Y0)/THETA1 = 1.000  
THETA2/THETA1 = .400

THETA 1 =  $\theta - X_0$   
THETA 2 =  $\theta - Y_0$

* B1	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
B1/B2 *	*****	*****	*****	*****	*****	*****	*****	*****	*****
.6 *	.0000								
.7 *	.0000	.0000	.0000						
.8 *	.0000	.0000	.0000	.0000					
.9 *	.0000	.0000	.0000	.0000	.0000				
1.0 *	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0 *	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0 *	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0 *	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5.0 *	.0018	.0006	.0002	.0001	.0000	.0000	.0000	.0000	.0000
6.0 *	.0099	.0047	.0022	.0010	.0004	.0001	.0000	.0000	.0000
7.0 *			.0086	.0049	.0026	.0013	.0006	.0002	.0001
8.0 *					.0083	.0051	.0029	.0016	.0008
9.0 *						.0081	.0052	.0032	
10.0 *									.0079

STRESS DISTRIBUTION - Weibull  
STRENGTH DISTRIBUTION - Weibull



THETA 1 =  $\theta_x - x_0$   
THETA 2 =  $\theta_y - y_0$

16

[illegible]

**STRESS DISTRIBUTION - Weibull**  
**STRENGTH DISTRIBUTION - Weibull**

$(X_0 - Y_0)/\theta_{12} = 1.000$   
 $\theta_{12}/\theta_{11} = .364$

$\theta_1 = \theta_x - X_0$   
 $\theta_2 = \theta_y - Y_0$

* B1	* B2	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
.6	*	.0000								
.7	*	.0000	.0000							
.8	*	.0000	.0000	.0000						
.9	*	.0000	.0000	.0000	.0000					
1.0	*	.0000	.0000	.0000	.0000	.0000				
2.0	*	.0000	.0000	.0000	.0000	.0000	.0000			
3.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000		
4.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	
5.0	*	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6.0	*	.0057	.0027	.0011	.0004	.0001	.0000	.0000	.0000	.0000
7.0	*			.0054	.0028	.0013	.0006	.0002	.0001	.0000
8.0	*					.0052	.0029	.0015	.0008	.0003
9.0	*						.0050	.0030	.0017	.0007
10.0	*									.0049

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull

1X0 - Y01/10-1A1 = 1.000  
 THETA2/THETA1 = .933

THETA 1 =  $\theta_x - x_0$   
 THETA 2 =  $\theta_y - y_0$

B1/B2	***** 01 *****									
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
.1	.0000									
.2	.0000	.0000	.0000							
.3	.0000	.0000	.0000	.0000	.0000					
.4	.0000	.0000	.0000	.0000	.0000	.0000				
.5	.0000	.0000	.0000	.0000	.0000	.0000	.0000			
.6	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000		.0000
.7	.0010	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.8	.0032	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.9	.0070	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.0124	.0003	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2.0			.0072	.0008	.0000	.0000	.0000	.0000	.0000	.0000
3.0					.0055	.0011	.0002	.0000	.0000	.0000
4.0							.0044	.0013	.0003	.0000
5.0								.0039	.0014	

STRESS DISTRIBUTION - Weibull  
 STRENGTH DISTRIBUTION - Weibull



**APPENDIX 3 MATHEMATICAL THEORIES OF ANALYTICAL EXPRESSION**

## A-3 MATHEMATICAL THEORIES OF ANALYTICAL EXPRESSION

### A-3.1 INTRODUCTION

#### A-3.1.1 The Concept of a Random Variable

Interference Theory is concerned with the interplay of two variables  $X$  and  $Y$  called the strength and stress respectively. Each variable is considered to arise as a consequence of performing some action and measuring the resulting value of the variable. Unlike other such problems frequently considered in engineering, however, one cannot predict with certainty what value of the variable will result as a consequence of a given action. For example, one cannot predict with certainty the strength of a manufactured part prior to performing a strength test on it. One feels from experience that the strength will lie in some finite interval or that in the past the average strength has been some known value. But that is quite different than knowing with certainty what strength this part will have. Hence it is convenient to consider both strength and stress to be random variables--variables whose values are not known with certainty prior to performing some test.

As is usual in studying random variables one associates with the possible values that the random variable can take, a set of numbers called the probability that the random variable takes less than that value. The function,  $F(y)$  that assigns these numbers is called a distribution function and its derivative, if it exists for all  $y$ ,

$$f(y)dy = dF(y)$$

is called a probability density function. From data analysis of the type performed in Section 6 of this report one estimates the density function or distribution function from given data. Hence one starts the mathematical study of interference assuming that  $X$ , the strength and  $Y$ , the stress, are random variables with known distribution or density functions,  $F(x)$  and  $G(y)$  or  $f(x)$  and  $g(y)$  respectively.

#### A-3.1.2 Interference Theory and Random Variables

Interference theory is concerned with the problem of determining the probability of failure of a part which is subjected to a stress  $Y$  and which has a strength  $X$ . It is assumed that both  $X$  and  $Y$  are random variables with known probability density functions. One says failure occurs whenever stress exceeds strength. Hence, the probability that failure occurs

is equivalent to the probability that stress exceeds strength. In symbols:

$$\Pr(\text{failure}) = \Pr(Y \geq X) .$$

### A-3.2 Determination of Probability of Failure

It is clear from A-3.1.2 that to determine the probability of failure one needs to explore the probability that one random variable, called stress, exceeds another random variable, called strength. In practical application it is to be expected that the random variables are independent of each other in the sense that knowledge of one does not allow one to predict the other any more closely than would the absence of such knowledge. In symbols one would say that the random variables  $X$  and  $Y$  are independent if

$$\Pr(X|Y) = \Pr(X) .$$

Roughly in words, this statement says that the probability of  $X$  is the same whether one knows the exact value of  $Y$  ( $\Pr(X|Y)$ ) or not, ( $\Pr(X)$ ) .

There are four main ways to determine the probability of failure from the above considerations. In any given case we will use the form most easily calculated.

a. One can fix attention on some particular value of one of the random variables, say  $Y$  and determine the probability that the other random variable does not exceed this fixed value, say  $y$  . The probability that  $X$  does not exceed a fixed, given value of  $Y$  is written as

$$(1) \qquad \Pr(X \leq y | Y = y) .$$

In terms of density and distribution functions this is equivalent to

$$\int_0^y f(x) dx$$

for those cases where  $X$  takes only non-negative values. If one now multiplies (1) by the probability that  $Y$  is in the neighborhood of  $y$  ,

one obtains a joint probability function

$$P(X \leq y; y < Y \leq y + dy) = \int_0^y f(x)g(y)dx .$$

The probability that  $X \leq Y$  for any value that the random variable  $y$  can take on is given by

$$(2) \quad P(X \leq Y) = \int_0^{\infty} \int_0^y f(x)g(y)dx dy$$

in the case the random variable  $Y$  is distributed on the non-negative axis. Since failure occurs whenever  $X \leq Y$ , formula (2) gives the probability of failure sought. It is expressed in terms of the double integral of the known density functions.

b. One can define a new dummy variable

$$Z = X - Y .$$

Since  $X$  and  $Y$  are random variables their difference,  $Z$  is a random variable. Further, if  $X$  and  $Y$  are distributed on  $(0, \infty)$ ,  $Z$  is distributed on  $(-\infty, \infty)$ . The probability of failure then is equivalent to the probability that  $Z$  is non-positive,  $\Pr(Z \leq 0)$ . The problem then is to find  $h(z)$ , the probability density function for  $Z$ . From this the desired probability of failure can be obtained in a simple fashion.

To motivate the study of interest, let us solve for the following simple problem in detail. Assume  $X$  has a probability density function  $f(x) = 1/6$  and  $Y$  is identically distributed. Both are distributed on the integer 1, 2, ... 6. So formally

$$f(x) = \frac{1}{6} \text{ for } x = 1, 2, 3, 4, 5, 6$$

$$= 0 \text{ elsewhere}$$

and  $Y$  is independent and identically distributed.



Now consider a table of  $X$  and  $Y$  values and the difference.  
 $X - Y = Z$ .

	X value	1	2	3	4	5	6
	1	0	1	2	3	4	5
	2	-1	0	1	2	3	4
Y value	3	-2	-1	0	1	2	3
	4	-3	-2	-1	0	1	2
	5	-4	-3	-2	-1	0	1
	6	-5	-4	-3	-2	-1	0

For  $X = 1$  and  $Y = 1$ ,  $Z = 0$ . The probability that  $X = 1$  and  $Y = 1$  is  $1/36$  since  $X$  and  $Y$  are independent. The probability associated with each cell in the above table is  $1/36$ . Now notice that if  $Z = 0$  then  $X = 1$ ;  $Y = 1$  or  $X = 2$ ;  $Y = 2$  or  $X = 3$ ;  $Y = 3$  etc. Hence the probability that  $Z = 0$  is given by  $1/36 + 1/36 + 1/36 + 1/36 + 1/36 + 1/36 = 1/6$ . If now, we let  $h(z)$  = the probability that  $X - Y = Z$  for fixed  $Z$  then  $h(z)$  is desired probability density function. From the above discussion, it is clear that  $f(x)f(y)$  is the probability that both  $X$  and  $Y$  take on desired values. In every case  $X = Z + Y$  for the  $X, Y, Z$  of interest. Hence  $f(y+z)f(y)$  is the probability that  $X = Z + Y$  and  $Y = Y$  for any  $Y$  and fixed  $Z$ . The above probability is the joint distribution of  $Y, Y + Z$ , say  $g(y, z)$ . It is well known that to get the marginal distribution  $h(z)$  from  $g(y, z)$  one merely "sums over all  $y$ ." One must remember that both  $X, Y$  are distributed on some interval (1, 2 ... 6 in this example) and hence the sum must be over "Permissible values of  $Y$ ." Let us see what these are in this example.

$X$  is distributed on 1, 2 ... 6 and  $f(z+y)$  is the probability distribution of  $X$ . Hence,  $z+y$  cannot exceed 6 nor fall below 1. Thus at the upper limit  $z+y = 6$  and at the low limit  $z+y = 1$ . Or  $y = 6 - z$  and  $y = 1 - z$ . Now let us look at the  $Y, Z$  plane. (See the Figure A-3.1).

Clearly  $g(y, z)$  can be summed only over the  $y$  values defined in the rectangle. But below the  $y$  axis this means  $y$  is summed from  $1 - z$  to 6 and above the  $y$  axis,  $y$  is summed from 1 to  $6 - z$ . Hence we must consider two parts of the sum as follows.

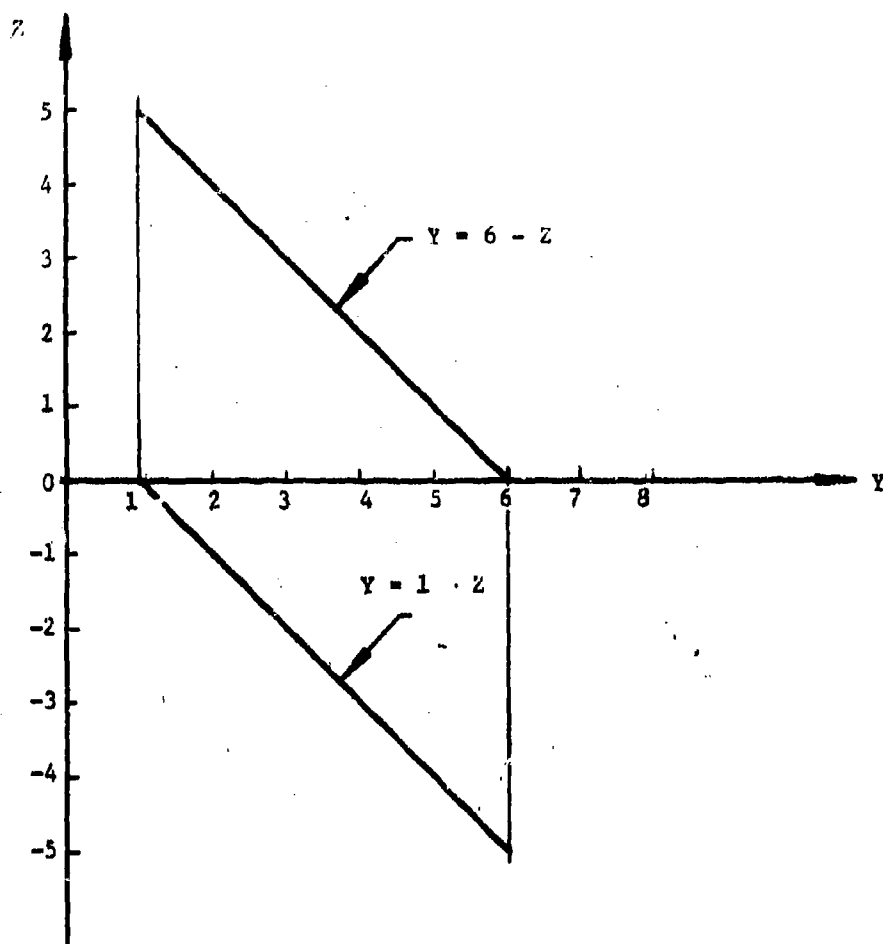


Figure A - 3.1 Permissible  $y$  Values  
for  $Z = X - Y$

$$\begin{aligned}
h(z) &= \sum_y f(z+y) f(y) \\
&= \sum_{y=1-z}^0 f(z+y) f(y) & \text{if } 0 > z \geq -5, \\
&= \sum_{y=1}^{6-z} f(z+y) f(y) & \text{if } 0 \leq z \leq 5.
\end{aligned}$$

(Note that  $Z = 0$  could be in either sum--but not both--arbitrarily it has been put into the second.)

Now recall  $f(x) = 1/6$  for all  $x = 1, 2 \dots 6$  and similarly for  $y$ . Hence

$$\begin{aligned}
h(z) &= \sum_{y=1-z}^0 1/36, & 0 > z \geq -5, \\
&= \sum_{y=1}^{6-z} 1/36, & 0 \leq z \leq 5.
\end{aligned}$$

From this it follows that:

$$\begin{aligned}
h(0) &= 1/36 [1 + 1 + 1 + 1 + 1 + 1] = 6/36, \\
h(1) &= 1/36 [1 + 1 + 1 + 1 + 1] = 5/36, \\
h(2) &= 1/36 [1 + 1 + 1 + 1] = 4/36, \\
h(3) &= 1/36 [1 + 1 + 1] = 3/36, \\
h(4) &= 1/36 [1 + 1] = 2/36, \\
h(5) &= 1/36 [1] = 1/36,
\end{aligned}$$

and

$$\begin{aligned}
 h(-1) &= 1/36 [1 + 1 + 1 + 1 + 1] &= 5/36, \\
 h(-2) &= 1/36 [1 + 1 + 1 + 1] &= 4/36, \\
 h(-3) &= 1/36 [1 + 1 + 1] &= 3/36, \\
 h(-4) &= 1/36 [1 + 1] &= 2/36, \\
 h(-5) &= 1/36 [1] &= 1/36.
 \end{aligned}$$

A picture of  $h(z)$  is given in Figure A-3.2.

The probability of failure in this case is given by

$$\sum_{z=-5}^0 h(z) = 1/2.$$

Having solved the foregoing simple problem one is able to generalize. Because of the special nature of the interference problem we assume:  $X$  has probability density function  $f(x)$ ;  $Y$  has probability density function  $g(y)$  and both are distributed on  $(0, \infty)$ . Note that we are allowing  $f$  and  $g$  to be different. Hence we have dropped the assumption of identically distributed random variables although we continue to assume that they are independent.

As in the previous work it is clear that the only difficulty in finding  $h(z)$  is in finding the correct limits on the integrals. The probability arguments are trivial. Since we have assumed that both  $X$  and  $Y$  are distributed on  $(0, \infty)$  we can give a complete solution to this problem. For consider the  $y, z$  plot again. Since  $x=0$  is the minimum value that  $X$  can take we must have  $y + z = 0$  or  $y = -z$  as the lower bound of the area to be considered. Since  $x = \infty$  is the maximum value that  $x$  can take there is no upper bound on area. Hence all values of permissible  $y$ 's are included in the area above. These are shown in Figure A-3.3,

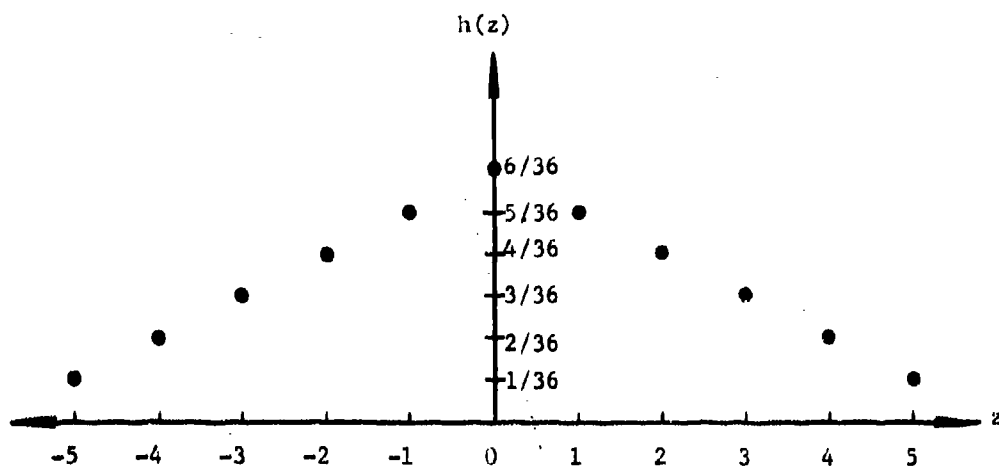


Figure A - 3.2 Probability density function  $h(z)$

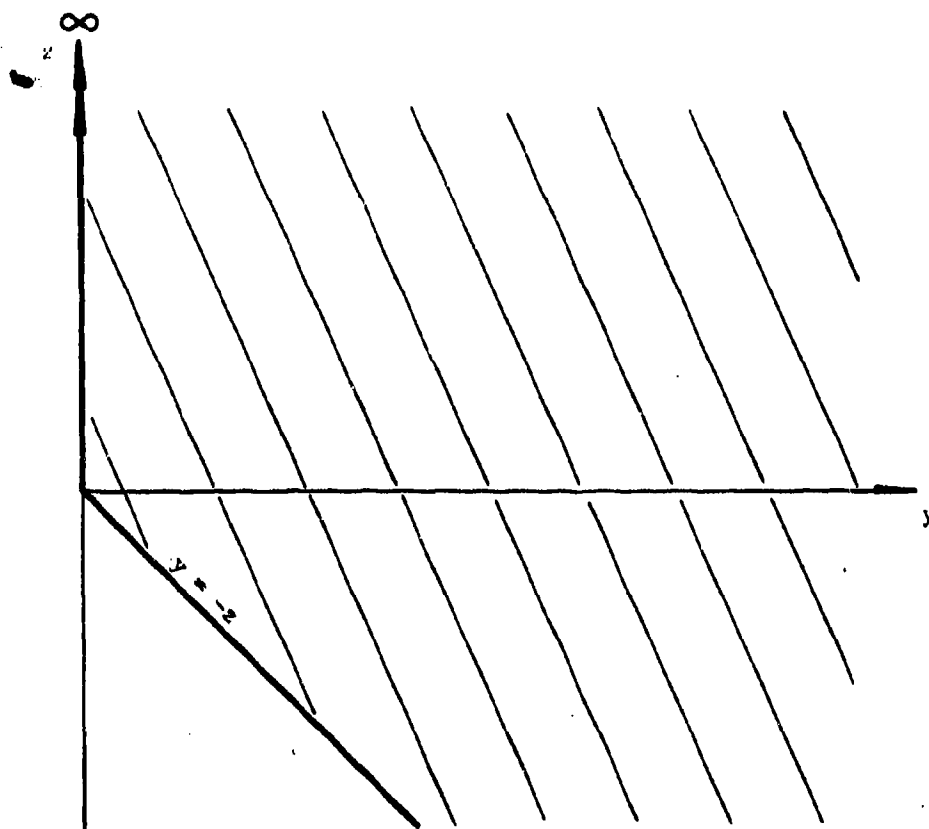


Figure A - 3.3 Area of integration for the difference of two non-negative random variables

and one can pursue the probability arguments precisely as in the previous section to show:

$$\begin{aligned}
 (3) \quad h(z) &= \int_{-\infty}^{\infty} f(z+y)g(y)dy, \\
 &= \int_0^{\infty} f(z+y)g(y)dy, \quad z \geq 0, \\
 &= \int_{-z}^{\infty} f(z+y)g(y)dy, \quad z \leq 0.
 \end{aligned}$$

Clearly this solves the problem in general for  $n=2$ . Extensions to other domains for  $X, Y$  follow quite readily.

It follows then that with the above formulation one need not resort to Monte Carlo simulation. At worst one must evaluate, numerically, the above integrals. In some cases the  $h(z)$  can be obtained in closed form. In the often considered case in which  $X$  and  $Y$  are normally distributed, the probability density function of  $z$  is known to be normal. Hence  $\Pr(z \leq 0)$  is obtainable from tables of the normal curve. We will show this to be true in A-3.3.1.

c. From the definition of a probability distribution function one sees from formula (2) that the probability of failure can be expressed as:

$$(4) \quad \Pr(X \leq Y) = \int_0^{\infty} F(y)g(y)dy$$

where  $F(y)$  is the probability distribution function of  $X$  evaluated at the point  $y$ . Formula (4) is convenient to use when  $F(y)$  is easily determined as in the case of strength's that are Weibull distributed.

An equivalent representation obtainable from Formula (2) is

$$(5) \quad \Pr(X \leq Y) = \int_0^{\infty} [1-G(x)] f(x)dx$$

where  $G(x)$  is the distribution function of the random variable  $Y$ . Again this is convenient to work with in some cases such as stresses that are Weibull distributed.

Since

$$\int_0^{\infty} f(x)dx = 1 ,$$

formula (5) can also be written as

$$(6) \quad \Pr(X \leq Y) = 1 - \int_0^{\infty} G(x)f(x)dx .$$

Each of the formulas (2), (3), (4), (5), (6) are easier to work with in special cases than the others. In developing the examples and tables in this report we have chosen the particular integral that appeared easiest for the cases considered.

d. A fourth method that can be used to evaluate the probability of failure is to reconsider the equation

$$Z = X + (-Y) .$$

From this it is clear that  $Z$  is the sum of two independent random variables  $X$  and  $-Y$ . It is well known in probability theory that the Laplace transform for the density function for the sum of two independent random variables is given by the product of the Laplace transforms of the density functions of the individual variables. Hence if  $H^*(s)$  is taken for the Laplace transform of  $h(z)$ , then

$$H^*(s) = F^*(s)G^*(-s)$$

where  $F^*(s)$  and  $G^*(-s)$  are respectively the Laplace transforms of  $f(x)$  and  $g(-x)$ .

This method of finding the probability density function of  $Z$  and thence the probability of failure has not been used in this work.



### A-3.3 Some Examples

#### A-3.3.1 Normally Distributed Strength (X), Normally Distributed Stress (Y)

It is well known that if X and Y are normally distributed with mean values  $\mu_X$  and  $\mu_Y$  and variances  $\sigma_X^2$  and  $\sigma_Y^2$  then  $Z = X - Y$  is normally distributed with mean value  $\mu_Z = \mu_X - \mu_Y$  and variance  $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$ . Consequently, the probability of failure will be given by the area under the normal curve whose mean and variance are  $\mu_Z$  and  $\sigma_Z^2$  respectively. The area is to found on the interval  $(-\infty, 0)$ . We proceed to prove these remarks to exemplify the ideas developed in A-3.2 part b.

The normal density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

Since it is easiest to develop the results using method b in Section A-3.2, we consider the random variable,  $Z = X - Y$ . It is easy to see in this case that the probability density function of Z, say  $h(z)$  is given by

$$h(z) = \frac{1}{\sqrt{2\pi}\sigma_X\sigma_Y} e^{-\frac{(y-\mu_Y)^2}{2\sigma_Y^2}} e^{-\frac{(z+y-\mu_X)^2}{2\sigma_X^2}} dy.$$

After laborious algebraic manipulation completing the square of the exponent and using the fact that

$$\int_{-\infty}^{\infty} e^{-r^2/2} dr = \sqrt{2\pi},$$

one is able to show

$$h(z) = \frac{1}{\sqrt{2\pi}(\sigma_X^2 + \sigma_Y^2)} e^{-(z - (\mu_X - \mu_Y))^2 / 2(\sigma_X^2 + \sigma_Y^2)} \quad -\infty < z < \infty.$$

That is, Z is normally distributed with mean value  $\mu_X - \mu_Y$  and variance  $\sigma_X^2 + \sigma_Y^2$ . From this it follows immediately that the probability of failure,  $\Pr(Z \leq 0)$ , is the integral of the normal curve over  $(-\infty, 0)$ .

### A-3.3.2 Exponentially Distributed Strength (X) and Exponentially Distributed Stress (Y)

Because of the simplicity of the integrals involved one can use this example to illustrate several of the methods discussed in Section A-3.2.

We take X to be distributed as

$$f(x) = ae^{-ax}, \quad 0 \leq x < \infty,$$

and Y to be distributed as

$$g(y) = be^{-by}, \quad 0 \leq y < \infty.$$

1. Using formula (2) in A-3.2 one fixes the value of one of the variables, say  $Y=y$ . For this fixed value one determines the probability that  $X \leq y$ . This is given by

$$\Pr(X \leq y | Y=y) = \int_0^y ae^{-ax} dx = 1 - e^{-ay}.$$

If we multiply this by

$$\Pr(y < Y \leq y + dy) = g(y)dy,$$

we obtain

$$\begin{aligned} \Pr(X \leq y | Y=y) \Pr(y < Y \leq y + dy) &= \Pr(X \leq y; y < Y \leq y + dy) \\ &= (1 - e^{-ay}) be^{-by} dy, \quad 0 \leq y < \infty. \end{aligned}$$

Then

$$\Pr(X \leq Y) = \int_0^{\infty} (1 - e^{-ay}) be^{-by} dy = 1 - \frac{b}{a+b},$$

which is the required probability of failure.

2. Using formula (3) of A-3.2 one must first find the probability density function of  $Z = X - Y$ . This is easily accomplished by using the formula

$$h(z) = \int_{-z}^{\infty} f(z+y) g(y) dy, \quad z \leq 0.$$

Since the probability of failure is equivalent to  $\Pr(Z \leq 0)$ , one has, using formula (3) of A-3.2 that

$$\int_{-\infty}^0 h(z) dz = \int_{-\infty}^0 \int_{-z}^{\infty} ae^{-a(z+y)} be^{-by} dy dz = \frac{a}{a+b} = 1 - \frac{b}{a+b},$$

which clearly agrees with the result found above.

3. Using formula (4) of A-3.2 one sees that

$$F(y) = \int_0^y ae^{-ax} dx = 1 - e^{-ay}.$$

Hence from formula (4) of A-3.2 one obtains the probability of failure as

$$\Pr(X \leq Y) = \int_0^{\infty} (1 - e^{-ay}) be^{-by} dy.$$

This integral, of course is

$$\Pr(X \leq Y) = 1 - \frac{b}{a+b},$$

which again agrees with the other results of this section.

#### A-3.3.3 Gamma Distributed Strength (X) and Gamma Distributed Stress (Y)

In some applications one finds typically that the probability density function for the random variable has a form as shown in Figure A-3.4, and that the gamma density function, given by the formula

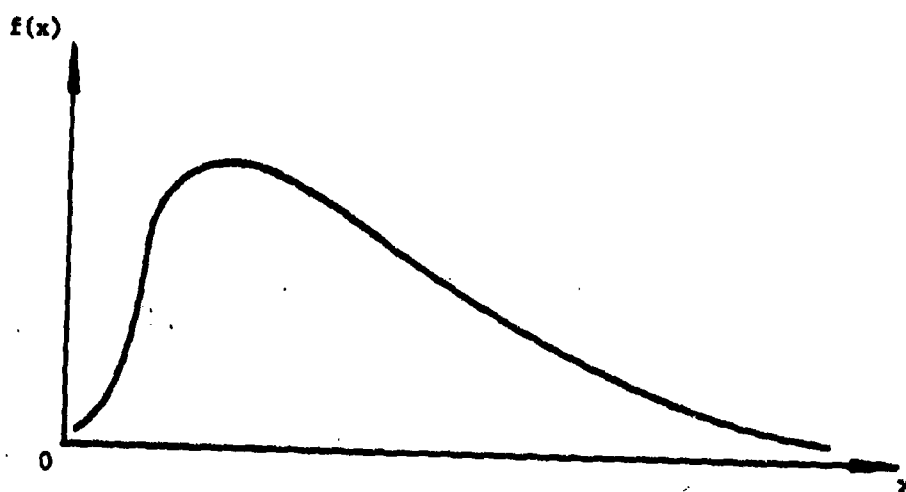


Figure A - 3.4 Possible Data Plot

$$(1) \quad f(x) = \frac{\lambda^n x^{n-1} e^{-\lambda x}}{\Gamma(n)}, \quad n > 0, \quad 0 \leq x < \infty, \quad \lambda > 0$$

can be used to closely fit the data.  $\lambda$  in the formula is a scale parameter.  $n$  is a shape parameter. For  $n = 1$  the gamma density function is the negative exponential discussed above. For large values of  $n$ , the gamma density can be approximated by the normal density. Hence the gamma function supplies a family of densities that roughly fall between the two cases previously discussed.

It has been shown in formula (3) of Section A-3.2 that if one considers the problem of determining the distribution of the difference variate  $Z$  (i.e.,  $Z = X - Y$ ) for given distributions for  $X, Y$ , then the density function for  $Z$  can be found from

$$(2) \quad \begin{aligned} h(z)dz &= \Pr[z \leq Z < z + dz] = \int_0^{\infty} f(z+y)g(y)dydz, \quad 0 \leq z < \infty, \\ &\int_{-z}^{\infty} f(z+y)g(y)dydz, \quad -\infty < z \leq 0. \end{aligned}$$

Hence one is interested in

$$\int_{-\infty}^0 h(z)dz = \int_{-\infty}^0 \int_{-z}^{\infty} f(z+y)g(y)dydz.$$

Equivalently one is interested in

$$\int_0^{\infty} h(z)dz = \int_0^{\infty} \int_0^{\infty} f(z+y)g(y)dy,$$

from which  $P(Z \leq 0) = 1 - \int_0^{\infty} h(z)dz$ . Here we suppose

$$(3) \quad f(x) = \frac{1}{\Gamma(m)} x^{m-1} e^{-x}, \quad 0 \leq x < \infty,$$

$$(4) \quad g(y) = \frac{1}{\Gamma(n)} y^{n-1} e^{-y}, \quad 0 \leq y < \infty.$$

(Later we shall extend this definition of the gamma function to include the scale parameter  $\lambda$  in formula 1.)

Straight forward substitution of (3) and (4) into (2) leads to

$$h(z) = \frac{1}{\Gamma(m)\Gamma(n)} \int_0^{\infty} (z+y)^{m-1} e^{-(z+y)} y^{n-1} e^{-y} dy, \quad z \geq 0.$$

The substitution  $v = y/z$  lead to

$$h(z)dz = \frac{dz}{\Gamma(m)\Gamma(n)} z^{m+n-1} e^{-z} \int_0^{\infty} v^{n-1} (1+v)^{m-1} e^{-2zv} dv, \quad z \geq 0.$$

The integral can be expressed in terms of the well known confluent hypergeometric function  $\Gamma(n)U(n, n+m, 2z)$  and the function  $h(z)dz$  can be expressed in terms of the well known Whittaker function  $W_{k,m}(2z)$ . In the special case  $m = n$  the Whittaker function can be expressed in terms of the Bessel function  $K_m(X)$ . Hence in general  $h(z)dz$  can be found in terms of well known functions.

The above results define the density function for  $Z \geq 0$ . The probability that  $Z \geq 0$  is given by

$$\int_0^{\infty} h(z)dz.$$

From the above discussion this is equivalent to

$$\int_0^{\infty} W_{k,m}(x)dx$$

where  $W_{k,m}(x)$  is the Whittaker function. From the definition of  $h(z)dz$  this is also equivalent to the double integral

$$\frac{1}{\Gamma(m)\Gamma(n)} \int_0^{\infty} z^{m+n-1} e^{-z} dz \int_0^{\infty} v^{n-1} (1+v)^{m-1} e^{-2zv} dv.$$

\* The expression of  $h(z)dz$  in terms of  $K_m(x)$  was first found by Pearson, et al<sup>13</sup>. The general result of  $h(z)dz$  in terms of the Whittaker function was first found by Kullback<sup>14</sup>. Our results follow directly from the definition of these functions<sup>15</sup>.

The last formulation is easiest to work with.

Interchanging the order of integration and noting that the integral involving  $z$  is by definition

$$\frac{\Gamma(m+n)}{(1+2v)^{m+n}}$$

leads to the single integral

$$\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^\infty \frac{(1+v)^{m-1} v^{n-1}}{(1+2v)^{m+n}} dv,$$

If now one takes  $u = v/(1+2v)$  it follows directly that the integral can be written as:

$$(5) \quad \Pr(Z \geq 0) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^{1/2} (1-u)^{m-1} u^{n-1} du.$$

This integral is the well known incomplete beta function  $B_{1/2}(m,n)$ . Hence, finally

$$\Pr(Z \geq 0) = \frac{1}{B(m,n)} B_{1/2}(m,n)$$

where  $B(m,n) = [\Gamma(m+n)/\Gamma(m)\Gamma(n)]^{-1}$ . It follows directly that the probability of failure is given by  $1 - \Pr(Z \geq 0)$ .

In all the previous results we have taken the gamma distribution in the form

$$f(x) = \frac{1}{\Gamma(m)} x^{m-1} e^{-x}, \quad 0 \leq x < \infty.$$

A simple generalization occurs if one admits the scale parameters  $\lambda, \mu$ . It is easy to show that the resultant probability density function is given as

$$f(x) = \frac{\lambda^m}{\Gamma(m)} x^{m-1} e^{-\lambda x}, \quad \lambda > 0, 0 \leq x < \infty, m > 0,$$

$$g(y) = \frac{\mu^n}{\Gamma(n)} y^{n-1} e^{-\mu y}, \quad \mu > 0, 0 \leq y < \infty, n > 0.$$

If one introduces this into Equation (2) for  $h(z)dz$  it follows by previously used methods that

$$h(z)dz = \frac{\lambda^m \mu^n}{\Gamma(m)\Gamma(n)} z^{m+n-1} e^{-\lambda z} \int_0^\infty (1+v)^{m-1} v^{n-1} e^{-(\lambda+\mu)zv} dv, \quad 0 \leq z \leq \infty,$$

which leads to  $h(z)dz$  being expressed in terms of the Whittaker function with argument  $(\lambda+\mu)z$  instead of  $2z$  as previously.

From the above one has, again using the previous methods,

$$\int_0^\infty h(z)dz = \frac{r^n \Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^\infty \frac{(1+v)^{m-1} v^{n-1}}{[1 + (1+r)v]^{m+n}} dv,$$

where  $r = \mu/\lambda$ . The change of variable  $u = rv/(1 + (1+r)v)$  allows one to express the above integral as

$$\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^{\frac{r}{1+r}} \frac{r}{1+r} (1-u)^{m-1} u^{n-1} du$$

which involves  $r$  only in the limit of integration. Hence  $P(Z \geq 0)$  can be expressed as the incomplete beta function whose truncation occurs at  $r/(1+r)$  instead of  $1/2$  as found in formula (5).

Therefore,  $B(m, n)$

$$(6) \quad \Pr(Z \geq 0) = \frac{\frac{r}{1+r}}{B(m, n)}$$

#### Special Cases

1. It is clear that for  $\lambda = \mu$ ,  $r = 1$  and  $r/(1+r) = 1/2$ . Hence all of the preceding work involving  $\lambda = \mu = 1$  holds for  $\lambda = \mu \neq 1$ .
2. If  $m = n = 1$  then for  $\lambda = \mu = 1$ ,  $\Pr(Z \geq 0) = 1/2$  and in general for  $\lambda = \mu \neq 1$  it is clear from the above that this holds. But this is expected since in the case  $m = n$ , both  $X$  and  $Y$  are negative exponentially distributed and for



$\lambda = \mu$  they are identically distributed no matter what  $\lambda$  or  $\mu$  are. Hence this case corresponds to taking the difference between two identical negative exponential variates and as one would expect  $P(Z \geq 0) = 1/2$  for any choice of  $\lambda$  and  $\mu$  for which  $\lambda = \mu$ .

3. If in 1, above,  $m = n = 1$  but  $\lambda \neq \mu$  then it follows that

$$P(Z \geq 0) = \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} \int_0^{\frac{r}{1+r}} du,$$

$$P(Z \geq 0) = \frac{r}{1+r}.$$

$$\text{The probability of failure} = 1 - \frac{r}{1+r}.$$

4. If  $m = 1$ ,  $n \neq 1$  then

$$\Pr(Z \geq 0) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^{\frac{r}{1+r}} \frac{r}{1+ru} n-1 du = \frac{n\Gamma(n)}{\Gamma(n)} \left( \frac{r}{1+r} \right)^n \frac{1}{n}.$$

$$\text{Thus the probability of failure is } 1 - \left( \frac{r}{1+r} \right)^n.$$

$$\text{In the special case } r = 1, \text{ this gives } 1 - (1/2)^n.$$

5. If  $m \neq 1$ ,  $n = 1$  then.

$$\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^{\frac{r}{1+r}} (1-u)^{m-1} du = 1 - \left( \frac{r}{1+r} \right)^m.$$

$$\text{Thus the probability of failure is } \left( \frac{r}{1+r} \right)^m \text{ which gives } (1/2)^m \text{ in the case } r = 1.$$

The incomplete beta function has been tabulated<sup>16,17</sup>. From these tables one can determine other probabilities of failure from formula (6).

### A-3.3.4 Weibull distributed Strength (X) and Weibull distributed Stress (Y)

The Weibull density function arises often in reliability studies (see Section 6 of this report). It is defined by the formula

$$f(x) = \frac{b}{\theta - x_0} \left( \frac{x - x_0}{\theta - x_0} \right)^{b-1} e^{-\left( \frac{x - x_0}{\theta - x_0} \right)^b} dx, \quad x_0 \leq x < \infty.$$

$b$  is called the slope,  $\theta$  is the characteristic value (characteristic strength for example) and  $x_0$  is a location parameter for the left end point of the distribution. Graphs of the distribution are given in Section 6. Plotting the Weibull on  $\ln-x$  vs.  $\ln \ln 1 / (1 - F(x))$  paper one finds the distribution to plot as a straight line. (see Section 6)

For purposes of interference theory the Weibull density function is rather difficult to work with. The probability of failure can not be obtained in closed form as we have been able to do in the case of the negative exponential densities. Neither is the integral expressible, except in certain cases in terms of well-known and tabulated functions as is true of the gamma and normal densities. Hence we derive an integral expression for the probability of failure when  $X$  and  $Y$  are both Weibull distributed random variables in this section. In the cases for which the resulting integral is well-known we give the results for future use. In section A-4.1 we will discuss numerical evaluations of the integral used to obtain the tabulation of the probability of failure given in the tables in section A-2.2.

We take

$$f(x) = \frac{b_x}{\theta'_x} \left( \frac{x - x_0}{\theta'_x} \right)^{b_x-1} e^{-\left( \frac{x-x_0}{\theta'_x} \right)^{b_x}}, \quad 0 \leq x < \infty.$$

$$g(y) = \frac{b_y}{\theta'_y} \left( \frac{y - y_0}{\theta'_y} \right)^{b_y-1} e^{-\left( \frac{y-y_0}{\theta'_y} \right)^{b_y}}, \quad 0 \leq y < \infty.$$

In these cases we have taken  $\theta'_x = \theta_x - x_0$  and  $\theta'_y = \theta_y - y_0$ . Then using the formulas (4), (5), (6) of section A-3.2 one obtains:

$$\Pr(Y \geq X) = \Pr(Y \geq x | X = x) \Pr(X = x).$$

$$\Pr(Y \geq x | X = x) = \int_x^\infty \frac{b_y}{\theta'_y} \left( \frac{y - y_0}{\theta'_y} \right)^{b_y-1} e^{-\left( \frac{y-y_0}{\theta'_y} \right)^{b_y}} dy = e^{-\left( \frac{x-y_0}{\theta'_y} \right)^{b_y}}.$$

The joint probability  $\Pr(Y \geq x \quad x \leq X \leq x + dx)$  is then given by

$$\frac{b_x}{\theta'_x} \left( \frac{x - x_0}{\theta'_x} \right)^{b_x-1} e^{-\left( \frac{x-x_0}{\theta'_x} \right)^{b_x}} e^{-\left( \frac{x-y_0}{\theta'_y} \right)^{b_y}} dx.$$

The integral of this expression is then the probability of failure desired. Let

$$u = \left( \frac{x-x_0}{\theta'_x} \right)^{b_x}; \quad du = \frac{b_x}{\theta'_x} \left( \frac{x-x_0}{\theta'_x} \right)^{b_x-1} dx; \quad u^{1/b_x} \theta'_x + x_0 = x$$

$$\Pr(\text{failure}) = \int_0^\infty e^{-u} e^{-\left( \frac{\theta'_x}{\theta'_y} u^{1/b_x} + \frac{x_0 - y_0}{\theta'_y} \right)^{b_y}} du.$$

In table A-2.2 we choose to work with the integral

$$(1) \quad \int_0^\infty e^{-u} e^{-\left( \frac{\theta'_x}{\theta'_y} u^{1/b_x} + \frac{x_0 - y_0}{\theta'_y} \right)^{b_y}} du,$$

with identifiers for the table taken to be:

$$\begin{aligned} \frac{x_0 - y_0}{\theta'_x} &= \frac{x_0 - y_0}{\text{Theta 1}}, \\ \frac{\theta'_y}{\theta'_x} &= \frac{\text{Theta 2}}{\text{Theta 1}}, \\ \frac{b_x}{b_y} &= \frac{B1}{B2}, \end{aligned}$$

Three special cases can be computed in terms of well-known functions using integral (1). We derive these here for use in checking the tables developed in section A.2.2.

Case 1.  $b_x = b_y = 1$ .

We have already considered this case for  $x_0 = y_0 = 0$ ,  $\theta'_x/\theta'_y = 1$ , since the Weibull's are then simply identical exponentials. If  $x_0 \neq y_0$ , one has from (1) that the probability of failure is:

$$\int_0^\infty e^{-u} e^{-\left[ \frac{\theta'_x}{\theta'_y} u + \left( \frac{x_0 - y_0}{\theta'_y} \right) \right]} du.$$

This integral can be expressed in closed form as

$$\frac{e^{-\frac{|x_0 - y_0|}{\theta'_y}}}{1 + \frac{\theta'_x}{\theta'_y}}, \text{ for any } \frac{\theta'_x}{\theta'_y}.$$

For  $x_0 = y_0$  and  $\theta'_x = \theta'_y$  this expression gives the probability of failure as  $1/2$ . For  $x_0 = y_0$ ,  $(\theta'_x)/\theta'_y \neq 1$  one can check this with the results given in section A-3.3.2.

For comparison with table A-2.2 we use the equivalent form

$$\frac{e^{-\frac{\theta'_x}{\theta'_y} \left( \frac{x_0 - y_0}{\theta'_x} \right)}}{1 + \frac{\theta'_x}{\theta'_y}}.$$

Case 2.  $b_x = 1$ ,  $b_y = 2$ .

In this case the integral to be evaluated is:

$$\int_0^\infty e^{-u} e^{-\left( \frac{\theta'_x}{\theta'_y} u + \frac{x_0 - y_0}{\theta'_y} \right)^2} dy.$$

To evaluate this integral we will expand the square on the exponent to give the integral in the form

$$\int_0^\infty e^{-(au^2 + 2bu + c)} du,$$

which can be expressed in terms of the error function. If this is done one finds that

$$a = \left( \frac{\theta'_x}{\theta'_y} \right)^2; b = \frac{1}{2} + \left( \frac{\theta'_x}{\theta'_y} \right)^2 \left( \frac{x_0 - y_0}{\theta'_y} \right); c = \left( \frac{x_0 - y_0}{\theta'_y} \right)^2 \left( \frac{\theta'_x}{\theta'_y} \right)^2.$$

It is well known (formula 7.4.2 of 'Ref. 15) that the integral (2) can be written in terms of the error function as:

$$(4) \quad \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{\left(\frac{b^2 - ac}{a}\right)} \operatorname{erfc} \left( \frac{b}{\sqrt{a}} \right).$$

Computer subroutines for the error function exist and thus this expression can be evaluated easily for given values of the parameters  $a$ ,  $b$ ,  $c$ . This will be discussed further in section A-4.1.5.

Case B.  $b_x = 2$ ,  $b_y = 1$ .

This case can be developed in a fashion similar to case 2. The integral to be evaluated is:

$$\int_0^\infty e^{-u} e^{-\left(\frac{\theta'_x}{\theta'_y} u^2 + \frac{x_0 - y_0}{\theta'_y}\right)} du.$$

This can be expressed as:

$$e^{-\left(\frac{\theta'_x}{\theta'_y}\right)\left(\frac{x_0 - y_0}{\theta'_x}\right)} \int_0^\infty 2re^{-r^2 - \frac{\theta'_x}{\theta'_y} r} dr$$

where  $r^2 = u$ . From this one expresses this integral as

$$(5) \quad e^{-\left(\frac{\theta'_x}{\theta'_y}\right)\left(\frac{x_0 - y_0}{\theta'_x}\right)} \left[ 1 - \frac{\theta'_x}{\theta'_y} \int_0^\infty e^{-(at^2 + 2bt + c)} dt \right]$$

where  $a = 1$ ,  $b = \theta'_x/\theta'_y$  and  $c = 0$ .

Thus the integral can be expressed in terms of the error function as above.

### A-3.3.5 Weibull distributed strength (X) and Normally distributed stress (Y).

As shown in Section 8 one is often interested in the case in which the strength has a Weibull distribution and the stress is normally distributed. In this section we take

$$f(x) = \frac{b}{\theta'} \left( \frac{x-x_0}{\theta'} \right)^{b-1} e^{-\left( \frac{x-x_0}{\theta'} \right)^b}, \quad x_0 \leq x < \infty.$$

$$g(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}, \quad -\infty \leq y < \infty.$$

In this case the probability of failure cannot be evaluated in closed form as was done for the negative exponential cases of section A-3.3.2 nor, except in special cases can the integral be reduced to well-known functions as was done in section A-3.3.3. In this section we will develop the integral that is used in the tables given in section A-2.1.

Since  $f(x)$  is truncated at  $x_0$  one must consider 2 cases.

Case 1. If  $y < x_0$

$$\Pr(X \leq y | Y = y) = 0.$$

Case 2. If  $y \geq x_0$

$$\begin{aligned} \Pr(X \leq y | Y = y) &= \int_{x_0}^y \frac{b}{\theta'} \left( \frac{x-x_0}{\theta'} \right)^{b-1} e^{-\left( \frac{x-x_0}{\theta'} \right)^b} dx, \\ &= 1 - e^{-\left( \frac{y-x_0}{\theta'} \right)^b}, \quad x_0 \leq y < \infty. \end{aligned}$$

Hence using formula (4) section A-3.2 one has

$$\begin{aligned} \Pr(Y \geq X) &= \int_{x_0}^{\infty} \left( 1 - e^{-\left( \frac{y-x_0}{\theta'} \right)^b} \right) \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy, \\ &= \frac{1}{\sqrt{2\pi}\sigma} \int_{x_0}^{\infty} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy - \frac{1}{\sqrt{2\pi}\sigma} \int_{x_0}^{\infty} e^{-\left( \frac{y-x_0}{\theta'} \right)^b - \frac{(y-\mu)^2}{2\sigma^2}} dy. \end{aligned}$$

The first integral is the area under the upper tail of the normal distribution. This area is usually denoted by:

$$1 - \Phi \left( \frac{x_0 - \mu}{\sigma} \right).$$

The second integral is troublesome because as it stands it contains five parameters which must be considered separately for computing formulas. The problem can be reduced in size by the change of variable

$$u = \frac{y - x_0}{\sigma'}, \quad du = \frac{dy}{\sigma'}.$$

Then the second integral becomes

$$(1) \quad \frac{1}{\sqrt{2\pi}} \left( \frac{\sigma'}{\sigma} \right) \int_0^\infty e^{-u^b} e^{-\frac{1}{2} \left( \frac{\sigma'}{\sigma} u + \frac{x_0 - \mu}{\sigma} \right)^2} du.$$

In this form there are only three free parameters,  $b$ ,  $\frac{\sigma'}{\sigma}$ , and  $\frac{x_0 - \mu}{\sigma}$ . Tables in section A-3.1 have been built using these three parameters. Two special cases serve as checks for the numerical analysis used later.

If  $b = 1$  or  $2$  the integral (1) can be evaluated in terms of the error function whose values have been tabulated.

Case 1:  $b = 1$

For  $b = 1$  one completes the square on the exponent inside the integral to obtain the integral in the form

$$(2) \quad \int_0^\infty e^{-(at^2 + 2bt + c)} dt$$

whose value in terms of the error function is well known (see Ref. 15, Formula 7.4.2). Here

$$a = \frac{1}{2} \left( \frac{\sigma'}{\sigma} \right)^2; \quad b = \frac{1}{2} \left[ \frac{\sigma'}{\sigma} \frac{x_0 - \mu}{\sigma} + 1 \right]; \quad c = \frac{1}{2} \left( \frac{x_0 - \mu}{\sigma} \right)^2.$$

Hence the probability of failure can be expressed in terms of: (a) the area under the normal curve  $1 - \Phi\left(\frac{x_0 - \mu}{\sigma}\right)$ ; (b) the error function. Since the area under the normal curve can also be expressed in terms of the error function, one can find the probability of failure completely from a computer routine that calculates the error function. In checking Table A-2.1 numerical values for the probability of failure have been computed from this computer subroutine. A further discussion of the checking procedure and results are given in A-4.1.7.

Case 2:  $b = 2$

For  $b = 2$ ; one can once again determine the value of integral (1) in terms of the error function. In terms of the parameters  $a, b, c$ , given in formula (2) above one finds

$$(3) \quad a = \frac{1}{2} \left[ \left( \frac{\sigma'}{\sigma} \right)^2 + 2 \right]; \quad b = \frac{1}{2} \left( \frac{\sigma'}{\sigma} \right) \left( \frac{x_0 - \mu}{\sigma} \right); \quad c = \frac{1}{2} \left( \frac{x_0 - \mu}{\sigma} \right)^2.$$

Again, as for  $b = 1$  above, values for the probability of failure were determined from the computer subrouting giving the error function values. The values thus determined were used to check Tables A-3.2.2.

In Table A-2.1 we choose to identify the necessary parameters in shorter form for typographical simplification. Hence we identify

$$b = B(x),$$

$$A = \frac{x_0 - \mu}{\sigma},$$

$$C = \frac{\sigma'}{\sigma}.$$



APPENDIX 4 TABLES OF THE INTEGRALS IN A-3.3.4 AND A-3.3.5

#### A-4 DISCUSSION OF THE EVALUATION OF INTEGRALS IN A-3.3.4 AND A-3.3.5

##### A-4.1 NUMERICAL ANALYSIS

Both of the integrals obtained in sections A-3.3.4 and A-3.3.5 must be evaluated using numerical methods. We discuss our approach to this problem in the following sections.

##### A-4.1.1 The Problem.

The integrals to be evaluated in A-3.3.4 are of the form

$$\int_0^{\infty} e^{-u} f(u) du,$$

where  $f(u)$  is a negative exponential with exponent of the form  $u^b$ .

The integrals to be evaluated in A-3.3.5 are of the form

$$\int_0^{\infty} e^{-u^b} f(u) du,$$

where  $f(u)$  is a negative exponential with exponent of the form  $u^2$ .

##### A-4.1.2 Method of Solution

Integrals of the form given in A-4.1.1 can be evaluated in several ways. We have chosen to use Simpson's rule with variable step sizes as discussed in sections A-4.1.4 and A-4.1.7. The approximation to the integral is given by

$$(1) \quad \frac{h^2}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n}) \right]$$

with a remainder term given by:

$$(2) \quad \frac{h^5}{90} f^{(iv)}(\xi_1) \quad ; \quad x_1 < \xi_1 < x_{1+1}.$$

In this formulation  $h$  is defined to be

$$h = (x_{1+1} - x_1)/2n$$

where  $n$  is the number of steps taken in the interval  $(x_1, x_{i+1})$ .

The values for the probability of failure were calculated using Simpson's rule as an approximation to the integral given in A-4.1.1. Simpson's rule was computed to  $10^{-6}$  using the University of Michigan IBM 7090 and the MAD language. Values thus obtained were rounded to  $10^{-4}$  as they appear in the table.

#### A-4.1.3 Properties of Simpson's Rule

The functions to be evaluated are reasonably well behaved. Tables of a few of the integrands are given in Tables A-4.1 - A-4.6 on the following pages. It is clear that the functions are monotone decreasing. They are probabilities and hence are bounded above by 1 and below by 0. The functions themselves decay quite rapidly.

Unfortunately, derivatives of the function for any value of  $b_x$  in those integrals given in section A-3.3.4 are asymptotically infinite at  $u \rightarrow 0$ . Derivatives up to order 4 of the functions for non-integral  $b < 4$  are asymptotically  $\infty$  for  $u \rightarrow 0$  for those integrals given in section A-3.3.5. Because of this property of the higher order derivatives, it was deemed best not to use numerical approximations such as Gauss - Laguerre or Gauss - Legendre methods whose error terms depend only on higher order derivatives. Instead Simpson's rule was chosen so that some attempt could be made to control errors by varying the step sizes as the higher order derivatives become large.

#### A-4.1.4 Error Analysis - Weibull - Weibull Case

Since the remainder term of Simpson's rule depends on the fourth derivative of the integrand, this derivative was determined. An analytic expression is given by the following. We are interested in the case:

1.  $b_y = 1, 2, \dots, 10;$
2.  $b_x = 1, 2, \dots, 10;$
3.  $\frac{x_0 - y_0}{\sigma_x} = 0, .25, .50, .75, 1.00;$
4.  $\frac{\sigma_x}{\sigma_y} = 1, 1.25, 1.50, \dots, 3.$

Parameters:  $B1 = 1, B2 = 1$   
 $\Theta_1 = 1, \Theta_2 = 1, X_0 - Y_0 = 0$

$u$	$F(u)$	$F^{(4)}(u)$
$1 \times 10^{-20}$	1.	16.
.5	.367879	5.88607
1.	.135335	2.16536
1.5	$4.97871 \times 10^{-2}$	.796593
2.	$1.83156 \times 10^{-2}$	.29305
2.5	$6.73795 \times 10^{-3}$	.107807
3.	$2.47875 \times 10^{-3}$	.03966
3.5	$9.11882 \times 10^{-4}$	$1.45901 \times 10^{-2}$
4.	$3.35463 \times 10^{-4}$	$5.3674 \times 10^{-3}$
4.5	$1.2341 \times 10^{-4}$	$1.97456 \times 10^{-3}$
5.	$4.53999 \times 10^{-5}$	$7.26399 \times 10^{-4}$
5.5	$1.67017 \times 10^{-5}$	$2.67227 \times 10^{-4}$
6.0	$6.14421 \times 10^{-6}$	$9.83074 \times 10^{-5}$
6.5	$2.26033 \times 10^{-6}$	$3.61653 \times 10^{-5}$
7.	$8.31529 \times 10^{-7}$	$1.33045 \times 10^{-5}$
7.5	$3.05902 \times 10^{-7}$	$4.89444 \times 10^{-6}$

Table A-4.1 Integrands and Their 4th Derivative  
for Weibull-Weibull Case

Parameters:  $B1 = 10, B2 = 1$   
 $\text{Theta } 1 = 1, \text{Theta } 2 = 1, X_0 - Y_0 = 1$

$u$ $1 \times 10^{-20}$	$F(u)$ .364219	$F^{IV}(u)$ $> 1 \times 10^6$
.1	.150419	$7.84551 \times 10^2$
.5	$8.77702 \times 10^{-2}$	1.63429
1.	$4.97871 \times 10^{-2}$	.168733
1.5	$2.89733 \times 10^{-2}$	$5.58241 \times 10^{-2}$
2.	$1.70471 \times 10^{-2}$	$2.59855 \times 10^{-2}$
2.5	$1.00925 \times 10^{-2}$	$1.36768 \times 10^{-2}$
3.	$5.99924 \times 10^{-3}$	$7.59447 \times 10^{-3}$
3.5	$3.57617 \times 10^{-3}$	$4.33376 \times 10^{-3}$
4.	$2.13626 \times 10^{-3}$	$2.51207 \times 10^{-3}$
4.5	$1.27819 \times 10^{-3}$	$1.47049 \times 10^{-3}$
5.	$7.65778 \times 10^{-4}$	$8.66458 \times 10^{-4}$
5.5	$4.59272 \times 10^{-4}$	$5.12927 \times 10^{-4}$
6.	$2.75691 \times 10^{-4}$	$3.04688 \times 10^{-4}$
6.5	$1.65615 \times 10^{-4}$	$1.81464 \times 10^{-4}$
7.	$9.95537 \times 10^{-5}$	$1.08298 \times 10^{-4}$
7.5	$5.98766 \times 10^{-5}$	$6.47386 \times 10^{-5}$

Table A-4.2 Integrands and Their 4th Derivative  
for Weibull-Weibull Case

Parameters:  $B1 = 10, B2 = 1$   
 $\Theta_1 = 1, \Theta_2 = 1, X_0 - Y_0 = 0$

$u$ $1 \times 10^{-20}$	$F(u)$	$F^{IV}(u)$
	.99005	$> 2.9 \times 10^{76}$
.1	.408882	2132.63
.5	.238584	4.44245
1.	.135335	.4588
1.5	$7.87577 \times 10^{-2}$	.151746
2.	$4.63389 \times 10^{-2}$	$7.06358 \times 10^{-2}$
2.5	$2.74344 \times 10^{-2}$	$3.71774 \times 10^{-2}$
3.	$1.63076 \times 10^{-2}$	$2.06439 \times 10^{-2}$
3.5	$9.72105 \times 10^{-3}$	$1.17804 \times 10^{-2}$
4.	$5.80696 \times 10^{-3}$	$6.82852 \times 10^{-3}$
4.5	$3.47449 \times 10^{-3}$	$3.99719 \times 10^{-3}$
5.	$2.0816 \times 10^{-3}$	$2.35528 \times 10^{-3}$
5.5	$1.24843 \times 10^{-3}$	$1.39428 \times 10^{-3}$
6.	$7.49405 \times 10^{-4}$	$8.28227 \times 10^{-4}$
6.5	$4.50188 \times 10^{-4}$	$4.93271 \times 10^{-4}$
7.	$2.70615 \times 10^{-4}$	$2.94384 \times 10^{-4}$
7.5	$1.62762 \times 10^{-4}$	$1.75978 \times 10^{-4}$

Table A-4.3 Integrands and Their 4th Derivative  
for Weibull-Weibull Case

Parameters:  $B = 1$ ,  $\frac{\theta}{\sigma} = 100$ ,  $\frac{X_0 - \mu}{\sigma} = -4$

$u$	$F(u)$	$F^{(4)}(u)$
$1 \times 10^{-25}$	$3.35463 \times 10^{-4}$	$5.39857 \times 10^6$
.05	.57695	$-1.20005 \times 10^8$
.1	$1.37807 \times 10^{-8}$	1503.39
.15	$4.5713 \times 10^{-27}$	$6.3861 \times 10^{-15}$
.2	0	0

Table A-4.4 Integrands and Their 4th Derivative  
for Weibull-Normal Case

Parameters:  $B = 1$ ,  $\frac{\theta}{\sigma} = 10$ ,  $\frac{X_0 - \mu}{\sigma} = -7$

$u$	$F(u)$	$F^{iv}(u)$
$1 \times 10^{-25}$	$2.28973 \times 10^{-11}$	$4.54295 \times 10^{-4}$
.05	$6.36523 \times 10^{-10}$	$9.13387 \times 10^{-3}$
.1	$1.37807 \times 10^{-8}$	.138616
.15	$2.32355 \times 10^{-7}$	1.57617
.2	$3.05113 \times 10^{-6}$	13.2852
.25	$3.12029 \times 10^{-5}$	81.6422
.3	$2.48517 \times 10^{-4}$	355.588
.35	$1.5415 \times 10^{-3}$	1037.02
.4	$7.44658 \times 10^{-3}$	1732.68
.45	$2.80154 \times 10^{-2}$	453.178
.5	.082085	-4619.66
.55	.187308	-9212.57
.6	.332871	-4007.44
.65	.460704	9516.3
.7	.496585	14600.1
.75	.416862	4041.9
.8	.272532	-7619.72
.85	.138761	-8057.04
.9	$5.50232 \times 10^{-2}$	-2207.48
.95	$1.69922 \times 10^{-2}$	1382.76
1.	$4.08677 \times 10^{-3}$	1540.39
1.05	$7.65436 \times 10^{-4}$	713.445
1.1	$1.11666 \times 10^{-4}$	206.265
1.15	$1.26862 \times 10^{-5}$	41.0759
1.2	$1.12245 \times 10^{-6}$	5.87556
1.25	$7.73443 \times 10^{-8}$	.617431
1.3	$4.15066 \times 10^{-9}$	$4.83271 \times 10^{-2}$
1.35	$1.73473 \times 10^{-10}$	$2.84342 \times 10^{-3}$
1.4	$5.64642 \times 10^{-12}$	$1.26576 \times 10^{-4}$

Table A-4.5 Integrands and Their 4th Derivative  
for Weibull-Normal Case



Parameters:  $B = 1$ ,  $\frac{\theta}{\sigma} = 10$ ,  $\frac{X_0 - \mu}{\sigma} = -7$

$u$	$F(u)$	$F^{IV}(u)$
1.45	$1.43133 \times 10^{-13}$	$4.28349 \times 10^{-6}$
1.5	$2.82576 \times 10^{-15}$	$1.10601 \times 10^{-7}$
1.55	$4.34466 \times 10^{-17}$	$2.18507 \times 10^{-9}$
1.6	$5.20239 \times 10^{-19}$	$3.31061 \times 10^{-11}$
1.65	$4.85151 \times 10^{-21}$	$3.8538 \times 10^{-13}$
1.7	$3.52352 \times 10^{-23}$	$3.45198 \times 10^{-15}$
1.75	$1.99296 \times 10^{-25}$	$2.38233 \times 10^{-17}$
1.8	$8.7792 \times 10^{-28}$	$1.26811 \times 10^{-19}$
1.85	$3.01185 \times 10^{-30}$	$5.21112 \times 10^{-22}$
1.9	0	0

Table A-4.5 Integrands and Their 4th Derivative  
for Weibull-Normal Case (continued).

Parameters:  $B = 1$ ,  $\frac{\theta'}{\sigma} = 10$ ,  $\frac{X_0 - \mu}{\sigma} = -10$

$u$	$F(u)$	$F^{iv}(u)$
$1 \times 10^{-25}$	$1.92875 \times 10^{-22}$	$1.73991 \times 10^{-14}$
.05	$2.40296 \times 10^{-20}$	$1.74943 \times 10^{-12}$
.1	$2.33155 \times 10^{-18}$	$1.35275 \times 10^{-10}$
.15	$1.76184 \times 10^{-16}$	$8.03111 \times 10^{-9}$
.2	$1.03685 \times 10^{-14}$	$3.65341 \times 10^{-7}$
.25	$4.75219 \times 10^{-13}$	$1.27031 \times 10^{-5}$
.3	$1.69628 \times 10^{-11}$	$3.3655 \times 10^{-4}$
.35	$4.71548 \times 10^{-10}$	$6.76653 \times 10^{-3}$
.4	$1.0209 \times 10^{-8}$	.102689
.45	$1.72133 \times 10^{-7}$	1.16765
.5	$2.26033 \times 10^{-6}$	9.84193
.55	$2.31157 \times 10^{-5}$	60.482
.6	$1.84106 \times 10^{-4}$	263.426
.65	$1.14197 \times 10^{-3}$	768.245
.7	$5.51656 \times 10^{-3}$	1283.6
.75	$2.07543 \times 10^{-2}$	335.722
.8	$6.08101 \times 10^{-2}$	3422.33
.85	.138761	-6824.84
.9	.246597	-2968.78
.95	.341298	7049.85
1.	.367879	10816.
1.05	.308819	2994.31
1.1	.201897	-5644.82
1.15	.102797	-5968.8
1.2	$4.07622 \times 10^{-2}$	-1635.34
1.25	$1.25881 \times 10^{-2}$	1024.41
1.3	$3.02756 \times 10^{-3}$	1141.10
1.35	$5.67086 \times 10^{-4}$	528.533
1.4	$8.27214 \times 10^{-5}$	152.805
1.45	$9.39813 \times 10^{-6}$	30.4298
1.5	$8.31529 \times 10^{-7}$	4.35272
1.55	$5.72981 \times 10^{-8}$	.497404

Table A-4.6 Integrands and Their 4th Derivative  
for Weibull-Normal Case

Parameters:  $B = 1$ ,  $\frac{\theta'}{\theta} = 10$ ,  $\frac{X_0 - \mu}{\sigma} = -10$

$u$	$F(u)$	$F^{IV}(u)$
1.6	$3.07488 \times 10^{-9}$	$3.58016 \times 10^{-2}$
1.65	$1.28512 \times 10^{-10}$	$2.10646 \times 10^{-3}$
1.7	$4.18297 \times 10^{-12}$	$9.37701 \times 10^{-5}$
1.75	$1.06036 \times 10^{-13}$	$3.17329 \times 10^{-6}$
1.8	$2.09338 \times 10^{-15}$	$8.1935 \times 10^{-8}$
1.85	$3.21861 \times 10^{-17}$	$1.61874 \times 10^{-9}$
1.9	$3.85403 \times 10^{-19}$	$2.45256 \times 10^{-11}$
1.95	$3.59409 \times 10^{-21}$	$2.85496 \times 10^{-13}$
2.	$2.61029 \times 10^{-23}$	$2.55729 \times 10^{-15}$
2.05	$1.47644 \times 10^{-25}$	$1.76487 \times 10^{-17}$
2.1	$6.5038 \times 10^{-28}$	$9.39437 \times 10^{-20}$
2.15	$2.23124 \times 10^{-30}$	$3.8605 \times 10^{-22}$
2.2	0	0

Table A-4. Integrands and Their 4th Derivative for Weibull-Normal Case (continued)

$$F(u) = e^{-u} e^{-\left\{ \frac{\theta'_x}{\theta'_y} \left( u \frac{1}{b_x} + \frac{x_0 - y_0}{\theta'_x} \right) \right\}^{b_y}}$$

Let

$$q(u) = e^{-u};$$

$$\sigma = \frac{\theta'_x}{\theta'_y} \left( u \frac{1}{b_x} + \frac{x_0 - y_0}{\theta'_x} \right),$$

$$h(u) = e^{-\sigma^{b_y}}.$$

$$F'(u) = q'(u)h(u) + q(u)h'(u) = -F(u) - F(u)[w]$$

where

$$w = b_y(\sigma)^{b_y-1} \left( \frac{\theta'_x}{\theta'_y} \right) (u) \left( \frac{1}{b_x} - 1 \right) \left( \frac{1}{b_x} \right),$$

and  $F'(u)$  denotes, as is usual, the first derivative of  $F(u)$  with respect to  $u$ .

$$F''(u) = -F'(u)[1+w] - F(u)[w']$$

where

$$w' = b_y(b_y-1)(\sigma)^{b_y-2} \left( \frac{\theta'_x}{\theta'_y} \right)^2 \left( \frac{1}{b_x} \right)^2 \left( u \frac{1}{b_x} - 1 \right)^2 \\ + b_y(\sigma)^{b_y-1} \left( \frac{\theta'_x}{\theta'_y} \right) \left( \frac{1}{b_x} \right) \left( \frac{1}{b_x} - 1 \right) \left( u \frac{1}{b_x} - 2 \right).$$

$$F'''(u) = -F''(u)[1+w] - 2F'(u)[w'] - F(u)[w'']$$

where

$$w'' = b_y(b_y-1)(b_y-2)(\sigma)^{b_y-3} \left( \frac{\theta'_x}{\theta'_y} \right)^3 \left( \frac{1}{b_x} \right)^3 \left( u \frac{1}{b_x} - 1 \right)^3 \\ + 3b_y(b_y-1)(\sigma)^{b_y-2} \left( \frac{\theta'_x}{\theta'_y} \right)^2 \left( \frac{1}{b_x} \right)^2 \left( \frac{1}{b_x} - 1 \right) \left( u \frac{2}{b_x} - 3 \right) \\ + b_y(\sigma)^{b_y-1} \left( \frac{\theta'_x}{\theta'_y} \right) \left( \frac{1}{b_x} \right) \left( \frac{1}{b_x} - 1 \right) \left( \frac{1}{b_x} - 2 \right) \left( u \frac{1}{b_x} - 3 \right).$$

$$\begin{aligned}
F^{iv}(u) &= -F'''(u)[1+w] - 3F''(u)[w'] - 3F'(u)[w''] - F(u)[w'''] \\
\text{where } w''' &= b_y(b_y-1)(b_y-2)(b_y-3)(\sigma)^{b_y-4} \left(\frac{\theta'_x}{\theta'_y}\right)^4 \left(\frac{1}{b_x}\right)^4 (u)^{\left(\frac{1}{b_x}-1\right)^4} \\
&+ 3b_y(b_y-1)(b_y-2)(\sigma)^{b_y-3} \left(\frac{\theta'_x}{\theta'_y}\right)^3 \left(\frac{1}{b_x}\right)^3 \left(\frac{1}{b_x}-1\right) (u)^{\left(\frac{1}{b_x}-2\right) \left(\frac{1}{b_x}-1\right)^2} \\
&+ 3(b_y)(b_y-1)(b_y-2)(\sigma)^{b_y-3} \left(\frac{\theta'_x}{\theta'_y}\right)^3 \left(\frac{1}{b_x}\right)^3 \left(\frac{1}{b_x}-1\right) (u)^{\left(\frac{3}{b_x}-4\right)} \\
&+ 3b_y(b_y-1)(\sigma)^{b_y-2} \left(\frac{\theta'_x}{\theta'_y}\right)^2 \left(\frac{1}{b_x}\right)^2 \left(\frac{1}{b_x}-1\right) \left(\frac{2}{b_x}-3\right) (u)^{\left(\frac{2}{b_x}-4\right)} \\
&+ b_y(b_y-1)(\sigma)^{b_y-2} \left(\frac{\theta'_x}{\theta'_y}\right)^2 \left(\frac{1}{b_x}\right)^2 \left(\frac{1}{b_x}-1\right) \left(\frac{1}{b_x}-2\right) (u)^{\left(\frac{2}{b_x}-4\right)} \\
&+ b_y(\sigma)^{b_y-1} \left(\frac{\theta'_x}{\theta'_y}\right) \left(\frac{1}{b_x}\right) \left(\frac{1}{b_x}-1\right) \left(\frac{1}{b_x}-2\right) \left(\frac{1}{b_x}-3\right) (u)^{\left(\frac{1}{b_x}-4\right)}.
\end{aligned}$$

Full exploration of this derivative appears to be unreasonable. One would hope that maximum values could be obtained as functions of the parameters so that step sizes appropriate to minimize the error term could be determined. Such appears to be hopeless from the form of the derivative. Instead, tables of the fourth derivative were obtained in an attempt to find where the derivative took maximum values, how large these maxima were and how they behaved with respect to the four parameters of interest. Some examples of these tables are given in Tables A-4.1, A-4.2, A-4.3. From examination of the derivative and its value at some points, it seems clear that for  $b_x \neq 1$ , the fourth derivative approaches infinity for  $u \rightarrow 0$ .  $b_x = 1$  was considered along with the other  $b_x$  values and not treated separately.  $b_y$  non-integer and less than 4 causes the derivative to become infinite for  $u \rightarrow 0$  if  $(x_0 - y_0)/\theta'_x = 0$ . In either event, it appears from the plots of the fourth derivative that it rapidly approaches zero for  $u > 0$ . It was therefore decided to evaluate the integral over 5 distinct intervals, (0,.01), (.01,1), (1,5), (5,10), (10,∞).

For the interval (0,.01), it did not appear feasible to make a full exploration of the fourth derivative to determine the optimum

step size. Instead the value of the integral was found using Simpson's rule with 50 and 500 steps within the interval (0,.01). The integrals did not differ by more than  $10^{-6}$ . Hence it was decided that in the interval (0,.01) one should take  $n = 50$ .

Values of  $n$  were determined for the intervals (.01,1), (1,5), (5,10) by looking at the maximum values of the fourth derivatives, as computed, in each of these intervals. From these observations,  $n$  was taken so that the maximum remainder in the interval was less than  $10^{-6}$ . The values of  $n$  used within these intervals were

<u>interval</u>	<u>n</u>
(.01,1)	50
(1,5)	20
(5,10)	10

The interval (10, $\infty$ ) was eventually eliminated as being "practically 0" based on the following:

Consider

$$g(u) = (au^x + ac)^y + u$$

for

$$a \geq 1, \quad u \geq 1, \quad 0 \leq c \leq 1, \quad .1 \leq x \leq 1, \quad 1 \leq y \leq 10.$$

For  $x = 1$  and any combination of the other parameters:

$$g(u) = (au + ac)^y + u = a^y u^y + ya^{y-1} u^{y-1} + \dots + u.$$

$$g(u) \geq a^y u + u = u(a^y + 1)$$

since over the values of  $u, y$  considered  $u^y \geq u$  with equality occurring when  $y = 1$ . Consequently

$$-g(u) \leq -u(a^y + 1) \leq -2u$$

and

$$e^{-g(u)} \leq e^{-u(a^y+1)} \leq e^{-2u}$$

and

$$\int_{10}^{\infty} e^{-g(u)} du \leq \int_{10}^{\infty} e^{-2u} du \leq 1/2e^{-20} \approx 1 \times 10^{-9}.$$

In a similar way it can be shown that for  $.1 \leq x < 1$ , but with  $xy \geq 1$  then the value of the integral for the interval  $(10, \infty)$  is less than  $1 \times 10^{-9}$ . For suppose  $.1 \leq x < 1$  and  $xy \geq 1$ . Then

$$\begin{aligned} g(u) &= (au^x + ac)^y + u \\ &= (au^x)^y + y(au^x)^{y-1}(ac)^y + \dots + u \\ &\geq a^y u^{xy} + u \geq a^y u + u \geq 2u \end{aligned}$$

since  $a \geq 1$ . Then as before

$$\int_{10}^{\infty} e^{-g(u)} du \leq \int_{10}^{\infty} e^{-2u} du \approx 1 \times 10^{-9}.$$

Hence for  $xy \geq 1$  and  $.1 \leq x \leq 1$  the contribution that the tail of the function makes to the integral is no bigger than  $10^{-9}$ .

For  $xy < 1$ , the above analysis is no longer valid since  $u^{xy} < u$ . However,

$$g(u) \geq a^y 10^{xy} + u$$

since

$$g(u) \geq (au^x)^y + u \geq a^y 10^{xy} + u, \text{ for } u \geq 10.$$

Also

$$10^{xy} \geq 10^{.1} = 1.258, \text{ for } xy \geq .1.$$

Hence

$$g(u) \geq 1.258a^y + u$$

and

$$-g(u) \leq -(1.258a^y + u)$$

$$e^{-g(u)} \leq e^{-(1.258a^y + u)}$$

$$\int_0^{\infty} e^{-g(u)} du \leq \int_{10}^{\infty} e^{-(1.258a^y + u)} du = e^{-1.258a^y + 10}.$$

Since  $a \geq 1$ ,  $e^{-1.258a^{10}+10} \leq e^{-11.258} \approx 1 \times 10^{-5}$ .

Hence if one identifies  $a = (\theta'_x)/(\theta'_y)$ ;  $c = (x_0 - y_0)/(\theta'_x)$ ;  $x = 1/(b_x)$  and  $y = b_y$ , one sees that no matter what the values of  $(x_0 - y_0)/\theta'_x \geq 0$ ,  $b_y/b_x \geq .1$ ,  $\theta'_x/\theta'_y \geq 1$  and  $b_y \geq 1$  the truncation error obtained by not including the integral in the interval  $(10, \infty)$ , is at most approximately  $1 \times 10^{-5}$ . In fact one finds easily that only one tabular value will have this much truncation error, and that will be the special case  $\theta'_x/\theta'_y = 1$  (the strength and stress distributions have the same characteristic values),  $b_y/b_x = .1$  (the slope parameter of the strength distribution is 10 times the slope of the stress distribution) and the two distributions have the same  $x_p$  points. For the values of  $b_y$  and  $b_x$  used in the tables, this set of conditions requires that the stress distribution be exponential (i.e.  $b_y = 1$ ). One expects this case to arise seldom enough to justify this moderate error. Consequently we truncate the integral at  $u = 10$  in all cases considered.

#### A-4.1.5 Conclusions Concerning Errors

In summary then the integral evaluated was

$$\int_0^{10} e^{-u} e^{-\left[\frac{\theta'_x}{\theta'_y} \left(u^{\frac{1}{b_x}} + \frac{x_0 - y_0}{\theta'_x}\right)^{b_y}\right]} du,$$

with parameters taken as

$$\frac{x_0 - y_0}{\theta'_x} = 0, .25, .50, .75, 1;$$

$$b_x = 1, 2, \dots, 10,$$

$$b_y = 1, 2, \dots, 10,$$

$$\frac{\theta'_x}{\theta'_y} = 1.00, 1.25, 1.50, 1.75, 2.00, \dots, 3.$$

The maximum truncation error (the error caused by not including the value of the integral in  $(10, \infty)$ ), is  $10^{-5}$  which occurs for one tabular value. For most of the table, the truncation error is  $1 \times 10^{-6}$  or less, and for  $b_y/b_x > .1$ , it is as small as  $1 \times 10^{-9}$ . The error of approximation using Simpson's rule was not determined precisely. Using the number of steps given on page 5, it appears that the maximum error of this type does not exceed,  $1 \times 10^{-6}$ . This conjecture was tested



once more by comparing the results obtained for  $b_x = b_y = 1$ ,  $b_x = 2$ ,  $b_y = 1$ ,  $b_x = 1$ ,  $b_y = 2$  as discussed in Section A-3.3.4. Tables A-4.7 and A-4.8 show the computed values of formula 4 in Section A-3.3.4 and formula 5 in Section A-3.3.4. These tabulated values can be compared to those given in Tables A-2.2. for the corresponding parameters. In no case was the disagreement found to exceed  $1 \times 10^{-6}$ . Hence the conjecture is valid in those cases where it can be tested. We conclude on the basis of this verification that the tables are correct to  $1 \times 10^{-4}$  after rounding.

#### A-4.1.6 A Note on Interpolation

One notes that the tabulated values are highly non-linear in general. Simple linear interpolation can produce errors of the order of  $10^{-2}$ . Hence higher order interpolation formulas should be used for more accuracy.

#### A-4.1.7 Error Analysis - Weibull - Normal Case

Since the remainder term of Simpson's rule depends on the fourth derivative of the integrand, this derivative was determined. An analytic expression for it is given by the following. We are interested in

$$(1) \quad b = 1, 2, \dots, 10;$$

$$(2) \quad \frac{\theta'}{\sigma} \geq 10;$$

$$(3) \quad -10 \leq \frac{x_0 - \mu}{\sigma} \leq 3.$$

$$F(u) = e^{-u^b} e^{-1/2 \left[ \frac{\theta'}{\sigma} u + \frac{x_0 - \mu}{\sigma} \right]^2}.$$

$$\text{Let} \quad g(u) = e^{-u^b}, \quad s = \frac{\theta'}{\sigma} u + \frac{x_0 - \mu}{\sigma}$$

$$h(u) = e^{-1/2 s^2}.$$

$$F'(u) = g'(u)h(u) + g(u)h'(u)$$

$$= -F(u)[w]$$

B1 = 1, B2 = 2

$X_0 - Y_0$   
Theta 1

	.00	.25	.50	.75	1.00
1.00	.545641	.422042	.295115	.185734	.104844
1.25	.475575	.335056	.201889	.103137	.044382
1.50	.420811	.268297	.136622	.054750	.017089
1.75	.377041	.216191	.091224	.027694	.005960
2.00	.341350	.174942	.059959	.013309	.001875
2.25	.311735	.141924	.038716	.006060	.000530
2.50	.286789	.115273	.024517	.002609	.000134
2.75	.265503	.093633	.015205	.001060	.000030
3.00	.247133	.075991	.009225	.000406	.000006

Theta 1  
Theta 2

Table A-4.7 Numerical Values of Formula 4  
of Section A-3.3.4

$$B1 = 2, B2 = 1$$

$\frac{X_0 - Y_0}{\text{Theta 1}}$

	.00	.25	.50	.75	1.00
1.00	.454358	.353855	.275582	.214623	.167149
1.25	.383170	.280333	.205096	.150051	.109780
1.50	.326107	.224130	.154042	.105871	.072764
1.75	.279900	.180717	.116680	.075334	.048639
2.00	.242128	.146858	.089073	.054026	.032768
2.25	.210974	.120209	.068493	.039026	.022236
2.50	.185063	.099057	.053021	.028380	.015190
2.75	.163345	.082135	.041300	.020767	.010442
3.00	.145007	.068496	.032355	.015283	.007219

Table A-4.8 Numerical Values of Formula 5  
of Section A-3.3.4

where

$$w = \frac{\theta'}{\sigma} s + b(u^{b-1}) .$$

$$F''(u) = -F'(u)(w) - F(u)(w')$$

where

$$w' = \left(\frac{\theta'}{\sigma}\right)^2 + b(b-1)(u^{b-2}) .$$

$$F'''(u) = -F''(u)(w) - 2F'(u)(w') - F(u)(w'')$$

where

$$w'' = b(b-1)(b-2)(u^{b-3}) .$$

$$F^{iv}(u) = -F'''(u)(w) - 3F''(u)(w') - 3F'(u)(w'') - F(u)(w''')$$

where

$$w''' = b(b-1)(b-2)(b-3)(u^{b-4}) .$$

One can see some behavior of the fourth derivative from this expression. First, if  $b$  is non-integer and  $b < 4$  the fourth derivative becomes infinite at zero as before. However, this is not true for all  $b$  (as it was for all  $b_x$  in the previous case), for if  $b$  is an integer, or if  $b > 4$ ,  $u^{b-4} \rightarrow 0$  for  $u \rightarrow 0$  except for  $b = 4$  in which case  $u^{b-4} = 1$  for all  $u$ . If the fourth derivative is not infinite at the origin, then the maximum error occurs away from the origin and is of the order of  $(\theta'/\sigma)^4$  for  $b > 4$  and is less than this for  $b$  integer and  $< 4$ . Since  $\theta'/\sigma$  can be as large as 100, the maximum error is of the order of  $10^8$ . The difficulty in using these facts lies in finding where this maximum occurs. In the neighborhood of the maximum steps of size  $10^{-3}$  will give maximum errors due to approximation on the order of  $10^{-9}$ .

To locate approximately where the maximum occurs, the fourth derivative was computed for selected values of  $u$ , and several values of the parameters. Examples of these calculations are given in tables A-4.4 - A-4.6. One notes from these plots that for  $(x_0 - \mu)/\sigma > 0$ , the maximum occurs near the origin and the derivative falls off rapidly to zero. For  $(x_0 - \mu)/\sigma < 0$ , such is not the case, and the  $u$  value at which the maximum occurs depends on the parameters. In all cases in which  $-10. \leq (x_0 - \mu)/\sigma < 0$ , the maximum occurs between .2 and 1.0. However, this range is too large to take steps of size  $10^{-3}$  along, and an approximation is needed to more precisely locate the  $u$ 's at which the maximum occurs as a function of the parameters. This will be discussed in detail later in this section.

For  $(x_0 - \mu)/\sigma > 0$ , the interval  $(0, 1)$  was explored with 50, 500 steps and the integrals were found to agree to  $10^{-6}$ . Hence 50 steps were taken in this interval for this case. The number of steps in the intervals  $(0, 1)$ ,  $(1, 5)$  were chosen to make the error term no larger than  $10^{-6}$ . The number of steps used then were:

<u>interval</u>	<u>n</u>
$(.1, 1)$	25
$(1, 5)$	10

The interval  $(5, \infty)$  was not computed for  $(x_0 - \mu)/\sigma > 0$  since it is "practically 0" as is shown in the following.

For  $u \geq 1, \quad b \geq 1, \quad \frac{x_0 - \mu}{\sigma} \geq 0 \quad \text{and} \quad \frac{\theta'}{\sigma} \geq 10,$

$$g(u) = u^b + \left(\frac{\theta'}{\sigma}u + \frac{x_0 - \mu}{\sigma}\right)^2 \geq u + \left(\frac{\theta'}{\sigma}\right)^2 u.$$

Hence  $-u - \left(\frac{\theta'}{\sigma}u + \frac{x_0 - \mu}{\sigma}\right)^2 \leq -[u + \left(\frac{\theta'}{\sigma}\right)^2 u]$

and  $e^{-u} e^{-\left(\frac{\theta'}{\sigma}u + \frac{x_0 - \mu}{\sigma}\right)^2} \leq e^{-[(\frac{\theta'}{\sigma})^2 + 1]u}.$

Hence 
$$\int_5^\infty e^{-g(u)} du \leq \int_5^\infty e^{-[(\frac{\theta'}{\sigma})^2 + 1]u} du = \frac{e^{-[(\frac{\theta'}{\sigma})^2 + 1]5}}{(\frac{\theta'}{\sigma})^2 + 1}.$$

Thus, even if  $\theta'/\sigma$  is as small as 2, which it is not, the area from  $(5, \infty)$  contributes less than  $1 \times 10^{-9}$  to the total area.

In any event (whether  $\frac{x_0 - \mu}{\sigma} \geq 0$  or not)

$$\left(\frac{\theta'}{\sigma}u + \frac{x_0 - \mu}{\sigma}\right)^2 \geq \left(\frac{\theta'}{\sigma}(5) + \frac{x_0 - \mu}{\sigma}\right)^2 \quad \text{for } u \geq 5.$$

Since we only consider cases in which  $\frac{\theta'}{\sigma} \geq 10$  and  $\frac{x_0 - \mu}{\sigma} \geq -10,$

$$\left(\frac{\theta'}{\sigma}(5) + \frac{x_0 - \mu}{\sigma}\right)^2 \geq 10.$$

Consequently

$$\int_5^\infty e^{-g(u)} du \leq e^{-20} \approx 2 \times 10^{-9}.$$

Thus the function error in any case is less than  $2 \times 10^{-9}$ . Of course it is much less than this. However, one sees that it is negligible.

For  $(x_0 - \mu)/\sigma < 0$ , the function in question is no longer a monotone decreasing exponential due to the  $\exp\{-1/2[\theta/\sigma(u) + (x_0 - \mu)/\sigma]^2\}$  term. Instead it has the properties of Tables A-4.4 - A-4.6 and the maximum point Y can be seen to shift as the parameters change.

For  $(x_0 - \mu)/\sigma < 1$ , the fourth derivative has large values (as large as  $10^8$ ) at some point  $u > 0$ . Precisely where that point  $u$  is and how large the neighborhood in which the fourth derivative remains largely depends on all the parameters.

By evaluating the fourth derivative for several values of the parameter, it was found that for fixed  $b$  one had two bounding functions for adjacent setting of the parameters. These bounding functions intersect at some value  $u^*$ , for  $u < u^*$ ,  $f_1^{iv}(u) > f_2^{iv}(u)$  while for  $u > u^*$ ,  $f_1^{iv}(u) < f_2^{iv}(u)$  where  $f_1^{iv}$  and  $f_2^{iv}$  depend on the parameters  $(x_0 - \mu)/\sigma$  and  $\theta/\sigma$ . It was found for

$$1. \quad \frac{x_0 - \mu}{\sigma} = -x, \quad x = 1, 2, \dots, 10$$

$$\frac{\theta}{\sigma} = 10(i+1), \quad i = 1, 2, \dots, 10$$

and

$$2. \quad \frac{x_0 - \mu}{\sigma} = -(x+1), \quad x = 1, 2, \dots, 10$$

$$\frac{\theta}{\sigma} = 10i, \quad i = 1, 2, \dots, 10$$

and

$$f_1^{iv}(u) = f^{iv}(u) \text{ with parameter set 1}$$

$$f_2^{iv}(u) = f^{iv}(u) \text{ with parameter set 2}$$

then for some  $u = u^*$

$$f_1^{iv}(u) > f_2^{iv}(u), \quad u < u^*,$$

$$f_2^{iv}(u) > f_1^{iv}(u), \quad u > u^*.$$

Furthermore, for any setting of the parameters in the intervals

$$[(x, x+1), (x+1, x+2)]$$

the fourth derivative for these parameters were dominated by either  $f_1^{iv}(u)$  or  $f_2^{iv}(u)$ . Hence for all  $u < u^*$  any setting of the parameters in the interval above,  $f_1^{iv}(u)$  was largest while for all  $u > u^*$  and any setting of the parameters in the interval above  $f_2^{iv}(u)$  was largest.

First, it was decided to evaluate the integral for  $-10 \leq (x_0 - u)/\sigma < 1$  over from subintervals,  $0 \leq u < x_1$ ,  $x_1 \leq u < x_2$ ,  $x_2 \leq u < x_3$ ,  $x_3 \leq u < 10$ . This was done to reduce as much as possible the size of the interval over which many steps had to be taken.

$x_1$  was taken to be that  $u$  such that  $f_1^{iv}(u) < 1 \times 10^3$  for  $u < x_1$  and  $f_1^{iv}(u) > 1 \times 10^3$  for  $u = x_1 + .05$ .

$x_2$  was taken to be that  $u$  such that  $f_2^{iv}(u) > 1 \times 10^3$  for  $x_1 < u \leq x_2$  while  $f_2^{iv}(u) < 1 \times 10^3$  for  $u = x_2 + .05$ .

$x_3$  was taken to be that  $u$  such that  $f_2^{iv}(u) > 1 \times 10^{-9}$  for  $x_2 < u \leq x_3$  while  $f_2^{iv}(u) < 1 \times 10^{-9}$  for  $u = x_3 + .05$ .

Since  $b = 1$  gave the maximum length of the intervals over which the error of approximation remained relatively large, the  $x_1$ ,  $x_2$ ,  $x_3$  were chosen for that  $b$  and used for all  $b_1$ .

It was found that  $x_1 < .3$ ,  $x_2 < 1.1$  and for  $f^{iv}(u) \rightarrow 10^9$ , the interval  $x_2 - x_1$  became small. Hence, in the interval  $(0, x_1)$  5 steps were taken, in  $(x_1, x_2)$ , 50 steps were taken and in each of  $(x_2, x_3)$  and  $(x_3, 10)$ , 5 steps were taken. For all of the values of the parameters studied, these intervals and steps gave

$$\frac{h^5}{90} f^{iv}(u) < 10^{-6}.$$

Hence, in all cases, the errors of approximation were less than  $1 \times 10^{-6}$ , and consequently, the maximum error of approximation is less than  $10^{-6}$ .

To check the logic used in attempting to reduce the errors of truncation and approximation, the formulas 2 and 3 of section A-3.3.5 were computed using the error function subroutine available on the University of Michigan 7090. These same integrals were evaluated

using Simpson's rule with the intervals discussed above. Comparisons can be made from Tables A-4.9-A-4.10 following with those of section A-2.1 for some selected values of the parameters. In no case do the failure probabilities differ by more than  $10^{-6}$ .

#### A-4.1.6 Conclusions Concerning Errors.

In summary the integral evaluated was

$$\int_0^{10} e^{-u^b} e^{-\left(\frac{u}{\sigma} + \frac{x_0 - \mu}{\sigma}\right)^2} du$$

with the parameters taken as

$$\begin{aligned} b &= 1, 2, \dots, 10; \\ \frac{x_0 - \mu}{\sigma} &= 10, 15, 20, 25, \dots, 100, \\ \frac{x_0 - \mu}{\sigma} &= 1, 1.2, 1.4, \dots, 3 \text{ and } -10, -9, \dots \end{aligned}$$

The maximum truncation error is less than  $10^{-9}$  for all values of the parameters.

The maximum approximation errors are less than  $10^{-6}$  using the steps noted in A-4.1.7.

The values of the integrals using Simpson's rule differs from the values obtained using the error function formula of section A-3.3.5 by no more than  $1 \times 10^{-6}$ .

Since the tabular values in Table A-2.1 have been found by rounding off the computed values, the tabulation are correct to  $\pm 5 \times 10^{-5}$ .

#### A-4.1.9 A Note on Interpolation

One notes that the tabulated values are non-linear. Simple linear interpolation can introduce significant errors (errors greater than  $10^{-4}$ ). Hence higher order interpolation formulas should be used for more accuracy.



B = 1

$\frac{X_0 - \mu}{\sigma}$

	-10	-5	-1	1	2	3
10	.63028	.39043	.09935	.00797	.00082	.00004
20	.39271	.22023	.05184	.00407	.00042	.00002
30	.28377	.15305	.03507	.00274	.00028	.00001
40	.22096	.11723	.02649	.00206	.00021	.00001
50	.18111	.09498	.02129	.00165	.00017	.00001
60	.15340	.07983	.01779	.00138	.00014	.00001
70	.13303	.06884	.01528	.00118	.00012	.00001
80	.11743	.06051	.01339	.00104	.00011	.00000
90	.10511	.05398	.01192	.00092	.00009	.00000
100	.09512	.04872	.01074	.00083	.00008	.00000

Table A-4.9 Numerical Values of Formula 2  
of Section A-3.3.5

$$B = 2$$

$$\frac{X_0 - \mu}{\sigma}$$

	-10	-5	-1
10	.62853	.22508	.01877
20	.22217	.06264	.00470
30	.10593	.02841	.00213
40	.06110	.01610	.00120
50	.03956	.01034	.00077
60	.02765	.00719	.00053
70	.02039	.00529	.00039
80	.01565	.00405	.00030
90	.01239	.00320	.00024
100	.01005	.00260	.00019

Table A-4.10 Numerical Values of Formula 3  
Using Parameters Given by Formula 3  
of Section A-3.3.5

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13. ABSTRACT This study addressed itself to the development of a practical engineering tool based on the Stress-Strength Interference Theory to be used for designing and quantitatively predicting the reliability of mechanical parts subjected to mechanical loading.  Fatigue data was gathered for various ferrous materials, heat treatments, surface conditions, etc. and was converted to strength data to obtain the scatter of strength at a given life. The Weibull distribution best fit the data and was found to be the most effective means of representing the strength distribution in the Interference Theory. For each material parameter studied, Weibull parameters were calculated and presented in tabular and graphic form.  The required stress distribution, in this study, is obtained by converting the stress spectrum, known or assumed, to a zero mean stress spectrum using the Goodman diagram. The required stress spectrum, expressed in terms of equivalent stress, is obtained using Miner's rule.  In the literature, it has generally been assumed that both stress and strength follow the normal distribution. The resulting interference distribution, being normal can be easily evaluated. In other cases when the distributions are not normal, Monte-Carlo techniques are recommended. In this study a method was developed to evaluate the complex integral resulting from the interference of two distributions using the IBM-7090 Computer. Tabulated interference values are presented for the stress-normal, strength-Weibull case and the stress-Weibull, strength-		

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